Physics 515, Electrodynamics III
Department of Physics, University of Washington
Spring quarter 2020
April 24, 2020, 11am
On-line lecture

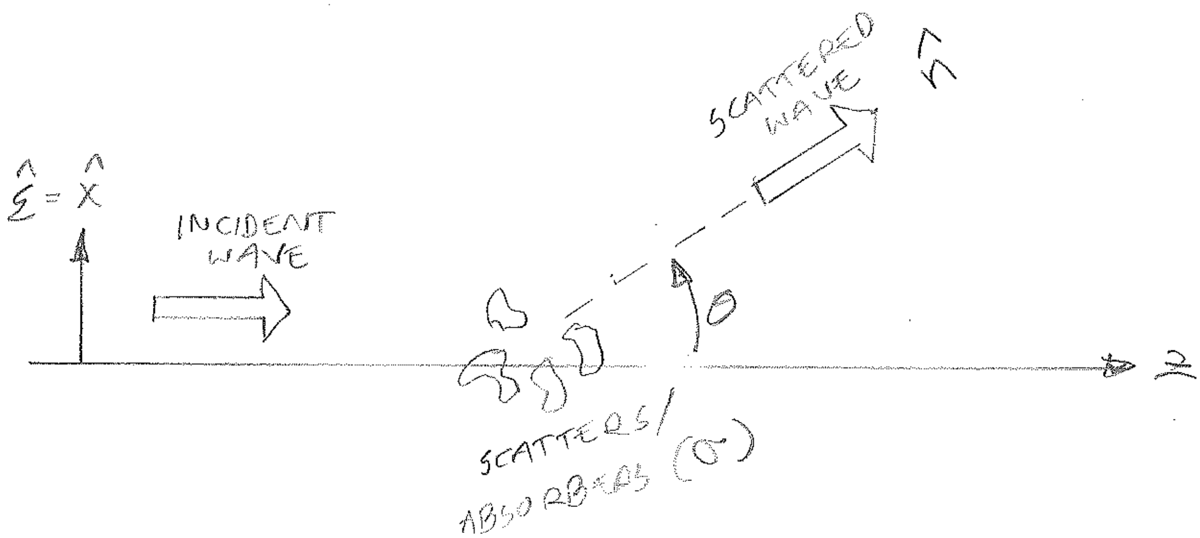
Administrative:

1. HW#3 due now (with some exceptions).
2. Office hours Wednesday after class at URL https://washington.zoom/us/j/712804010

Lecture:

Close-out J. chapter 10: Scattering & diffraction.
1. J. Chapter 10.11: Optical Theorem & optics.
a. Example: Absorbing disk.
Start J. chapter 11: Special theory of relativity.
1. J. Chapter 11.1-3: Historical review, Postulates of Special Relativity, Lorentz transformations.

J.C. 10.11 OPTICAL THEOREM



AT LARGE DISTANCES FROM THE SCATTERERS/ABSORBERS:

$$\vec{E} = E_0 \left\{ \hat{x} e^{ik\hat{z}\cdot\vec{r}} + \vec{F}(\theta,\phi)\frac{e^{ikr}}{kr} \right\}$$

$\vec{F}(\theta,\phi)$ IS THE VECTOR AMPLITUDE OF THE SCATTERED $\vec{E}$-FIELD RELATIVE TO E_0, THE MAGNITUDE OF THE INCIDENT $\vec{E}$-FIELD.

$$\vec{H} = \frac{1}{Z_0}\hat{n}\times\vec{E}.$$

THE GENERALIZED POYNTING'S THEOREM READS

$$-\iiint \sigma |\vec{E}|^2 dv = \qquad \text{a}$$

$$\frac{1}{Z_0} E_0^2 \left[\frac{1}{K^2} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} |\vec{F}|^2 \, d\Omega \qquad \text{b} \right.$$

$$\left. + \frac{2\pi i}{K^2} \hat{x} \cdot (\vec{F} - \vec{F}^*)_{\theta=0} \right] \qquad \text{c}$$

