Physics 515, Electrodynamics III
Department of Physics, University of Washington
Spring quarter 2020
May 20, 2020, 11am
On-line lecture

Administrative

1. You should be getting your graded homework back; if not let me know asap.
2. You should be getting your graded midterm back today; if not let me know asap.
3. Office hours Wednesday after class at URL https://washington.zoom/us/j/712804010

Lecture

Chapter 12: Lagrangian formalism of electrodynamics.
Chapter 13: Collisions, energy loss, and scattering of charged particles, Cherenkov and transition radiation.
1. J. C. 12.10: Stress tensor(s) and conservation laws: angular momentum.
2. J. C. 13.1-2. Rutherford scattering, energy loss through ionization.
3. J. C. 13.3 Density effect in collisional energy loss. Fermi asked: Why do ultra-relativistic particles have anomalously low energy loss?

BACK TO ANGULAR MOMENTUM.

WE ASSERTED ANGULAR MOMENTUM DENSITY

$M^{\alpha\beta\gamma} = T^{\alpha\beta}x^{\gamma} - T^{\alpha\gamma}x^{\beta}$ (J. EQN 12.109).

THIS, WE SAID, NEEDS TO HAVE $T^{\alpha\beta}$ SYMMETRIC IN ORDER TO CONSERVE ANGULAR MOMENTUM (SEE J. DISCUSSION P. 608). SO WE INTRODUCED A SYMMETRIZED STRESS TENSOR $\Theta^{\alpha\beta}$ (J. E 12.106). WITH

$$\Theta^{\alpha\beta} = \frac{1}{4\pi}\left\{g^{\alpha\mu}F_{\mu\lambda}F^{\lambda\beta} + \frac{1}{4}g^{\alpha\beta}F_{\mu\lambda}F^{\mu\lambda}\right\}$$

(J. EQN. 12.133).

THE ANGULAR MOMENTUM DENSITY IS THEN

$$M^{\alpha\beta\gamma} = \Theta^{\alpha\beta}x^{\gamma} - \Theta^{\alpha\gamma}x^{\beta}$$

(J. EQN. 12.117).

EXPLORE THE STRUCTURE OF $M^{\alpha\beta\gamma}$;

HOW IS $M^{\alpha\beta\gamma}$ RELATED TO ANGULAR MOMENTUM?

CONSIDER $M^{\alpha ij}$:

$$M^{0ij} = \Theta^{0i}x^j - \Theta^{0j}x^i$$

$$\Theta^{0i} = \frac{1}{4\pi}\vec{E}\times\vec{B} \quad \text{(J, EQN. 12.114)}$$

$$M^{0ij} = c\left(\wp^i x^j - \wp^j x^i\right)$$

WITH $\wp^i$ THE i^{th} COMPONENT OF THE LINEAR MOMENTUM DENSITY $\vec{\wp}$.

$$M^{0ij} = c\,\varepsilon^{ijk}\left[\vec{\wp}\times\vec{x}\right]_k$$

RECALL $\vec{x}\times\vec{\wp}$ IS THE ANGULAR MOMENTUM DENSITY $\vec{\mathcal{J}}$.

FOR LOCALIZED FIELDS, THE TOTAL FIELD ANGULAR MOMENTUM IS

$$\iiint M^{0ij}\,dV = -c\,\varepsilon^{ijk}\iiint\left[\vec{x}\times\vec{\wp}\right]_k dV$$

$$= -c\,\varepsilon^{ijk}L^k$$

SO, EVIDENTLY M^{0ij} EMBEDS THE ANGULAR-MOMENTUM DENSITY

$$M^{0ij} = \begin{pmatrix} 0 & g^{21} & g^{13} \\ -g^{21} & 0 & g^{23} \\ -g^{13} & -g^{23} & 0 \end{pmatrix}$$

EXERCISE: SHOW $\partial_\mu M^{\mu ij} = 0$ IMPLIES ANGULAR-MOMENTUM CONSERVATION. (HINT WRITE M^{kij}.)

WHAT ABOUT THE THREE COMPONENTS M^{00i}? THEY, TOO, ARE CONSERVED. (SEE JACKSON P. 12.19b). BUT THEY ARE SOMEWHAT OBSCURE CONSERVATION LAWS.

