Physics 515, Electrodynamics III
Department of Physics, University of Washington
Spring quarter 2020
May 22, 2020, 11am
On-line lecture

Administrative:
1. HW#7 due now (with some exceptions).
2. You should be getting your homework back; if not let me know.

Lecture:
J. Chapter 13: Collisions, Energy Loss, and Scattering of Charged Particles; Cherenkov and Transition Radiation.
1. J. C. 13.3 The density effect on ionization loss and aspects of Cherenkov and transition radiation.
2. J. C. 13.4 Cherenkov radiation.
3. Transition radiation.
4. Multiple scattering.

Next week: **J. C. 14.1 Liénard-Wiechert potentials II, 14.2 Larmor formula, 14.3 Angular distribution of radiation, J. 16.2 Radiation reaction, J. 16.3 Abraham-Lorentz formula.**

J.C. 13.3 THE DENSITY EFFECT (AND AN INTRO TO ČERENKOV & TRANSITION RADIATION).

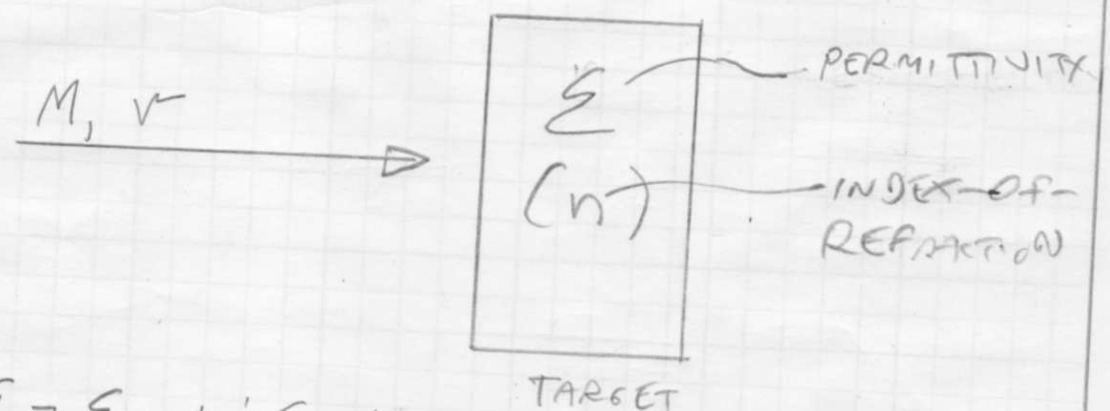
FERMI (~1940) WONDERED WHY ULTRARELATIVISTIC PARTICLES HAVE ANOMALOUSLY-LOW ENERGY LOSS.

ELECTROMAGNETISM INCLUDES THESE ENERGY-LOSS PROCESSES

- ATOMS CAN BE IONIZED OR EXCITED;
- EMISSION OF ČERENKOV RADIATION;
- EMISSION OF TRANSITION RADIATION.

(Q: WHAT OTHER EM PROCESSES ARE THERE?)

HERE'S A PICTURE OF THE PROJECTILE AND TARGET:



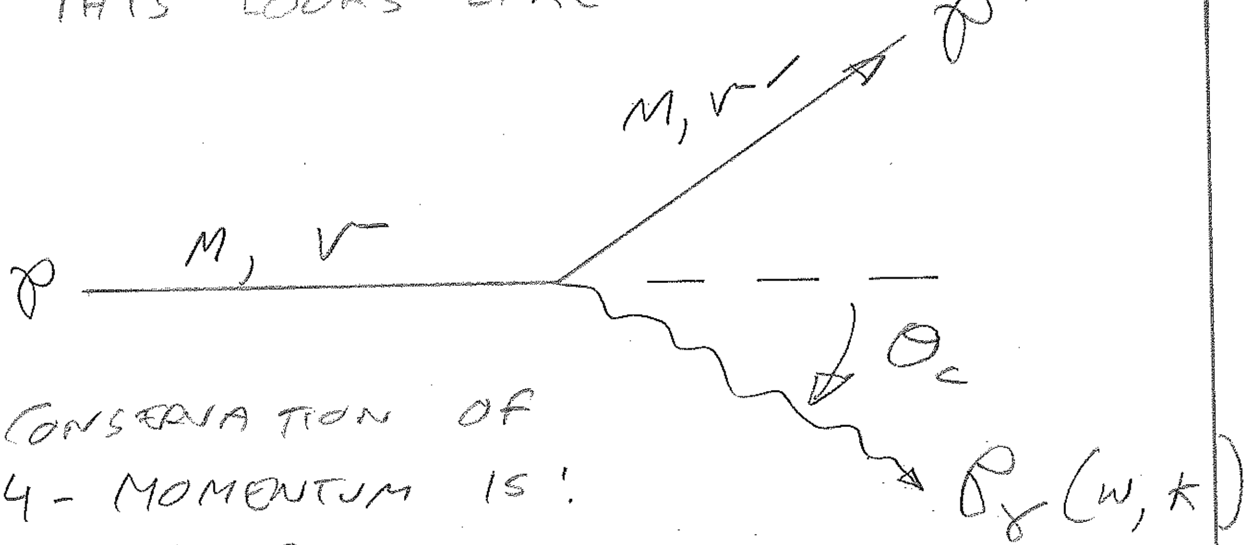
$\varepsilon = \varepsilon_R + i\varepsilon_I$;

$n = \varepsilon_R^2$

LET'S LOOK AT EACH SCATTER MICRO-SCOPICALLY: A PHOTON (REAL OR VIRTUAL) OF ENERGY $\hbar\omega$ AND MOMENTUM $\hbar k$ IS CREATED.

