Physics 515, Electrodynamics III
Department of Physics, University of Washington
Spring quarter 2020
May 27, 2020, 11am
On-line lecture

Administrative

1. You should be getting your graded homework back; if not let me know asap.

2. Office hours Wednesday after class 12:30 at URL https://washington.zoom/us/j/712804010

Lecture

Chapter 13: Collisions, energy loss, and scattering of charged particles, Cherenkov and transition radiation.
Chapter 14: Radiation by moving charges.

1. J. C. 13. Selected topics. Straggling. Passage of gamma rays through matter.

2. J. C. 14.1. Liénard-Wiechert potentials III.

3. J. C. 14.2. Larmor's formula for radiated power and the covariant generalization. Linear and circular particle accelerators.

BEFORE WE LEAVE J.C. 13, OTHER ISSUES REGARDING ENERGY LOSS

- STRAGGLING — THE IONIZATION ENERGY LOSS WE FOUND IS SOME MEAN VALUE; THE ACTUAL ENERGY LOSS FLUCTUATES. STRAGGLING: FOR SOME GIVEN ENERGY LOSS THE PATH LENGTH FLUCTUATES.

EXAMPLE

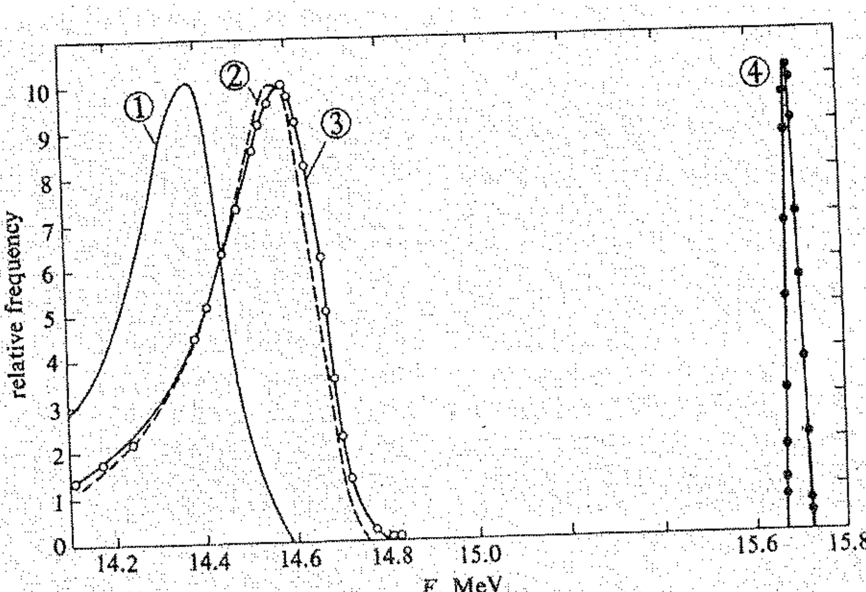


Figure 2-18 Energy distribution of an "unobstructed" electron beam and the calculated and experimental distributions of electrons that have passed through 0.86 g cm^{-2} of aluminum. (1) Landau theory without density correction; (2) Landau theory with Fermi density correction; (3) experiment; (4) incident beam. [E. L. Goldwasser, F. E. Mills, and A. O. Hanson, *Phys. Rev.*, **88**, 1137 (1952).]

WE LOOKED AT CHARGED PARTICLES GOING THROUGH MATTER, WHAT ABOUT

- GAMMA RAYS THROUGH MATTER
  - PHOTOELECTRIC EFFECT.

    A GAMMA RAY REMOVES AN INNER-SHELL ELECTRON.