FROM THE ABOVE, YOU CAN READ OFF CROSS-SECTIONS

$$\frac{d\sigma_{SCAT}}{d\Omega} = \frac{1}{K^2} |\vec{F}(\theta, \phi)|^2 \qquad \text{b}$$

$$\& \ \sigma_{SCAT} = \frac{1}{K^2} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} |\vec{F}|^2 d\Omega$$

$$\sigma_{ABS} = \frac{1}{Z_0} \frac{1}{E_0^2} \iiint \sigma |\vec{E}|^2 dv \qquad \text{a}$$

$$\sigma_{TOTAL} = \sigma_{SCAT} + \sigma_{ABS}$$

$$= \frac{2\pi}{iK^2} \hat{x} \cdot (\vec{F} - F^*)_{\theta=0} \qquad \text{c}$$

$$= \frac{4\pi}{K^2} \, \text{Im} (\vec{F})_{\theta=0}$$

THIS IS PROBABLY FAMILIAR FROM QUANTUM MECHANICS.

EXAMPLE : σ_{TOTAL} FOR ABSORBING DISK

RECALL FROM DIFFRACTION, VIA BABINET'S PRINCIPLE, THE ON-AXIS DIFFRACTION FIELD

$$E_{DISK}(z) = E_0 e^{ik\sqrt{a^2+z^2}} \quad \text{(ON-AXIS)}.$$

ALSO RECALL FROM THE DEFINITION OF $\vec{F}(\theta,\phi)$:

$$E_{DISK}(\theta,\phi) = E_0 F(\theta,\phi)\frac{e^{ikr}}{kr}$$

EVALUATE BOTH FOR $\theta \approx 0$:

$$E_{DISK}(z) \approx E_0 e^{ik\left(z+\frac{a^2}{z^2}+\cdots\right)}$$

$$E_{DISK}(\theta,\phi)\Big|_{\theta=0} = E_0 F(\theta,\phi)\Big|_{\theta=0}\frac{e^{ikz}}{kz}$$

WE IDENTIFY

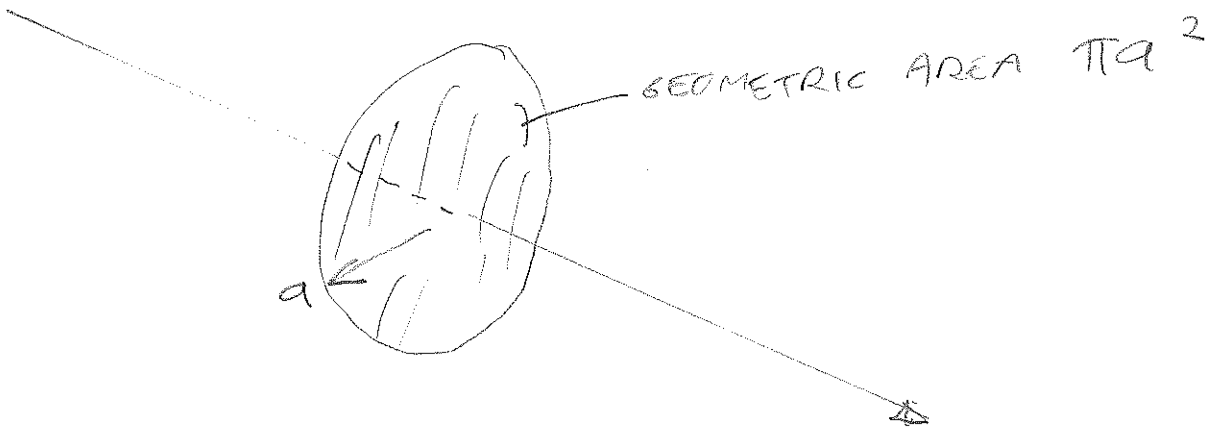
$$F\Big|_{\theta=0} \approx kz\, e^{ik\frac{a^2}{2z}}$$