ASSUMPTION: $\hbar\omega$ AND $\hbar k$ ARE TRUE EVEN FOR VIRTUAL-PHOTON EXCHANGE. (THIS IS A DETAIL FROM QFT.)

THIS LOOKS LIKE



CONSERVATION OF 4-MOMENTUM IS:

$$p' = p - p_\gamma$$

THIS GIVES

$$\omega = \vec{v}\cdot\vec{k} = vk\cos\theta_c.$$

Q: WHERE DID THIS COME FROM?

A: TAKE THE SQUARE OF P':

$$M^2 = (E - E_\gamma)^2 - (\vec{P} - \vec{P}_\gamma)^2$$

$$= E^2 + E_\gamma^2 - 2EE_\gamma - \vec{P}\cdot\vec{P} - \vec{P}_\gamma\cdot\vec{P}_\gamma + 2\vec{P}\cdot\vec{P}_\gamma$$

THE $E^2 - \vec{P}\cdot\vec{P}$ CANCELS M^2.

THE $E_\gamma^2 - \vec{P}_\gamma\cdot\vec{P}_\gamma$ GIVES ZERO.

THIS LEAVES

$$0 = -2EE_\gamma + 2\vec{P}\cdot\vec{P}_\gamma.$$

RECALL $E = \gamma Mc^2$, $P = \gamma M\vec{v}$,

$E_\gamma = \hbar\omega$, $\vec{P} = \hbar\vec{K}$, HENCE

$$\omega = \vec{v}\cdot\vec{K} = vK\cos\theta_c$$

IN MEDIA, THE PHOTON'S DISPERSION RELATION IS $\omega^2 = K^2c^2/\varepsilon$, SO

$$\sqrt{\varepsilon}\,\frac{v}{c}\cos\theta_c = 1$$

THERE'S A LOT OF PHYSICS IN THIS EQUATION.

OVERVIEW OF WHAT'S TO COME:

- FOR SMALL PHOTON ENERGIES (ENERGIES BELOW THE EXCITATION ENERGY OF THE MEDIA: THE "OPTICAL REGION"),

ε IS REAL AND >1;

HENCE $\sqrt{\varepsilon}\,\frac{v}{c}\cos\theta_c = 1$

WITH $v > \frac{c}{\sqrt{\varepsilon}}$ GIVES

$\cos\theta_c$ REAL AND REAL PHOTONS ARE EMITTED;
THIS IS ČERENKOV LIGHT EMITTED WHEN A CHARGED PARTICLE TRAVERSES A MEDIUM AT SPEEDS GREATER THAN THAT OF LIGHT IN THE MEDIUM.
(J, EQN. 13.47)

- FOR INTERMEDIATE PHOTON ENERGIES (~2 eV TO 5 keV), $\mathcal{E} = \mathcal{E}_R + i\mathcal{E}_I$ IS COMPLEX WITH $\mathcal{E}_R < 1$ AND $\mathcal{E}_I > 0$.

IN THIS CASE ONLY VIRTUAL PHOTONS ARE EXCHANGED BETWEEN THE INCIDENT PARTICLE AND ATOMS OF THE MEDIUM; THE ATOMS ARE EXCITED OR IONIZED, REPRESENTING ENERGY LOSS AND A NON-ZERO $\mathcal{E}_I$.

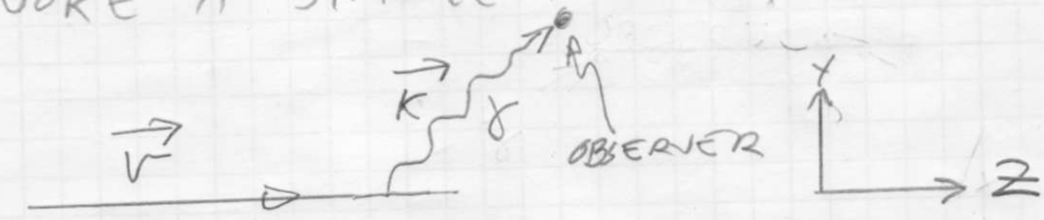
- IN THE X-RAY REGIME (FOR $E_\gamma \gtrsim 5$ keV) THE ABSORPTION IS SMALL: $\mathcal{E}_I \ll 1$ AND STILL $\mathcal{E}_R < 1$. EVEN THOUGH THERE IS ALMOST NO ABSORPTION, THE ČERENKOV VELOCITY

$$v = \frac{c}{\sqrt{\mathcal{E}}}$$ IS GREATER THAN C!