NOTE, HAVING ANGULAR MOMENTUM L^K IN A TENSOR SHOULDN'T BOTHER YOU TOO MUCH. THE CROSS PRODUCT HAS A SIMILAR STRUCTURE

$$\vec{A} \times \vec{B} = \vec{C} : \begin{pmatrix} 0 & C_{12} & C_{13} \\ -C_{12} & 0 & C_{23} \\ -C_{13} & -C_{23} & 0 \end{pmatrix}$$

WE CHOOSE TO WRITE THIS AS THE PSEUDO-VECTOR $\vec{C}$.

J. C. 13. COLLISIONS, ENERGY LOSS, SCATTERING OF CHARGED PARTICLES, ČERENKOV RADIATION AND TRANSITION RADIATION.

(SEE SPECIAL LECTURE.)
START WITH RUTHERFORD-LIKE SCATTERING. ASSUME FAST PROJECTILE $\beta = 1$. THE RESULTING INTERACTIONS OF THE PROJECTILE ARE CLASSIFIED INTO "HARD" AND "SOFT" COLLISIONS.

- ELASTIC COLLISIONS (e.g., TYPICAL ELECTRON INCIDENT ON A HEAVY NUCLEUS) DON'T USUALLY TRANSFER MUCH ENERGY; THE PARTICLE IS USUALLY DEFLECTED.

- INELASTIC COLLISIONS (e.g., HEAVY NUCLEUS ON MATTER) TRANSFERS A LOT OF ENERGY; THE PARTICLE ISN'T MUCH DEFLECTED.

RECALL RUTHERFORD-LIKE SCATTERING:
A CHARGED PARTICLE (ze), HISTORICALLY AN α-PARTICLE, TRAVERSES A MEDIUM WITH TARGETS OF ATOMIC NUMBER Z.

OCCASIONALLY, THE INCIDENT PARTICLE WILL ELASTICALLY COLLIDE WITH A NUCLEUS AND SCATTER OFF THE NUCLEAR COULOMB FIELD OF Z. THE FAMOUS RUTHERFORD RESULT IS

$$\frac{d\sigma}{d\Omega} = \frac{(Ze \cdot ze)^2}{(2mv^2)^2} \frac{1}{\sin^4(\theta/2)}$$

v THE PROJECTILE VELOCITY;
$\theta = 0$ THE FORWARD DIRECTION.

(J. EQN 13.1)

NUMERICALLY, FOR INCIDENT PROTONS

$$\frac{d\sigma}{d\Omega} \approx \frac{\frac{1}{10} Z^2}{E^2(\text{MeV}^2)} \frac{1}{\sin^4(\theta/2)} \times 10^{-26} \frac{\text{cm}^2}{\text{NUCLEUS}}$$

(N.B., $10^{-26} \frac{\text{cm}^2}{\text{NUCLEUS}} \approx \frac{(1\,\text{fm})^2}{\text{NUCLEUS}}$)

THIS EXPERIMENTAL AND ANALYTIC WORK OF RUTHERFORD, GEIGER, MARSDEN et al. (~1910-15) IS CRUDE

- NEGLECTS NUCLEAR FORCE; ONLY CONSIDERS COULOMB FORCE;
- NOT RELATIVISTIC;
- ASSUMES TARGET NUCLEUS IS FIXED;
- NEGLECTS FINITE SIZE OF NUCLEUS;
- NO QUANTUM EFFECTS.

NONETHELESS, IT WAS HUGELY IMPORTANT.

FURTHER, FOR EXTREMELY FORWARD DIRECTIONS, CORRESPONDING TO LARGE IMPACT PARAMETERS, THE NUCLEAR ELECTRIC CHARGE IS SCREENED BY ATOMIC ELECTRONS; THIS AS WELL REGULATES THE $1/\sin^4(\theta/2)$ DIVERGENCE.

WE'LL RETURN TO RUTHERFORD SCATTERING.

AS THE CHARGED PROJECTILE TRAVERSES A MEDIUM, IT IONIZES THE MEDIUM. WE'LL FIND THE ENERGY LOSS DUE TO THIS PROCESS.