$$= kz\left\{\cos k\frac{a^2}{2z} + i\sin k\frac{a^2}{2z}\right\}$$

$$\mathrm{Im}\,(F)_{\theta=0} = KZ \sin K\frac{a^2}{2Z}$$

$$\approx \tfrac{1}{2}K^2a^2 \quad (Z \gg Ka)$$

$$\sigma_{TOTAL} = \frac{4\pi}{K^2}\,\mathrm{Im}\,(f)_{\theta=0} \approx 2\times\pi a^2$$



Q: WHY IS $\sigma_{TOTAL} > \pi a^2$?
A: SCATTERING PLUS ABSORPTION.

Q: CAN YOU USE THE OPTICAL THEOREM TO EVALUATE THE COMPLEMENT OF THE ABSORBING DISK?

A: NO. THE PREMISE OF THE OPTICAL THEOREM IS VIOLATED; THERE IS NO POINT "DISTANT" FROM SOURCES.

WE ARE TRANSITIONING TO GAUSSIAN UNITS.

UP TO NOW WE'VE USED MKS UNITS. IT HAS THE ADVANTAGE ~~OF~~ USING COMMON UNITS LIKE VOLT, AMPERE, COULOMB, etc.

IT'S EASY TO TRANSLATE MKS TO GAUSSIAN UNITS IN VACUUM:

$$c\,B_{MKS} \longrightarrow B_{GAUSSIAN}$$

$$\varepsilon_0 \longrightarrow \frac{1}{4\pi}$$

$$\mu_0 \varepsilon_0 \longrightarrow \frac{1}{c^2}$$

THE TRANSLATION IN MEDIA IS MORE COMPLEX. SEE JACKSON APPENDIX 3.

JACKSON C. 11: REVIEW OF SPECIAL RELATIVITY.

SPECIAL THEORY OF RELATIVITY (EINSTEIN 1905). THIS HAS ITS ORIGINS IN E&M: MAXWELL'S EQUATIONS HAVE A SIMPLE RELATION TO SPECIAL RELATIVITY.

SEVERAL OBVIOUS COMMENTS. E&M FIELDS PROPAGATE IN VACUUM AT SPEED C; NO SUCH VELOCITY APPEARS IN NEWTONIAN MECHANICS. IN NEWTONIAN MECHANICS, THE LAWS OF MOTION ARE THE SAME IN ALL RIGID COORDINATE SYSTEMS MOVING WITH CONSTANT VELOCITY RELATIVE TO EACH OTHER: NO SO WITH E&M.

THIS IS ILLUSTRATED IN A SIMPLE EXERCISE IN MECHANICS.

CONSIDER MASS POINTS WITH A POTENTIAL DEPENDING ON SEPARATION.

$$m_i \ddot{\vec{r}}_i = -\sum_{j \neq i} \vec{\nabla}_i V(\vec{r}_i - \vec{r}_j)$$

CONSIDER A TRANSFORMATION TO PRIMED COORDINATES

$$x' = x - vt;\ y' = y;\ z' = z;\ t' = t,$$

WE'D THEN HAVE

$$m_i \ddot{\vec{r}}_i{}' = -\sum_{j \neq i} \vec{\nabla}_i{}' V(\vec{r}_i{}' - \vec{r}_j{}').$$

THIS LAST RESULT DEPENDS ON

$$t' = t$$

SUCH A GALILEAN TRANSFORMATION WAS CONSIDERED SELF EVIDENT, AND LAWS OF NEWTONIAN MECHANICS ARE INVARIANT UNDER GALILEAN TRANSFORMATIONS.