NO ČERENKOV EMISSION.

(THAT SAID, THERE MAY BE DISCONTINUITIES IN THE MATERIAL, LEADING TO TRANSITION RADIATION.)

TO CONTINUE FROM HERE, WE INVOKE A SIMPLE MODEL:



FROM $\omega = \vec{v}\cdot\vec{K}$, $\omega = v K_Z$

FROM $\frac{\omega^2}{K^2} = \frac{c^2}{\varepsilon}$, $K_Y{}^2 + K_Z{}^2 = \omega^2 \frac{\varepsilon}{c^2}$,

HENCE $K_Y = \frac{\omega}{v}\sqrt{\frac{v^2}{c^2}\varepsilon - 1}$

LETS CALL THE PHASE VELOCITY OF PHOTONS IN THE MEDIUM v_M

$v_M = c/\sqrt{\varepsilon}$ AND $\beta_M = \frac{v}{v_M}$,

AND $\gamma_M = \frac{1}{\sqrt{1-\beta_M{}^2}}$.

BASICALLY, WE'RE TAKING OUR USUAL EXPRESSIONS AND REPLACING THE SPEED OF LIGHT WITH v_M.

IN THIS NOTATION

$K_Y = \frac{\omega}{v}\sqrt{\beta_M{}^2 - 1}$ AND

$K_Y{}^2 + K_Z{}^2 = \frac{\omega^2}{v_M{}^2}$.

SINCE $\beta_M = \frac{c}{\sqrt{\varepsilon}}$, β_M CAN BE > 1.

WE'LL LOOK AT TWO CASES

- $\beta_M > 1$.

  HERE K_Y IS REAL,

  AND K_Z IS REAL.

  THE PHOTONS COMPRISE A WAVE OF FORM

  $$e^{i(\vec{K}\cdot\vec{r} - \omega t)}$$

- $\beta_M < 1$.

  HERE $K_Y = \frac{\omega}{v}\sqrt{\beta_M^2 - 1}$ IS

  PURE IMAGINARY. THIS REPRESENTS LOSSES.

  THE TRANSVERSE COMPONENT OF THE FIELD (TOWARDS $\hat{y}$) DOES NOT OSCILLATE, BUT INSTEAD IS DAMPED WITH ATTENUATION LENGTH

  $$e^{i(\vec{K}\cdot\vec{r} - \omega t)} \sim e^{-y/y_0},$$

  WITH $y_0 = \frac{v}{\omega}\gamma_M$ (FROM $K_Y^2 = \frac{\omega^2}{v^2}(\beta_M^2 - 1)$),

  $= \frac{\beta_M \gamma_M}{K}$ (FROM $\frac{v}{v_M} = \beta_M$; $\frac{\omega}{K} = v_M$).

NOTICE THE RANGE OF THE TRANSVERSE FIELDS GROWS AS $\beta_M \gamma_M$. THIS SIMPLE FORM FOR THE RANGE OF THE TRANSVERSE FIELDS IS WHY WE INTRODUCED β_M AND γ_M.

SINCE THE RANGE OF THE TRANSVERSE FIELD GROWS WITH PARTICLE VELOCITY, THE ENERGY LOSS DUE TO IONIZATION INCREASES (THE "RELATIVISTIC RISE" OF ENERGY LOSS).

IN TERMS OF THE USUAL (VACUUM) VARIABLES:

$$Y_0 = \frac{\beta}{k_0} \frac{1}{\sqrt{\frac{1}{\gamma^2} + (1+\varepsilon)\beta^2}}.$$

WE'LL CONSIDER THE ABOVE EXPRESSION IN TWO CASES:

$\varepsilon \gg 1$ AND $\varepsilon \ll 1$.

- $\varepsilon < 1$. THIS IS WHAT WE HAVE FOR ENERGY TRANSFER HIGH ENOUGH TO IONIZE THE MEDIUM. THERE IS LOSS BUT NO PHOTON PROPAGATION. WITH INCREASING VELOCITY, THE TRANSVERSE RANGE INCREASES, BUT THE DENOMINATOR OF Y_0 BECOMES:

$$\sqrt{\frac{1}{\gamma^2} + (1+\varepsilon)\beta^2} \xrightarrow[\beta \to 1]{} \sqrt{(1+\varepsilon)}.$$