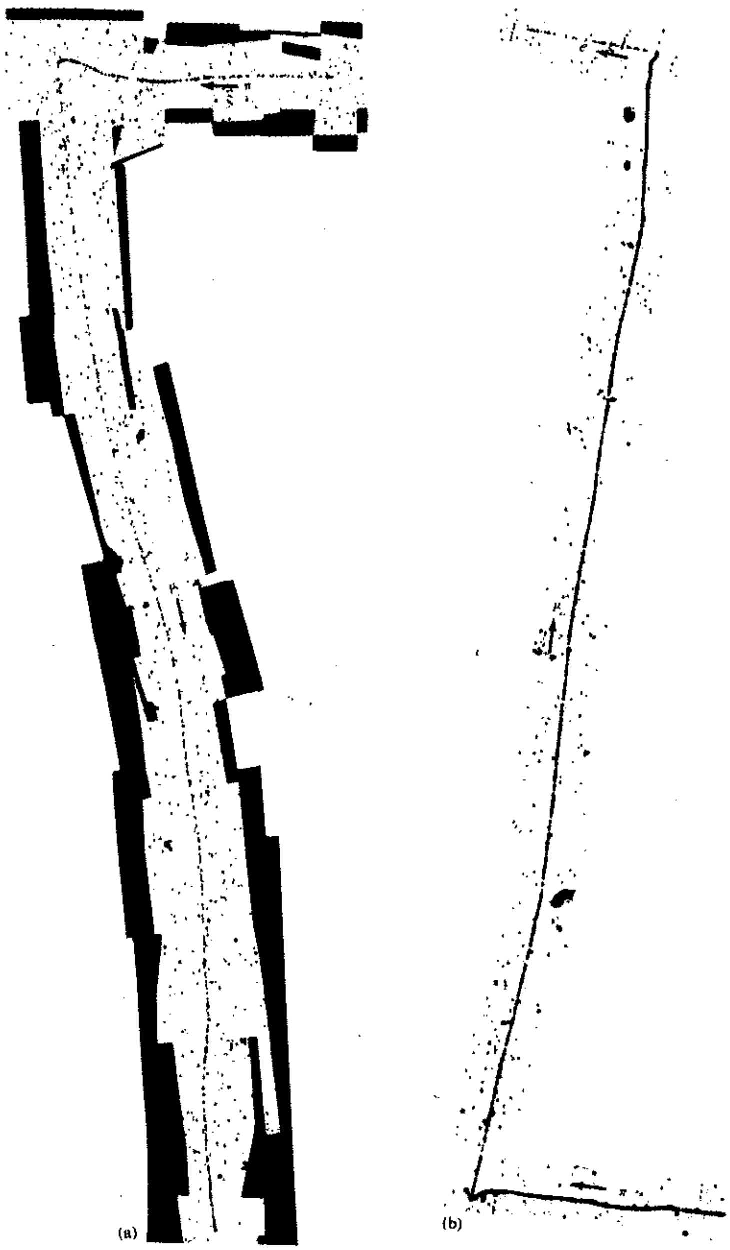


Figure 1-6 A pion from cosmic rays seen in a photograph emulsion. (a) First observation of the decay of a pion. [Lattes, Muirhead, Occhialini, and Powell, 1947.] (b) An early observation of the π–μ–e decay. The particles travel in the direction of increasing ionization. The range of the μ is 600 microns. [Courtesy Prof. C. F. Powell.]

TYPICALLY IONIZATION DOMINATES ENERGY LOSS (BUT NOT ALWAYS AT VERY HIGH AND VERY LOW ENERGIES).

SOMETIMES, ELECTRONS ARE DETACHED FROM ATOMS AS "DELTA RAYS". SOMETIMES ATOMS ARE EXCITED, BUT NOT IONIZED.

IN ANY CASE, ENERGY IS REMOVED FROM THE INCIDENT PARTICLE'S KINETIC ENERGY AND IT SLOWS DOWN.