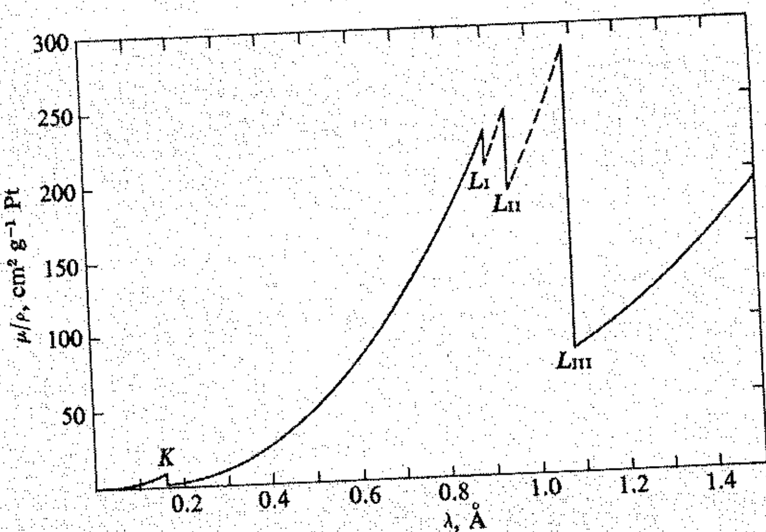
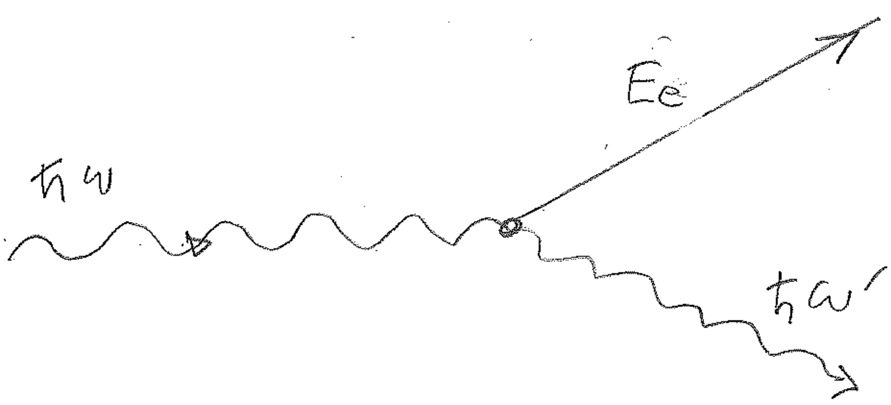
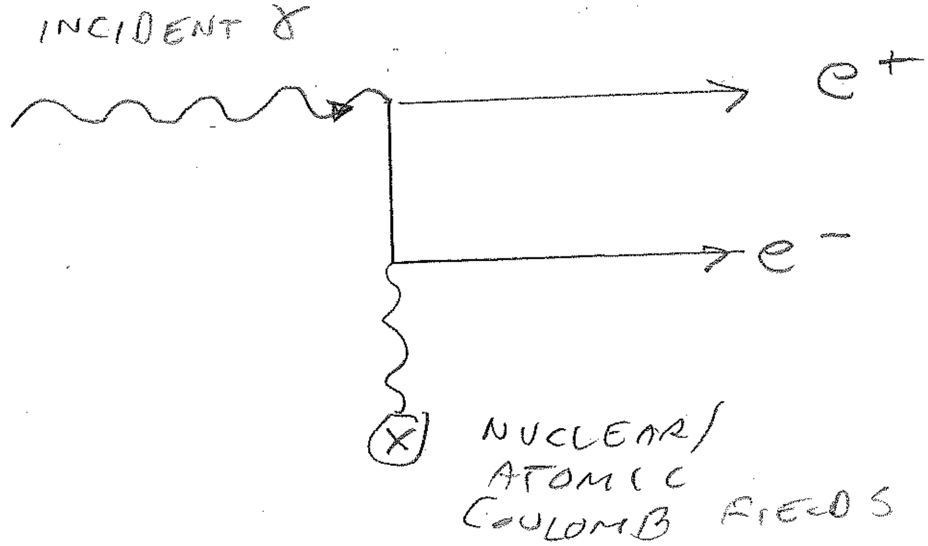


Figure 2-21 Mass absorption coefficient of platinum as a function of the wavelength of the incident X rays. [W. Bothe, in Geiger Scheel, *Handbuch der Physik*, Springer, Berlin, 1933.]