THIS PLATEAU IN Y_0 CORRESPONDS TO A SATURATION IN ENERGY LOSS VIA IONIZATION. THIS IS REACHED AT A VELOCITY AT WHICH THE TWO TERMS $\frac{1}{\gamma^2}$ AND $(1-\varepsilon)\beta^2$ ARE EQUAL. THAT IS,

$$(\beta\gamma)_{SAT} = \frac{1}{\sqrt{1-\varepsilon}}.$$

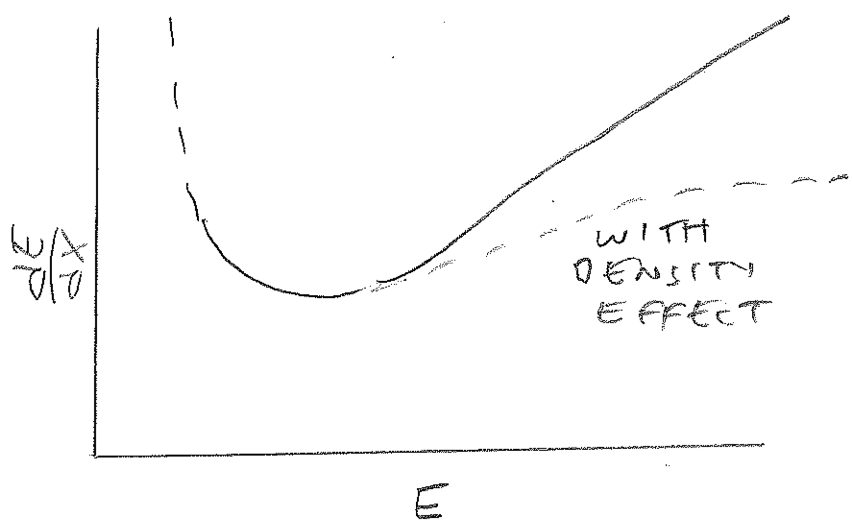
THIS RISE AND SATURATION IS CALLED THE "DENSITY EFFECT".

HOW IS IT RELATED TO DENSITY?

SINCE THE SUSCEPTIBILITY χ_E $(= \varepsilon - 1)$ PROPORTIONAL TO DENSITY ρ, SATURATION IS REACHED EARLIER IN DENSE MATERIALS

$$(\beta\gamma)_{SAT} \sim \frac{1}{\sqrt{\rho}}$$

THIS GIVES US JACKSON FIG 13.2:



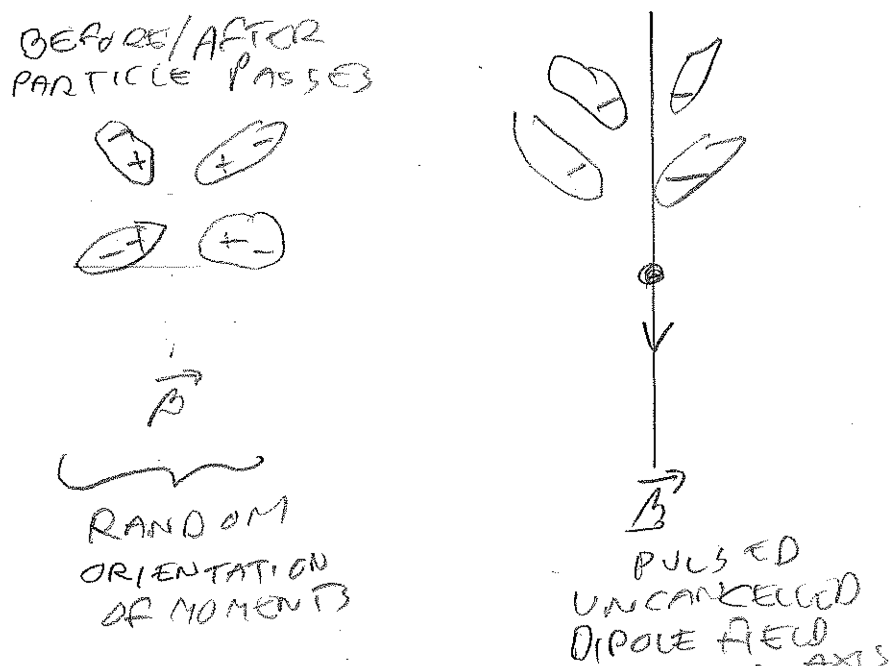
LOOSELY SPEAKING, THE SUSCEPTIBILITY χ_E OF THE MEDIUM WEAKENS THE FIELD AT AT A DISTANCE FROM THE TRAJECTORY, THEREBY DECREASING THE STOPPING POWER.

ANOTHER IMPORTANT EFFECT OF THE MEDIA SUSCEPTIBILITY: ČERENKOV RADIATION.

THIS WAS IMPLIED IN THE PREVIOUS DISCUSSION FOR PHOTON ENERGY BELOW THE EXCITATION ENERGY OF THE MEDIUM AND $v > c/\sqrt{\varepsilon}$ $(= c/n)$