THERE'S A CHARACTERISTIC ENERGY-LOSS PROFILE (A "BRAGG CURVE"):

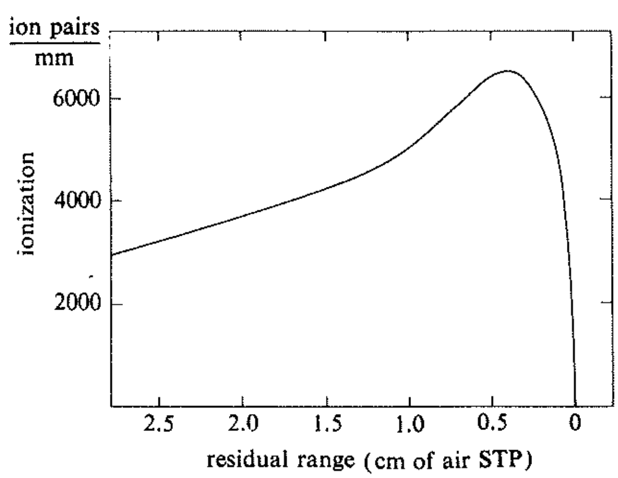
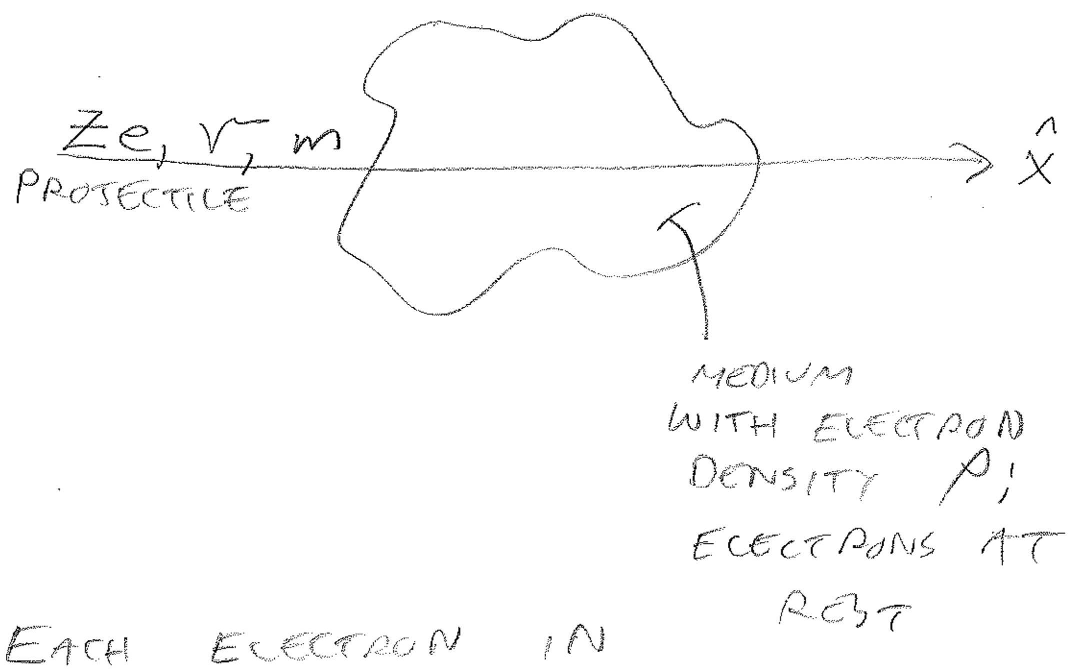


Figure 2-2 Bragg curve of an individual alpha particle. Ionization of an alpha particle, in ion pairs per millimeter, as a function of its residual range, according to experiments by M. G. Holloway and M. S. Livingston. [*Phys. Rev.*, **54**, 29 (1938).] In experiment $\rho_{air} = 1.184$ mg cm^{-3} (15°C, 760 mm Hg).

I THINK OF THE BRAGG CURVE AS THE ENERGY LOSS SINCE THE IONIZATION ENERGY IS APPROXIMATELY INDEPENDENT OF PROJECTILE ENERGY ABOVE SOME THRESHOLD.