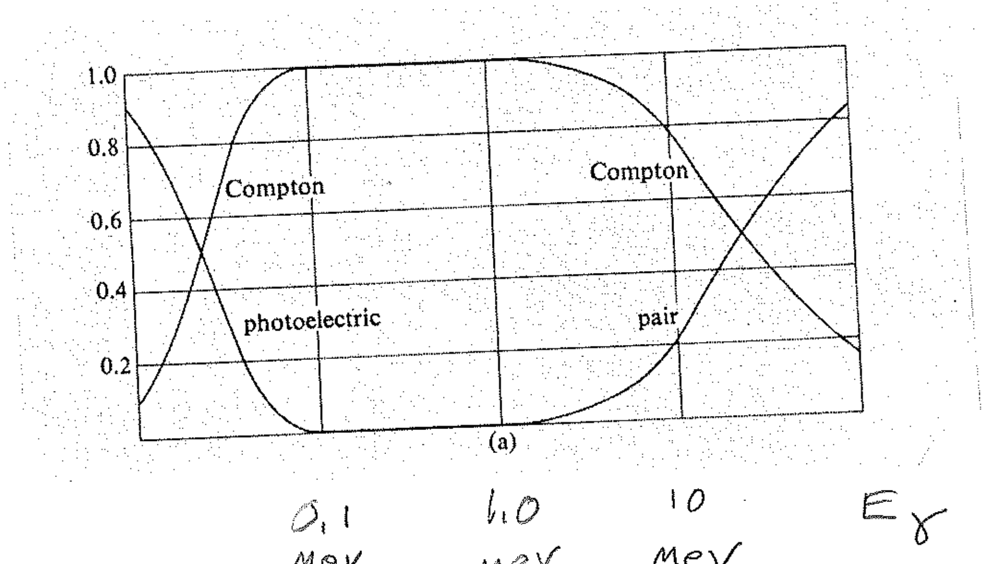
  - FREE-ELECTRON SCATTERING: THOMSON, RAYLEIGH, COMPTON.



∴ PAIR PRODUCTION



TYPICAL ENERGY-LOSS CONTRIBUTIONS



THIS BY NO MEANS TOUCHES ON ALL TOPICS IN ENERGY LOSS, BUT WE NEED TO MOVE ON. PROF. HENRY LUBATTI IS OUR LOCAL EXPERT.

J. C. 14.1

LIÉNARD WIECHERT POTENTIALS III.

THIS IS THE USUAL TREATMENT.

RECALL: IN THE COVARIANT FORMULATION OF ELECTRODYNAMICS WE STARTED WITH A STATIC-FRAME POTENTIAL

$$A_0^\mu = \left(0, \frac{e}{r_0}\right),$$

THEN WE SUPPOSED WE COULD EXTEND THIS TO AN ARBITRARY FRAME, YIELDING

$$A^\mu = \left(\frac{e\vec{v}}{s}, \frac{e}{s}\right)$$

WITH THE LIÉNARD-WIECHERT DENOMINATOR

$$s = r - \vec{r} \cdot \vec{\beta},$$

WITH $\vec{r}$ AND $\vec{\beta}$ THE RETARDED POSITION AND VELOCITY

A^μ IS MORE GENERAL: IT TAKES AS INPUT THE INSTANTANEOUS POSITION AND VELOCITY FOR A CHARGED-PARTICLE'S WORLD LINE,

WE ALSO VERY BRIEFLY LOOKED AT THE LORENTZ TRANSFORMATION OF A_0^μ.

J.C. 14.1 GOES BACK TO THE RETARDED POTENTIALS

$$\Phi(\vec{r},t) = \iiint \frac{[\rho(\vec{r}',t')]_{RET}}{|\vec{r}-\vec{r}'|}\, d\tau' \quad (CGS)$$

$$\vec{A}(\vec{r},t) = \frac{1}{c}\iiint \frac{[\vec{J}(\vec{r}',t')]_{RET}}{|\vec{r}-\vec{r}'|}\, d\tau' \quad (CGS)$$

IN MAKING THE TRANSITION FROM DISTRIBUTIONS ρ AND $\vec{J}$ TO A SINGLE CHARGED PARTICLE e, IT'S TEMPTING TO WRITE

$$\Phi(\vec{r},t) \not\longrightarrow \frac{e}{|\vec{r}-\vec{r}'|}$$

WITH $\vec{r}'$ THE RETARDED POSITION, AND

$$\vec{A}(\vec{r},t) \longrightarrow \frac{e\vec{v}}{|\vec{r}-\vec{r}'|}.$$

THIS, HOWEVER IS WRONG. IT'S INDEED TRUE THAT, WITH SOME SUBTLETIES, THE DENOMINATOR $|\vec{r}-\vec{r}'|$ CAN BE INTERGRATED AS SHOWN,

$$\text{BUT } \Phi(\vec{r},t) = \iiint \frac{[\rho(\vec{r}',t')]_{RET}}{|\vec{r}-\vec{r}'|} dv' \not\leftrightarrow \frac{e}{|\vec{r}-\vec{r}'|}$$

THAT IS,

$$\iiint [\rho(\vec{r}',t)]_{RET} \, dv' \neq e$$

WHEN THE CHARGE IS MOVING, EVEN FOR $\rho(\vec{r},t)$ REPRESENTING A POINT CHARGE.