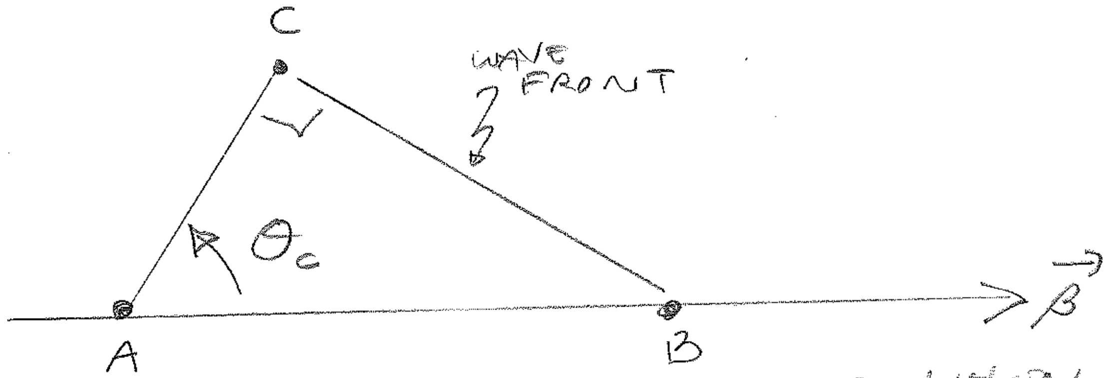
THE SITUATION IS A CHARGED PARTICLE MOVING THROUGH A MEDIUM WITH INDEX n (AND PHASE VELOCITY c/n).

THE MICROSCOPIC PICTURE IS



IN GENERAL, THE RADIATED WAVELETS FROM ALL PARTS OF THE TRAJECTORY HAVE NO PARTICULAR PHASE RELATIONSHIP; THIS IS FOR SLOW-MOVING PROJECTILES.

HOWEVER IF $v > c/n$, IT'S POSSIBLE FOR ALL WAVELETS TO BE IN PHASE. THIS IS SEEN IN A HUYGENS CONSTRUCTION (J. FIG. 13.5)



THE WAVE FRONT OCCURS WHEN A PARTICLE GOES FROM $A \to B$ IN THE TIME IT TAKES LIGHT TO GO FROM $A \longrightarrow C$.

ΔT IS THE TIME IT TAKES FOR THE PARTICLE TO GO FROM $A \to B$.

$$d(A \to B) = \beta c \, \Delta T$$

MEANWHILE THE WAVELET EMITTED AT A TRAVELS A DISTANCE

$$d(A \to C) = \frac{c}{n} \Delta T.$$

HENCE

$$\cos\theta_c = \frac{c/n \, \Delta T}{\beta c \, \Delta T} = \frac{1}{\beta n};$$

THE WAVE FRONT IS A CONE. THIS IS THE STANDARD "BOW-WAVE" SHOCK CONSTRUCTION.

GENERAL COMMENTS

- FOR A MEDIUM OF INDEX n, THE CONDITION $\cos\theta_c < 1$ IMPLIES A MINIUM $\beta = 1/n$ FOR RADIATION TO OCCUR (THE "ČERENKOV THRESHOLD"). AT THIS MINIMUM β $\cos\theta_c = 1$ AND THE RADIATION IS BEAMED FORWARD. THE ANGLE θ_c IS RELATED TO β.

- FOR ULTRA-RELATIVISTIC PARTICLES $\beta \approx 1$ AND $\cos\theta_c \approx 1/n$ IS THE MAXIMUM EMISSION ANGLE.

- THE RADIATION OCCURS MAINLY IN THE VISIBLE AND NEAR-VISIBLE PART OF THE SPECTRUM.

- THE LIGHT-INTENSITY DISTRIBUTION IN θ IS SHARP: THE CONE IS WELL-DEFINED.

- THE POLARIZATION IS STRANGE: FIRSTLY, INSIDE THE CONE IT POINTS TOWARDS THE PARTICLE AND THE ELECTRIC FIELD ON THE CONE POINTS AWAY FROM THE CHARGE.

THE NUMBER OF QUANTA EMITTED PER UNIT TRACK LENGTH IS SURPRISINGLY DIFFICULT TO CALCULATE (SEE, e.g., JACKSON 1975),

APPROXIMATELY

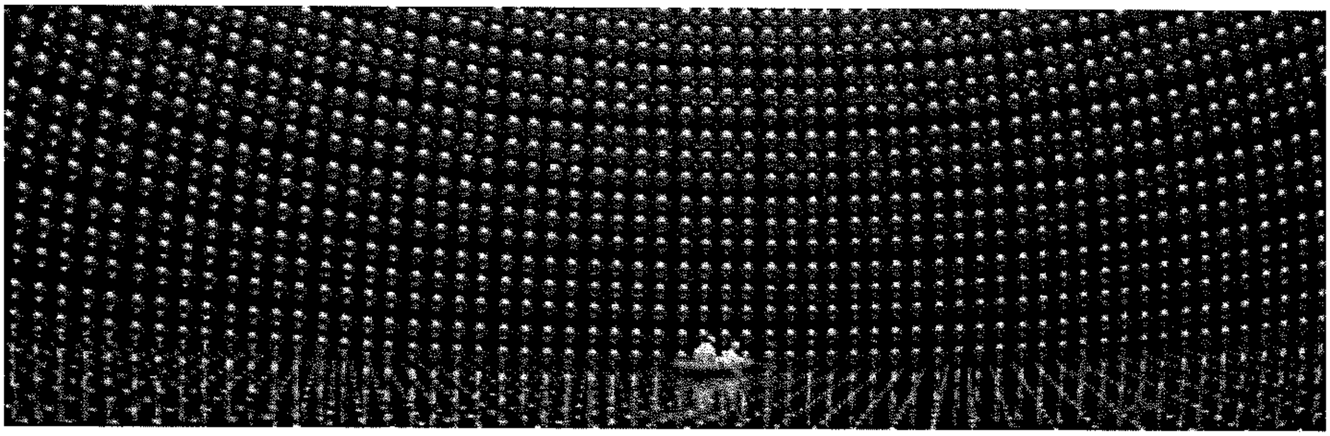
$$\frac{d^2 N_\gamma}{dl\, df} \sim e^2 \sin^2\theta$$