J. & 13.1-2 ENERGY LOSS DUE TO IONIZATION



EACH ELECTRON IN THE MEDIUM EXERTS A FORCE ON THE PROJECTILE $Ze \cdot e/r^2$ (CGS).

ASSUME THE PROJECTILE IS SO ENERGETIC THAT ITS TRAJECTORY IS NOT AFFECTED: IMPULSE APPROXIMATION.

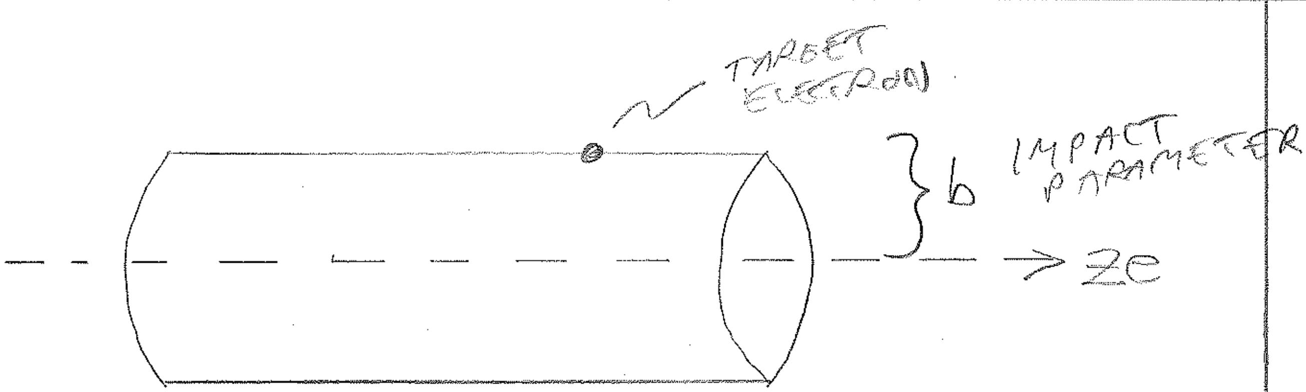
SUCH IMPULSIVE FORCES IMPLY THE FORCE LASTS SUCH A SHORT TIME THAT THE TARGET ELECTRONS PICK UP MOMENTUM WITHOUT MUCH CHANGING THEIR POSITION DURING THE SCATTER.

IT FOLLOWS THE SYSTEM IS "IN-OUT" SYMMETRIC: THE IMPULSE ON A SCATTERING ELECTRON IS PERPENDICULAR TO THE TRAJECTORY.

THE RESULTING IMPULSE IS

$$\Delta P_\perp = \int_{-\infty}^{+\infty} e E_\perp \, dt = \int_{-\infty}^{+\infty} e E_\perp \frac{dx}{v}$$

WE'LL EVALUATE THIS INTEGRAL VIA GAUSS'S LAW:



THE FLUX THROUGH THE CYLINDER IS

$$\iint \vec{E}\cdot\hat{n}\,da = 4\pi r^2 \frac{ze}{r^2}$$

= SAME AS SPHERE OF RADIUS r CENTERED ON PROJECTILE.

$$= \int_{-\infty}^{+\infty} E_\perp \, 2\pi b \, dx$$

$$\Delta P_\perp = \int_{-\infty}^{+\infty} e E_\perp \frac{dx}{v} = \frac{2 e z e}{b v};$$

THE ELECTRON'S KINETIC ENERGY IS

$$\frac{(\Delta P_\perp)^2}{2 m_e} = \frac{2}{m_e}\left(\frac{e z e}{b v}\right)^2$$

SINCE THERE ARE

$$\rho\, 2\pi\, b\, db\, dx$$

ELECTRONS IN A LENGTH dx OF THE CYLINDER OF SHELL b TO $b+db$, THE ENERGY LOSS PER LENGTH IS

$$-\frac{dE}{dx} = \rho\, 2\pi \int_{b_{MIN}}^{b_{MAX}} \frac{(\Delta p_\perp)^2}{2m_e}\, b\, db$$

$$= \rho\, 4\pi \frac{(eze)^2}{m_e v^2} \int_{b_{MIN}}^{b_{MAX}} \frac{db}{b}$$

THIS IS THE ENERGY LOSS PER LENGTH DUE TO IONIZATION (ALSO CALLED THE "STOPPING POWER" OF THE MEDIUM).