BUT MAXWELL'S EQUATIONS ARE DIFFERENT. THE FORM OF THE WAVE EQUATIONS CHANGES UNDER A GALILEAN TRANSFORMATION. THIS SEEMS TO IMPLY THAT THE LAWS OF E&M ARE DIFFERENT IN DIFFERENT FRAMES. AND IN PARTICULAR $c^2 \neq 1/\epsilon_0 \mu_0$ IN ALL FRAMES.

HERE'S WHAT WAS DONE. IF WE ASSUME NEWTONIAN KINEMATICS, PLUS THE EXTRA TERMS INTRODUCED BY THE GALILEAN CORRECTIONS, THERE'S A SPECIAL "ETHER" FRAME IN WHICH MAXWELL'S EQUATIONS ARE VALID AND LIGHT HAS SPEED c.

IN THIS SITUATION WHERE GALILEAN RELATIVITY APPLIES TO NEWTONIAN MECHANICS, BUT NOT MAXWELL E&M, WE'RE FORCED TO CHOOSE AMONG:

1. A "RELATIVITY" PRINCIPLE EXISTS FOR MECHANICS, BUT NOT ELECTRODYNAMICS; THERE'S AN "ETHER" FRAME IN ELECTRODYNAMICS.

2. A "RELATIVITY" PRINCIPLE EXISTS FOR BOTH MECHANICS AND ELECTRODYNAMICS; MAXWELL'S FORMULATION OF ELECTRO-DYNAMICS IS INCORRECT.

3. A "RELATIVITY" PRINCIPLE EXISTS FOR BOTH MECHANICS AND ELECTRODYNAMICS; NEWTON'S FORMULATION OF MECHANICS IS INCORRECT.

ONLY EXPERIMENT CAN SORT THIS OUT. OF COURSE, AS YOU WELL KNOW, EXPERIMENT POINTS TO (3) IN THE FORM OF SPECIAL RELATIVITY.

THE EXPERIMENTS FALL INTO THREE CLASSES:

1. ATTEMPT TO LOCATE A PREFERRED FRAME IN ELECTRODYNAMICS;

2. ATTEMPT TO FIND DEVIATIONS FROM MAXWELL ELECTRODYNAMICS;

3. ATTEMPT TO FIND DEVIATIONS FROM NEWTONIAN MECHANICS.

OVER 100 YEARS AGO, (1) SEEMED OBVIOUSLY CORRECT.

a. TROUTON & NOBLE (TORQUES);
b. MICHELSON & MORLEY (INTERFERENCE);

LED TO CONCLUSION NO PREFERRED FRAME FOR E&M. BUT CARE NEED FOR "LOCAL" ETHER.

STILL ALLOWED WERE "EMISSION THEORIES": e.g.,

a. THE SPEED OF LIGHT REMAINS C RELATIVE TO THE ORIGINAL SOURCE (RITZ);

b. THE SPEED OF LIGHT REMAINS C RELATIVE TO A BOUNCE OFF A MIRROR

; etc., etc.

a. WAS PARTICULARLY TRICKY; IT WAS FINALLY ATTACKED VIA LIGHT FROM BINARY STARS (de SITTER).

SO FAR, EXPERIMENTALLY, ONLY (3) SURVIVES (LEAVE E&M ALONE, MODIFY NEWTONIAN MECHANICS; THERE'S A RELATIVITY PRINCIPLE FOR BOTH). IN PARTICULAR

- THE EXISTENCE OF THE "ETHER" IS EXCLUDED (WITH SOME "ESCAPE HATCHES" FOR LOCAL ETHER THEORIES).
- MODIFICATIONS TO MAXWELL ELECTRODYNAMICS, SUCH AS THE "RITZ EQUATIONS" IN EMISSION THEORIES,
- THE LAWS OF NEWTONIAN MECHANICS NEED TO BE MODIFIED.

THE MODIFICATION IS "SPECIAL RELATIVITY".