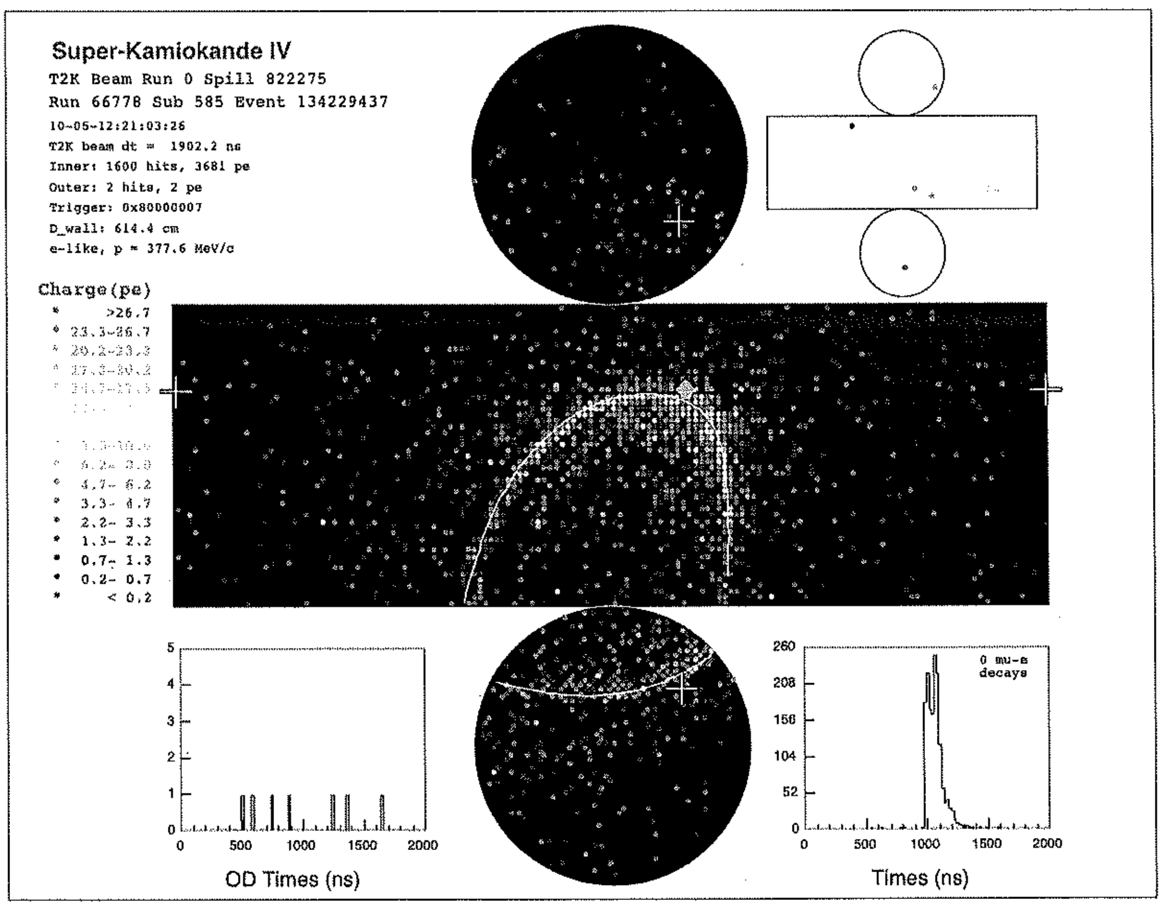
IN THE OPTICAL,

$$\frac{dN_\gamma}{dl} \approx 700 \sin^2\theta / \text{cm}$$

RECALL: θ CONTAINS INFORMATION ABOUT β! THE GEOMETRY OF THE CONE IS OFTEN USED TO INFER β.

THE "SUPER K" NEUTRINO DETECTOR
(FIGS. COURTESY JEFF WILKES)





YET ANOTHER EFFECT OF MEDIA SUSCEPTIBILITY: TRANSITION RADIATION.