WHY THIS IS SO IS THAT THE INTEGRAL $\iiint [\vec{J}(\vec{r}',t')]_{RET} \, dv'$ HAS THE INTEGRAND EVALUATED ~~AT~~ DIFFERENT TIMES.

YOU CAN IMAGINE THE OPERATION OF PERFORMING THIS INTEGRAL ON RETARDED SOURCES AS EVALUATING THE INTEGRAND OVER A SPHERE CENTERED ON THE OBSERVER; LARGER SPHERE RADII ARE FARTHER BACK IN TIME.

THE CHARGE, HOWEVER, IS MOVING DURING THE INTEGRATION. THE CHARGE DENSITY WOULD THEREFORE APPEAR MORE OR LESS DENSE THAN THEY SHOULD SO AS TO

GIVE THE STATIC VALUE FOR THE TOTAL CHARGE.

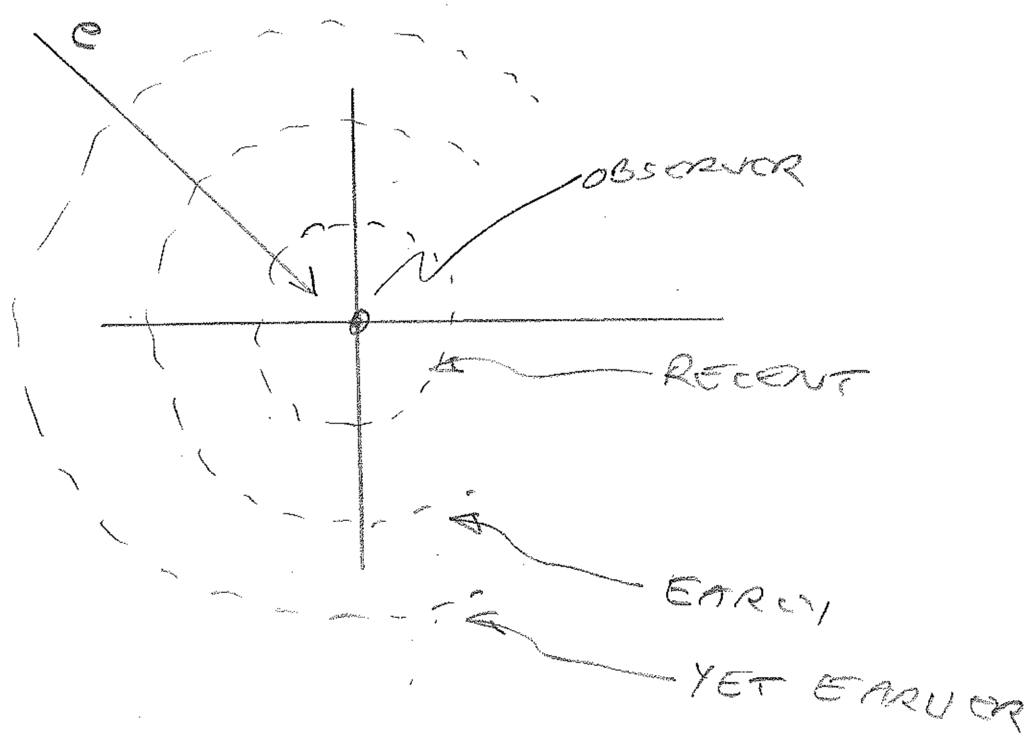
AN EXAMPLE OF THIS IS THE CENSUS: THE OBSERVER IS IN THE MIDDLE OF THE COUNTRY AND COLLECTS COUNTS FROM A RING OF CENSUS-TAKERS CONVERGING AT SOME SPEED ON THE OBSERVER.