THERE ARE MANY WAYS TO EXPRESS EINSTEIN'S POSTULATES OF SPECIAL RELATIVITY; HERE'S ONE WAY*:

- THE LAWS OF ELECTRODYNAMICS, (INCLUDING THE SPEED OF LIGHT), AS WELL AS THE LAWS OF MECHANICS, ARE THE SAME IN ALL INERTIAL FRAMES.

- IT IS IMPOSSIBLE TO DEVISE AN EXPERIMENT TO DETERMINE A STATE OF "ABSOLUTE" MOTION, OR TO DETERMINE FOR ANY PHYSICAL PHENOMOMENON A PREFERRED INERTIAL FRAME HAVING SPECIAL PROPERTIES.

* EINSTEIN 1905.

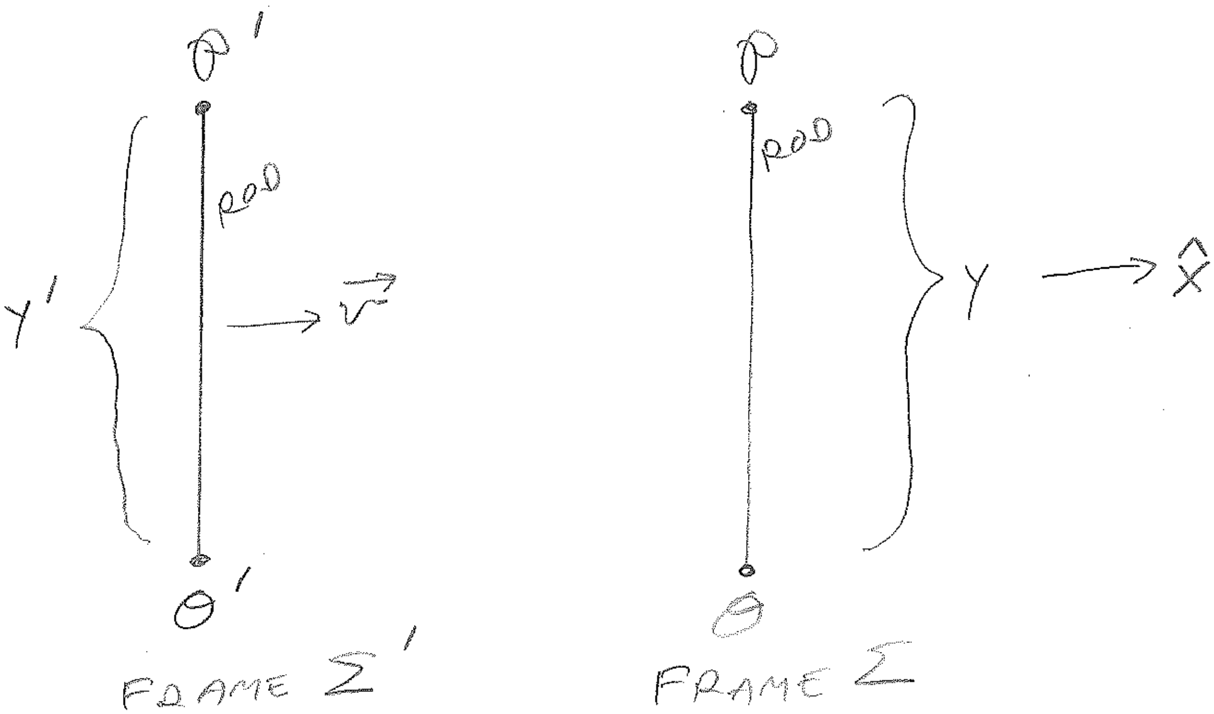
EINSTEIN 1905 INTRODUCED TWO "AUXILARY PRINCIPLES":

- "INVARIANCE OF THE SENSE OF TIME"; TIED TO ENTROPY;

- "INVARIANCE OF PROPER QUANTITIES"; ALL "STATIONARY" OBSERVERS OBSERVE THE SAME LENGTH AND TIME.