NOW WE NEED CARE IN EVALUATING THE LIMITS b_{MIN} AND b_{MAX}. FOR INSTANCE, $b_{MAX} \to \infty$ VIOLATES THE IMPULSE APPROXIMATION SINCE DISTANT COLLISIONS LAST A LONG TIME

SO, WHAT ARE SENSIBLE VALUES OF b_{MIN} & b_{MAX}?

- REGARDING b_{MAX}.
  RECALL BIGGER b_{MAX} LEADS TO LONGER DURATION.
  - TARGET ELECTRONS ARE BOUND IN ATOMIC ORBITS.
    RECALL THE ADIABATIC PRINCIPLE FROM QUANTUM MECHANICS: YOU CAN'T INDUCE A TRANSITION FROM ONE STATE TO ANOTHER VIA A TIME-DEPENDENT PERTURBATION WHEN THE CHANGE IN THE PERTURBATION IS SMALL OVER THE PERIOD τ OF THE SYSTEM.

    SO THEN, HOW MUCH TIME IS THE DURATION OF THE PERTURBATION?

    $\sim b/v$, SO

    $$b/v < \tau = \frac{1}{\langle f \rangle}$$

    WITH $\langle f \rangle$ SOME AVERAGE OF FREQUENCIES OF THE ATOM (SAY, ENERGY LEVELS).
    (J. EQN 13.8).

NOW FOLD IN RELATIVITY: THE SCATTERING DURATION IS SHORTENED BY γ:

$$b_{MAX} \approx \gamma \frac{v}{\langle f \rangle}$$

- REGARDING b_{MIN}.
  - IN AN ELASTIC COLLISION, YOU CAN'T CHANGE THE ELECTRON'S MOMENTUM BY MORE THAN $2mv$

  THIS IMPLIES (FROM $\Delta p_\perp = \frac{2Ze^2}{bv}$)

$$b_{MIN} = \frac{Ze^2}{mv^2} > \frac{Ze^2}{mc^2} = Zr_0$$

WITH r_0 THE CLASSICAL ELECTRON RADIUS (WE'VE SEEN THIS IN THOMSON SCATTERING)

$$r_0 \approx 10^{-13}\ cm.$$

(- ANOTHER LINE OF ATTACK: FROM QUANTUM MECHANICS, YOU CAN LOCALIZE THE ELECTRON TO ITS de BROGLIE WAVELENGTH AS SEEN BY THE PROJECTILE:

$$b_{MIN} > \frac{\hbar}{p} = \frac{\hbar}{\gamma m v}. \Bigg)$$

THESE LIMITS GIVE THE BETHE-BLOCH IONIZATION FORMULA

$$-\frac{dE}{dx} = 4\pi \frac{(ze)^2(e^2)}{mv^2} \rho \ln \frac{\gamma m v^2}{h\langle f \rangle}$$

(JACKSON EQN. 13, 14; 1^{ST} TERM).

WITH $h\langle f \rangle$ AN ENERGY, WHICH IS THE CHARACTERISTIC ENERGY OF IONIZATION IN THE MEDIUM. THERE'S AN INDUSTRY OF COMPUTING $h\langle f \rangle$. (N.B., I JUST LOOK IT UP; IT'S TYPICALLY WELL MEASURED.)

e.g., BLOCH (1933) SUGGESTED $h\langle f \rangle \equiv I = Bz$ VIA A THOMAS-FERMI ATOM MODEL.

IN A FAMOUS CALCULATION, BETHE ADDED A RELATIVISTIC CORRECTION

$$-\frac{dE}{dx} = 4\pi \frac{(ze)^2 (e)^2}{\gamma m v} \rho \left\{ \ln \frac{2\gamma m v^2}{I} - \beta^2 \right\}.$$

J. EQN. 13.14.

EXAMPLE. FOR LIQUID H_2

$I \approx 20$ eV.