THE CORRECT TOTAL POPULATION DIFFERS FROM THE OBSERVER'S TOTAL POPULATION DEPENDING ON WHETHER PEOPLE ARE MOVING TOWARDS OR AWAY FROM THE OBSERVER WHEN THE RING OF CENSUS-TAKERS PASS THEM.

THE CORRECTION TERM IS

$$\rho \vec{v} \cdot \vec{r}$$

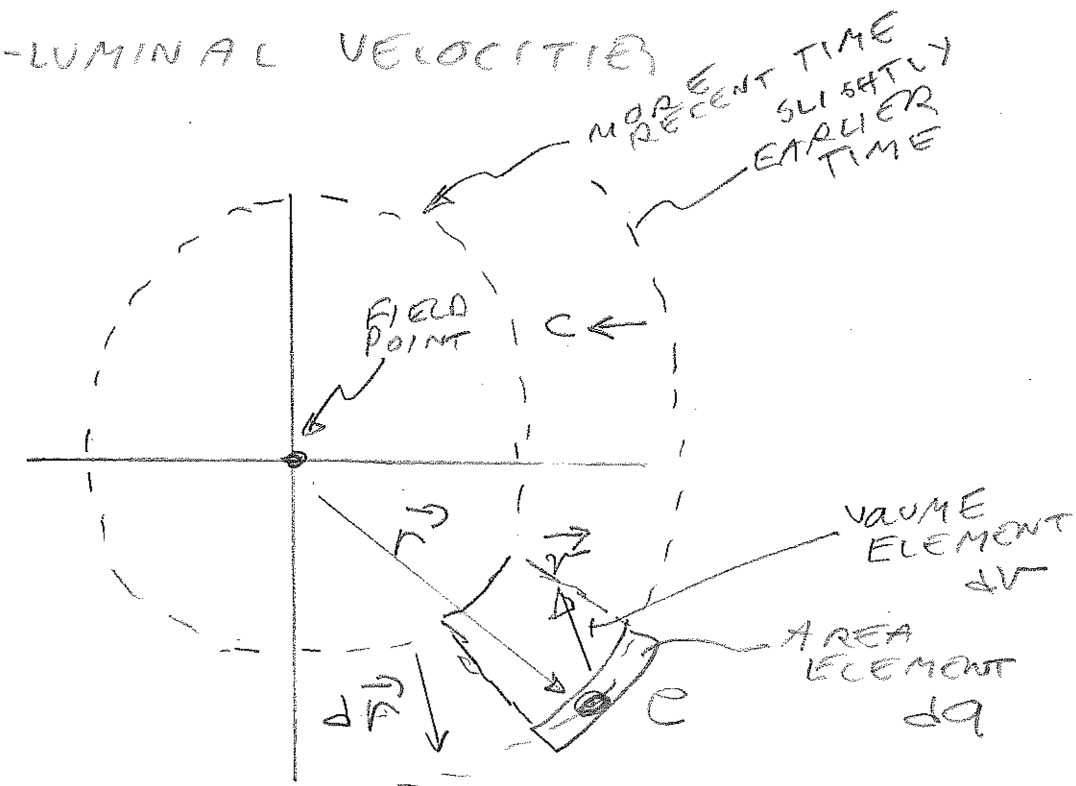
<u>EXAMPLE</u> OBVIOUS, BUT DANGEROUS, EXAMPLE! A CHARGE APPROACHING AN OBSERVER AT THE SPEED OF LIGHT. THE CHARGE ARRIVES AT THE OBSERVER AT TIME t.



IN THIS CASE THE CHARGE IS ALWAYS ON THE INTEGRAND "SPHERE" AND

$$\phi(0,t) = \iiint [\rho(\vec{r}', t')]_{RET}\, dr' \longrightarrow \infty$$

LET'S DO THIS MORE CAREFULLY, ALSO TAKE THE CHARGES TO HAVE SUB-LUMINAL VELOCITIES



THIS IS A CHARGE e MOVING AT VELOCITY $\vec{v}$ SOME TIME IN THE PAST. $\vec{r}$ IS THE DISPLACEMENT VECTOR FROM OBSERVER TO RETARDED POSITION OF THE CHARGE.