NOW, WE'LL FIND THE LORENTZ TRANSFORMATIONS RESULTING FROM THESE POSTULATES VIA "GEDANKEN" EXPERIMENTS.

1. GEDANKEN EXPERIMENT #.
PERPENDICULAR MEASURING STICKS.



- CONSTRUCT RODS TO HAVE SAME LENGTHS IN THEIR REST FRAMES (THEIR "PROPER FRAMES").

- ARGUE O' AND P' CROSS THE OTHER ROD SIMULTANEOUSLY IN BOTH SYSTEMS (AND O AND P CROSS THE OTHER ROD SIMULTANEOUSLY IN BOTH SYSTEMS).

THE CROSSING IS SIMULTANEOUS IN BOTH FRAMES. AT THE CROSSING TIME, STATIONARY OBSERVERS IN BOTH FRAMES RECORD THE FOLLOWING POSSIBILITIES FOR THE POSITIONS OF THE END OF THE ROD:

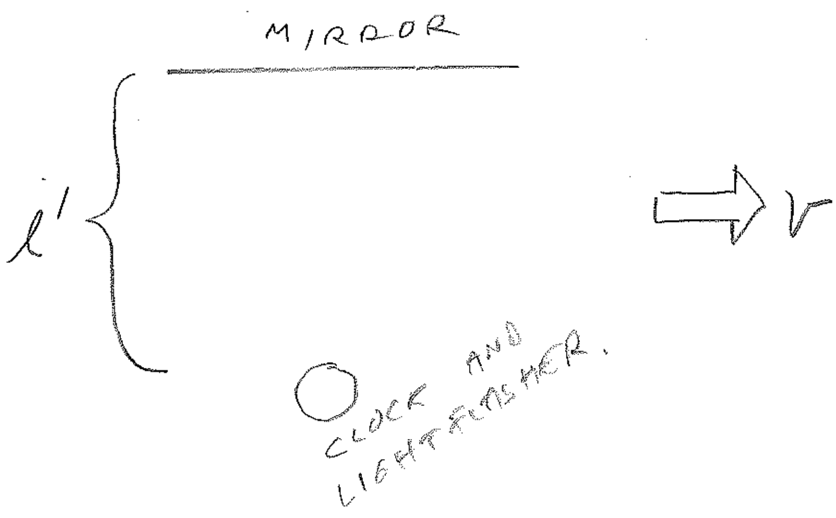
$$OP < O'P' \, ;$$
$$OP > O'P' \, ;$$
$$OP = O'P' \, .$$

BUT BOTH SYSTEMS ARE EQUIVALENT, SO EITHER OF THE FIRST 2 CHOICES WOULD DETERMINE ABSOLUTE MOTION (VIOLATING A POSTULATE). HENCE

$$Y' = Y \quad (\text{AND } Z' = Z).$$

THE TRANSVERSE LENGTHS TRANSFORM THE SAME AS FOR THE GALILEAN TRANSFORMATION.