AT VERY LOW PROJECTILE ENERGIES (SPEEDS COMPARABLE TO CLASSICAL ORBIT SPEEDS. e.g., FOR H_2 THIS SPEED $\sim c/137$), THE INCIDENT PARTICLE NEUTRALIZES ITSELF BY CAPTURING ELECTRONS FOR MORE AND MORE OF THE TIME, THERE'S THUS A RAPID FALLOFF OF THE BRAGG CURVE NEAR THE END OF A PARTICLE'S RANGE.

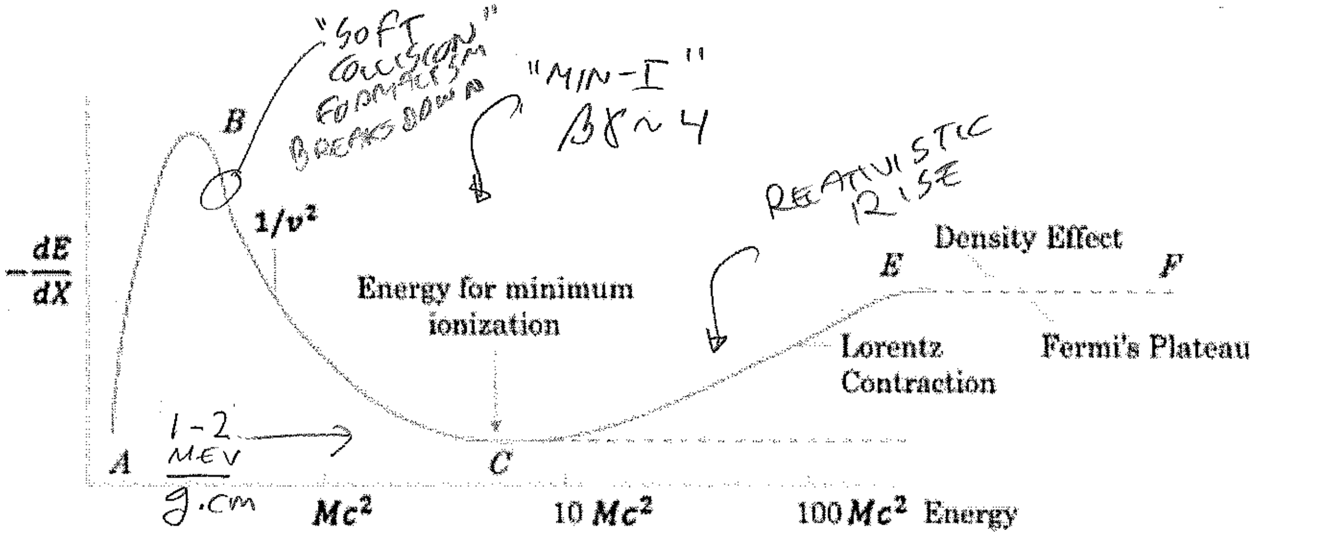
THERE ARE MANY IMPROVEMENTS TO THE IONIZATION FORMULA.

ON THE OTHER HAND: AT VERY HIGH INCIDENT ENERGIES, SAY $\gamma > 5$, IONIZATION ACTUALL INCREASES.

- THERE'S A RELATIVISTIC CONTRACTION/INCREASE OF THE TRANSVERSE COULOMB FIELD.
  - INCREASES b_{MAX} (γ)
  - DECREASES b_{MIN} $(1/\gamma)$
  - AS WE'LL SEE LATER, AT HIGH-ENOUGH SPEEDS, SIGNIFICANT AMOUNTS OF ENERGY IS RELEASED AS ČERENKOV RADIATION.

THE OVERALL EFFECT IS A SLOW RISE IN ENERGY LOSS BY IONIZATION UNTIL A PLATEAU NEAR $\gamma \approx 100$: THE $\frac{dE}{dx}$ NEAR THIS PLATEAU IS PERHAPS $1.2 - 1.4 \left.\frac{dE}{dx}\right|_{MIN.}$

GENERAL BEHAVIOR OF IONIZATION ENERGY LOSS:



NOTICE AT HIGH ENERGY "DENSITY EFFECT".
WE'LL RETURN TO THIS J.C. 13.3