THE SIMPLEST CASE IS WHEN THE CHARGE IS AT REST (RETARDED $\vec{v} = 0$). THE AMOUNT OF CHARGE THE SPHERE WILL CROSS IN TIME dt IS

$$[\rho]_{RET}\, da\, dr \quad (dr = v\,dt)$$

HOWEVER, IF $\vec{v} \neq 0$, THE AMOUNT OF CHARGE THE SPHERE WILL CROSS IN TIME dt IS LESS THAN

$$[\rho]_{RET}\, da\, dt \text{ BY THE AMOUNT}$$

$$[\rho]_{RET}\, da\, (\vec{v} \cdot \hat{r})\, dt$$

(THIS IS A VARIANT OF THE CONVECTIVE DERIVATIVE.)

HENCE, THE AMOUNT OF CHARGE CONTRIBUTED IN THIS TIME dt IS

$$de = [\rho]_{RET} \underbrace{dv'}_{da\,dr} - [\rho]_{RET}\, \vec{v} \cdot \hat{r} \frac{dv'}{c}$$

WE CAN COLLECT TERMS CONTAINING $[\rho]_{RET}$:

$$[\rho]_{RET}\, dv' = \frac{de}{1 - \vec{v} \cdot \hat{r}/c}, \text{ OR}$$

$$\frac{[\rho]_{RET}\, dv'}{r} = \frac{de}{r - \vec{v} \cdot \vec{r}/c}$$

THIS LEADS TO RETARDED POTENTIALS

$$\Phi(\vec{r},t) = \iiint \frac{de}{r - \vec{v}\cdot\vec{r}/c}$$

$$\vec{A}(\vec{r},t) = \iiint \frac{\vec{v}\cdot de}{r - \vec{v}\cdot\vec{r}/c}$$

THIS IS CONCEPTUALLY TRICKY. BY WRITING THIS AS AN INTEGRAL OVER de, IT SEEMS THE CHARGE e HAS STRUCTURE. NONETHELESS, WE CONTINUE.

LET THE CHARGE e APPROACH A POINT CHARGE. IN THIS CASE THE DENOMINATOR APPROACHES A CONSTANT AND CAN BE TAKEN OUT FROM UNDER INTEGRATION. THE REMAINING INTEGRAL IS

$$e = \iiint de .$$

HENCE WE REDISCOVER THE LIÉNARD-WIECHERT POTENTIALS

$$\Phi(\vec{r},t) = \frac{e}{r - \vec{r}\cdot\vec{v}/c} = \frac{e}{s}$$

$$\vec{A}(\vec{r},t) = \frac{1}{c}\,\frac{e\vec{v}}{r - \vec{r}\cdot\vec{v}/c} = \frac{1}{c}\,\frac{e\vec{v}}{s}.$$

$$s = [r]_{RET} - [\vec{r}]_{RET}\cdot[\vec{v}]_{RET}/c$$

NOTICE THAT EVEN THOUGH WE STARTED WITH A CHARGE DISTRIBUTION, THESE EXPRESSIONS ARE INDEPENDENT OF THE CHARGES MICROSCOPIC STRUCTURE.

YOU COULD USE THESE POTENTIALS TO FIND THE RESULTING $\vec{E}$ AND $\vec{B}$ FIELDS. YOU RECOVER

$$E(\vec{r},t) = e\left[\frac{\hat{n}-\vec{\beta}}{\gamma^2(1-\vec{\beta}\cdot\hat{n})^3 r^2}\right]_{RET} + \frac{e}{c}\left[\frac{\hat{n}\times\{(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\}}{(1-\vec{\beta}\cdot\hat{n})^3\, r}\right]_{RET}$$

$$B(\vec{r},t) = \left[\hat{n}\times\vec{E}\right]_{RET}$$

JACKSON EQN.S. 14.13-14.

J.C. 14.2

LARMOR'S (POWER) FORMULA AND ITS RELATIVISTIC GENERALIZATION

LARMOR FORMULA: RADIATED POWER OBSERVED IN FRAME WITH $v \ll c$.