2. GADENKEN EXPERIMENT 2: CLOCKS.

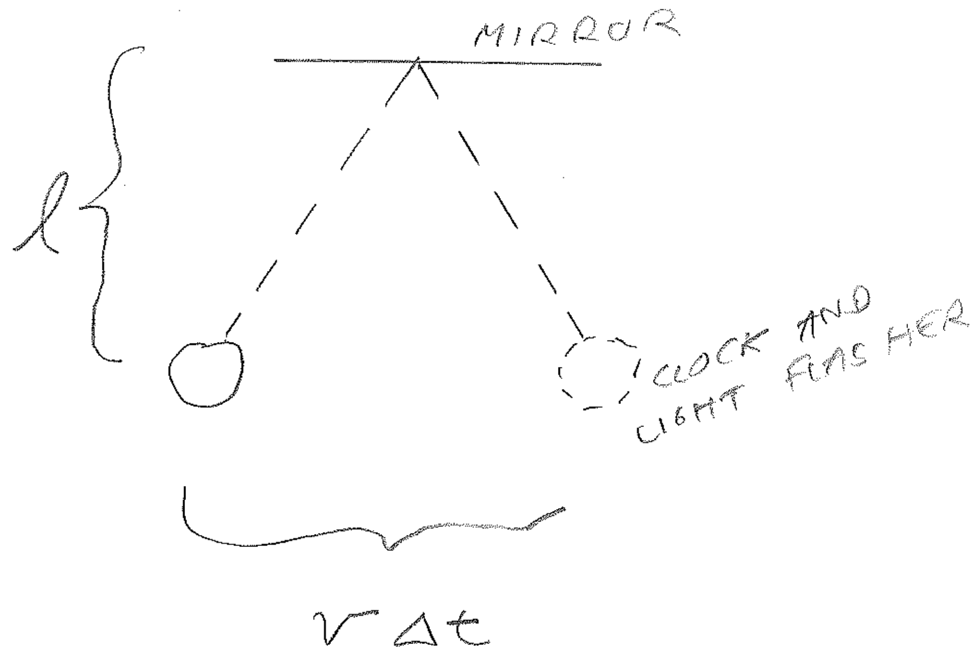


FRAME Σ'

- A LIGHT FLASH IS EMITTED IN Σ' PERPENDICULAR TO $\vec{v}$ AND RETURNS AFTER BOUNCING OFF THE MIRROR TO THE CLOCK ("TICK — TOCK").

FOR AN OBSERVER AT REST IN Σ', THE "TICK-TOCK" TIME INTERVAL IS $\Delta t' = 2\frac{l'}{c}$.

IN FRAME Σ, THIS LOOKS LIKE



WITH Δt THE "TICK-TOCK" TIME INTERVAL.

$$\Delta t = 2\frac{\ell}{c}\frac{1}{\sqrt{1-\beta^2}} \quad ; \quad \beta = v/c$$

"MOVING CLOCKS RUN SLOW"

3. GEDANKEN EXPERIMENT 3.
PARALLEL METER STICKS.

START WITH ROD OF PROPER LENGTH ℓ' IN FRAME Σ'

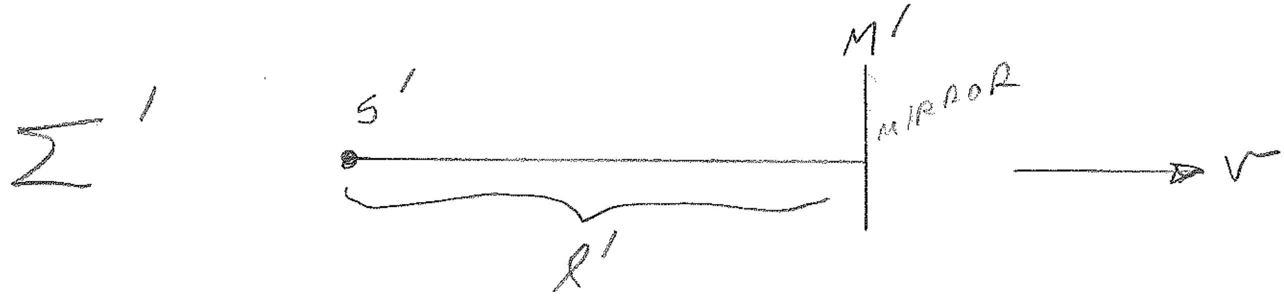
IN FRAME Σ, ITS LENGTH ℓ WOULD BE THE DISTANCE BETWEEN ITS ENDS MEASURED SIMULTANEOUSLY. THIS IS CONCEPTUALLY TRICKY.