THE SITUATION IS AN ULTRA-RELATIVISTIC CHARGED PARTICLE CROSSING THE INTERFACE BETWEEN, SAY, VACUUM AND A CONDUCTOR HAVING PLASMA FREQUENCY

$$\omega_p^2 = 4\pi \rho_E \frac{e^2}{m}.$$

{A HANDY FORMULA FOR ω_p IS

$$\hbar \omega_p = 29 \left(\frac{\rho}{1000} \frac{Z}{A} \right)^{1/2} eV$$

WHERE ρ IS THE DENSITY IN kg/m^3.}

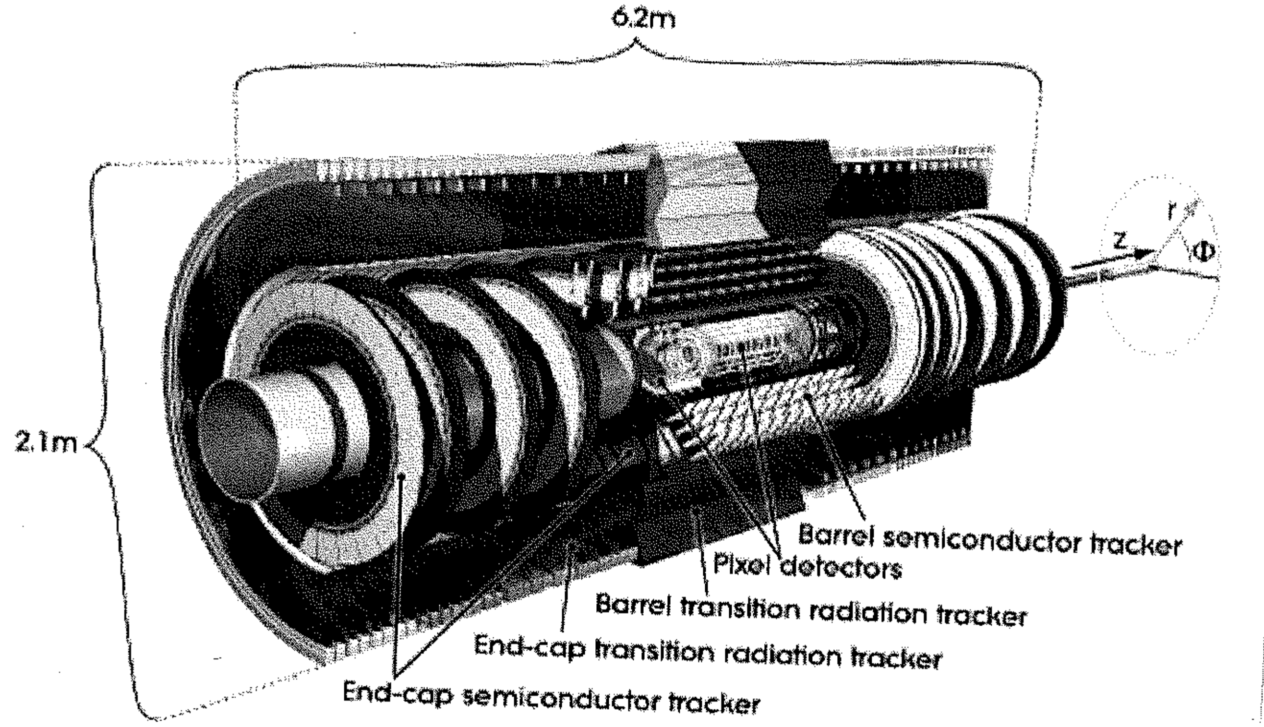
X-RAYS (TRANSITION RADIATION) ARE EMITTED IN A NARROW FORWARD CONE.

THE ENERGY RADIATED PER INTERFACE IS $E \approx \frac{2}{3} \left(\frac{e^2}{c} \right) \omega_p \gamma$ J. EQN. 13.87.

THE RADIATED ENERGY IS PROPORTIONAL TO THE PARTICLE ENERGY: THIS CAN BE USED TO INFER γ.

A TYPICAL DETECTOR HAS 100'S OF FOILS.

THE ATLAS INNER DETECTOR (CERN).
(COURTESY ANNA GOUSSIOU.)



MORE ENERGY-LOSS EFFECTS:

MULTIPLE SCATTERING

MULTIPLE SCATTERING.
ASSUME THE SCATTERING ANGLES θ OBEY $\theta \ll \Omega$, WITH Ω AN ANGLE WHERE IT'S STATISTICALLY LIKELY THE LARGE SCATTERING ANGLE IS DUE TO A SINGLE COLLISION.
$\longrightarrow$ SMALL-ANGLE SINGLE SCATTERS.

EARLIER, FOR IONIZATION, WE FOUND

$$\frac{\Delta p}{p} = 2 z Z \frac{e^2}{b v} \frac{1}{p}$$

THIS IS WHAT GIVES SMALL DEFLECTIONS IN A SINGLE COLLISION

NOT SURPRISINGLY, FOR MANY SCATTERS THE TOTAL SCATTER ANGLE Θ IS

$$\langle \Theta^2 \rangle = \sum_i^n \langle \theta_i^2 \rangle$$

THAT IS, ADD THE DEFLECTIONS IN QUADRATURE.
AND THE MOST PROBABLE VALUE OF Θ, Θ_x, Θ_y IS ZERO.