RECALL THE RADIATED ($1/r$) PART OF THE LIÉNARD-WIECHERT-DERIVED $\vec{E}$ AND $\vec{B}$ FIELDS ARE

$$\vec{E}(\vec{r},t) = \frac{e}{c}\left[\frac{\hat{n}\times(\hat{n}\times\dot{\vec{\beta}})}{r}\right]_{RET} \qquad (CGS)$$

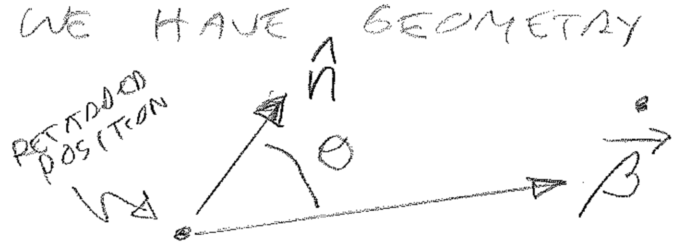
$$\vec{B}(\vec{r},t) = \left[\hat{n}\times\vec{E}\right]_{RET} \qquad (CGS)$$

RECALL THE POYNTING VECTOR

$$\vec{S} = \frac{c}{4\pi}\,\vec{E}\times\vec{B} \sim |\vec{E}|^2\,\hat{n} \qquad (CGS)$$

$$\sim |\dot{\vec{\beta}}|^2 - |\hat{n}\cdot\dot{\vec{\beta}}|^2$$

WE HAVE GEOMETRY



, SO

$$\vec{S} \sim \sin^2\theta$$

BECAUSE THE TOTAL RADIATED POWER P IS

$$P = \oiint \vec{S} \cdot d\vec{a} = \oiint \vec{S} \cdot \hat{n} \; r^2 d\Omega$$

WE HAVE

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3} \left| \dot{\vec{\beta}} c \right|^2 \sin^2\theta$$

J. EQN. 14.22

THE LARMOR FORMULA

ON INTEGRATING

$$P = \frac{2}{3} \frac{e^2}{c^3} \left| \dot{\vec{\beta}} c \right|^2$$

THESE ARE COMMONLY USED.

RELATIVISTIC GENERALIZATION OF LARMOR'S FORMULA

WE FOLLOW OUR USUAL PATH FOR WRITING A COVARIANT EXPRESSION. ENERGY IS A "TIME" COMPONENT OF MINKOWSKI MOMENTUM. SO TREAT THE RADIATED LARMOR AS A "TIME" COMPONENT.

FROM ABOVE: $P \sim |\dot{\vec{v}}|^2 \rightarrow \frac{1}{m^2} \left| \frac{d\vec{p}}{dt} \right|^2$

SO, MAYBE, WE CAN GENERALIZE THIS TO

$$\frac{1}{m^2} \left(\frac{dp_\mu}{d\tau} \frac{dp^\mu}{dt} \right).$$

EXERCISE: SHOW $P = -\frac{2}{3}\frac{e^2}{c^3}\frac{1}{m^2}\left(\frac{dP_\mu}{d\tau}\frac{dP^\mu}{d\tau}\right)$

REDUCES TO $\frac{2}{3}\frac{e^2}{c^3}\frac{1}{m^2}\left|\frac{d\vec{p}}{dt}\right|^2$. (CGS)

OUTLINED IN J. EQN. 14.25

THE RELATIVISTIC GENERALIZATION IN A PARTICULAR FRAME IS

(J. EQN. 14.26)

$$P = \frac{2}{3}\frac{e^2}{c^3}\gamma^6\left[|\dot{\vec{\beta}}|^2 - |\vec{\beta}\times\dot{\vec{\beta}}|^2\right]$$

VIA $E = \gamma mc^2$, $\vec{p} = \gamma m\vec{v}$.

THIS IN LIÉNARD'S 1898 FORMULA.

YOU HAVE HOMEWORK PROBLEMS EXPLORING LINEAR- AND CIRCULAR-PATHS.

THE USUAL APPLICATION IS TO LINEAR AND CIRCULAR ACCELERATORS. IN AN ACCELERATOR, AN EXTERNAL AGENT SUPPLIES POWER dE/dt (OR dE/dx) AND THE PARTICLE RADIATES POWER P.

EXAMPLE: LINEAR ACCELERATOR (JACKSON EQN. 14.29).