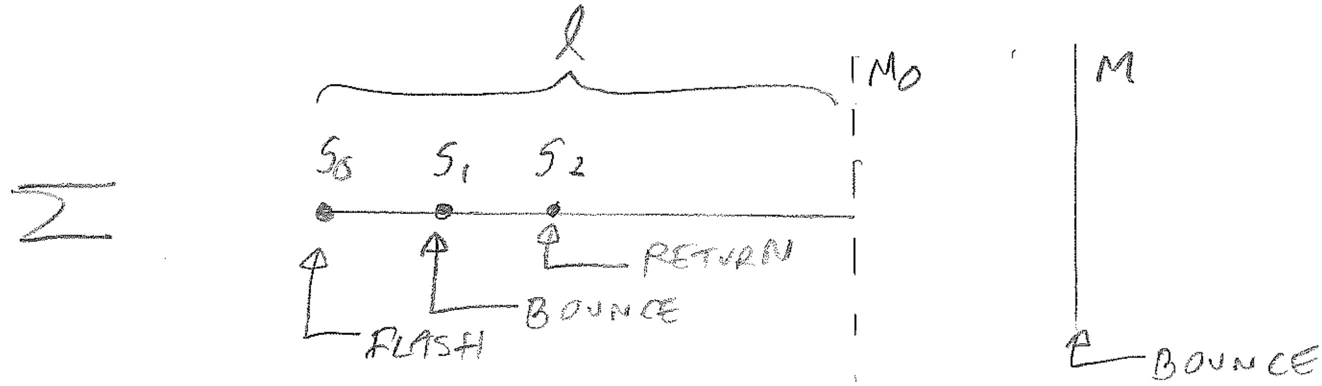
ONE WAY TO DO THIS IS TO PUT A LIGHT-FLASHER AT ONE END OF THE ROD, AND A MIRROR AT THE OTHER END.

IN Σ', THE TIME INTERVAL BETWEEN FLASH AND RETURN IS

$$\Delta t' = 2\frac{\ell'}{c}.$$

IN Σ, THIS IS MORE COMPLICATED; WE'LL DRAW PICTURES.





THE SOURCE S' WAS AT S_0 AT THE TIME OF EMISSION, AND THE MIRROR M' WAS AT M_0 AT THE TIME OF EMISSION.

AT THE TIME OF REFLECTION, M' MOVED TO M, AND THE FLASH RETURNS TO S' WHEN S' IS AT S_2

THE TIME INTERNAL Δt BETWEEN FLASH AND RETURN IS THEREFORE MEASURED BETWEEN S_0 AND S_2;' THIS PUTS US IN THE PREVIOUS CASE OF CLOCKS.

WE ALSO NEED TO BE CAREFUL OF WHAT WE MEAN BY THE DISTANCE ℓ! THIS IS THE DISTANCE $S_0 M_0$ (OR EQUIVALENTLY $S_1 M$).

SINCE M_0 MOVES TOWARDS M WITH SPEED v, WHILE THE PULSE MOVED FROM S_0 TO M WITH SPEED c, WE HAVE

$$\underbrace{S_0 M}_{\text{DISTANCE}} = \underbrace{\ell}_{S_0 M_0} + \frac{v}{c} M S_2$$

$$= \frac{\ell}{1-\beta}$$

SIMILARLY, WHEN THE SOURCE MOVED FROM S_1 TO S_2 WITH SPEED v, THE PULSE TRAVELED FROM M TO S_2 WITH SPEED c.

$$M S_2 = \frac{\ell}{1+\beta}$$

THE TIME INTERVAL IS THE "TO AND FROM" DISTANCE DIVIDED BY C:

$$\Delta t = \frac{S_0 M + M S_2}{c}$$

RECALL IN Σ': $\Delta t' = 2\frac{\ell'}{c}$.

ALSO RECALL FROM GEDENKEN EXPERIMENT 2 $\Delta t' = \Delta t \frac{1}{\sqrt{1-\beta^2}}$.

HENCE $\ell = \ell' \sqrt{1-\beta^2}$

"MOVING METER-STICKS SHRINK"

"LORENTZ CONTRACTION".