ASSUME θ_x AND θ_Y ARE GAUSSIAN-DISTRIBUTED (THE "GAUSSIAN REGION" OF SCATTER).

THAT MEANS THE PROBABILITY OF FINDING A TOTAL DEFLECTION BETWEEN θ_x AND $\theta_x + d\theta_x$ IS

$$P(\theta_x)\,d\theta_x = \frac{e^{-\theta_x^2/\langle\theta_x^2\rangle}}{\sqrt{\pi\langle\theta\rangle^2}}\,d\theta_x$$

RECALL $\langle\theta^2\rangle = \sum_i \langle\theta_i^2\rangle$

AND $\frac{\Delta P_i}{P_i} \approx \theta_i = \frac{2zZe^2}{b v P_i}$

INTEGRATE THE ABOVE OVER (ALLOWED) IMPACT PARAMETERS b WITH TARGET DENSITY ρ:

$$\langle\theta^2\rangle = 2\pi\rho X \int_{b_{MIN}}^{b_{MAX}} \frac{2zZe^2}{b v P_i\,(\sim P)}\, b\,db$$

ρ IS THE NUMBER OF TARGET NUCLEI PER CM^3

X IS THE TARGET THICKNESS.

THIS IS

$$\langle \theta^2 \rangle = \frac{8\pi \rho x z^2 Z^2 e^4}{v^2 p^2} \ln \frac{b_{MAX}}{b_{MIN}}$$

WE DISCUSSED b MAX BEFORE. WE'LL TAKE b_{MAX} WHERE THE NUCLEAR CHARGE IS COMPLETELY SCREENED BY ATOMIC ELECTRONS

$$b_{MAX} = \frac{r_0}{Z^{1/3}}$$

(r_0 IS THE BOHR OR CLASSICAL ATOMIC SIZE.).

FOR b_{MIN}, WE REQUIRE THE SCATTERED ANGLE IS $\ll 1$.

FROM $\theta_i \simeq \frac{\Delta p_i}{p_i} = \frac{2zZ}{bvp} e^2$

$$b_{MIN} = \frac{2zZe^2}{vp}$$

THIS GIVES

$$\langle \Theta^2 \rangle \approx Z^2 x \Big/ (KE)^2$$ FOR THE SCALING.

(FOR LEAD) $$\langle \Theta^2 \rangle \approx 600 \frac{x\,[cm]}{(E\,[MeV])^2}$$

(FOR AIR) $$\langle \Theta^2 \rangle \approx 7000 \frac{x\,[cm]}{(E\,[keV])^2}$$

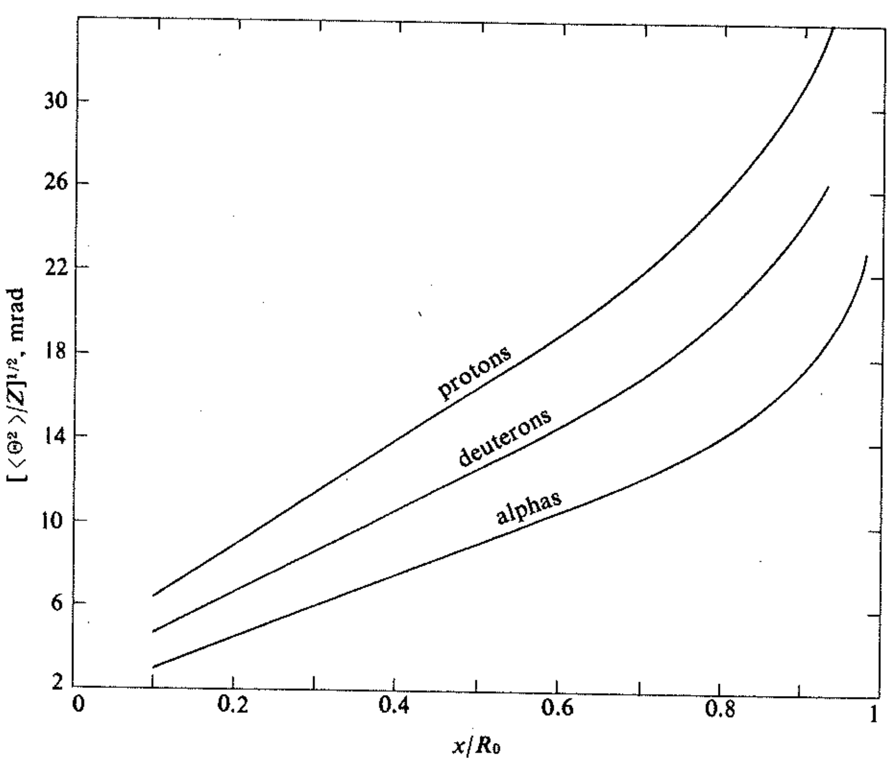


Figure 2-16 Multiple-scattering angle (unprojected) versus fraction of range traversed by protons, deuterons, and alpha particles. [R. H. Milburn and L. Schecter, UCRL 2234 (rev. ed.).]