1. SHOW FOR 1D MOTION THE POWER IS

EUCLIDEAN MOMENTUM

$$P = \frac{2}{3}\frac{e^2}{c^3}\frac{1}{m^2}\left(\frac{dp}{dt}\right)^2 \quad \text{(CGS)}$$

WITH $\frac{dp}{dt} = \frac{dE}{dx}$ (1D)

$$P = \frac{2}{3}\frac{e^2}{c^3}\frac{1}{m^2}\left(\frac{dE}{dx}\right)^2 \quad \text{(CGS)}$$

RESULTING IN THE RATIO OF EXTERNAL- TO RADIATED- POWER

$$\frac{P}{dE/dt} = \frac{2}{3}\frac{e^2}{c^3}\frac{1}{m^2}\frac{1}{v}\frac{dE}{dx}$$

$$= \frac{2}{3}\frac{e^2}{c^3}\frac{1}{m^2}\frac{1}{\beta c}\frac{dE}{dx}$$

$$\xrightarrow[\beta \to 1]{} \frac{2}{3}\frac{e^2/mc^2}{mc^2}\frac{dE}{dx} \quad \text{(J. EQN. 14.29)}$$

WITH $e^2/mc^2 \approx 3 \times 10^{-13}$ cm,

$mc^2 \sim$ ELECTRON MASS,

THE RATIO IS USUALLY, BUT NOT ALWAYS, SMALL.

EXAMPLE: CIRCULAR ACCELERATOR.

RECALL J. EQN. 14.25

$$-\frac{dP_\mu}{d\tau}\frac{dP^\mu}{d\tau} = \left|\frac{d\vec{P}}{d\tau}\right|^2 - \beta^2\left|\frac{dP}{dt}\right|^2.$$

HERE WE CAN'T IGNORE $\frac{d\vec{P}}{d\tau}$.

THE PHYSICAL SITUATION IS THE PARTICLE MOMENTUM IS CONSTANTLY CHANGING ALONG THE ORBIT SO $\frac{d\vec{P}}{d\tau}$ IS LARGE. HOWEVER THE PARTICLE ENERGY IS MORE-OR-LESS CONSTANT (SOME EXTERNAL AGENT MAINTAINS THE ENERGY).

THAT IS:

$$\left|\frac{d\vec{P}}{d\tau}\right| \quad \left(= \gamma\omega|\vec{P}|\right) \gg \frac{1}{c}\frac{dE}{d\tau}$$

WE KEEP THE FIRST TERM:

$$P = \frac{2}{3}\frac{e^2}{c^3}\frac{1}{m^2}\gamma^2\omega^2|\vec{P}|^2,$$

WITH $\omega = v/r$ (r THE ORBIT RADIUS)
$= \beta c/r$.

WE HAVE $P = \frac{2}{3} e^2 c \frac{1}{r^2} \beta^4 \gamma^4$

(LIÉNARD 1898). J, EQN. 14.32.

THERE'S A NIFTY FORM J, EQN 14.33

THE ENERGY PER REVOLUTION FOR ULTRA-RELATIVISTIC ELECTRONS IS

$$\Delta E(\text{MeV}) = 9 \times 10^{-2} \frac{\{E[\text{GeV}]\}^4}{r[\text{METERS}]}$$

WHAT IF YOU WANTED TO CONVERT THE CERN LHC COLLIDER TO A TeV e^+e^- COLLIDER?

$$\Delta E(\text{MeV}) = 9 \times 10^{-2} \frac{\{10^3\}^4}{\{9.3 \times 10^3\}}$$

$=$ FEW TeV ... YIKES!

THERE'S A SIMILAR EXPRESSION FOR THE RADIATED POWER OF A CIRCULAR ACCELERATOR J. EQN. 14.34

$$P[\text{WATTS}] = 10^6 \Delta E[\text{MeV}]\ J[\text{AMP}]$$

WITH J THE BEAM CURRENT.

FOR THE CONCEPTUAL 200 NLC,

$J \sim 100\,\mu A$.

$$P[\text{WATTS}] = 10^6 \cdot 10^8\ 10^{-4}$$

$$= 10^{10} \text{ WATTS}$$ YIKES!!

YOU'D NEED THE FULL OUTPUT OF A NUCLEAR POWER PLANT.

ALSO NOTICE AGAIN FOR CIRCULAR ACCELERATORS J. EQN.S 14.31-2

$P \sim \gamma^4$.

AS BEAM ENERGY GOES UP, THE RADIATED POWER GROWS FAST.