Physics 515, Electrodynamics III
Department of Physics, University of Washington
Spring quarter 2020
April 17, 2020, 11am
On-line lecture

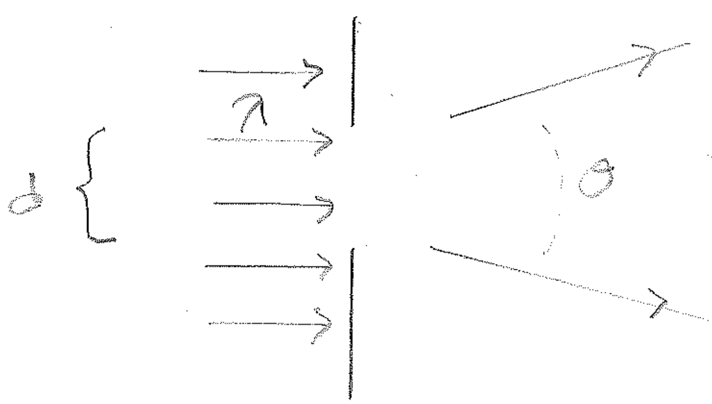
Administrative:
1. HW#2 due now.
2. Office hours Wednesdays after class at
URL: https://washington.zoom/us/j/712804010

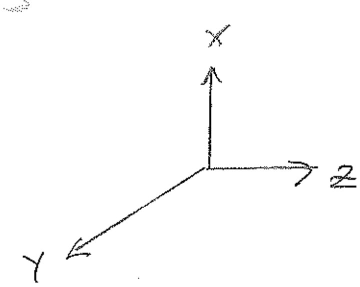
Lecture: **J. Chapter 10: Scattering & diffraction.**
1. J. Chapter 10.5: Scalar Diffraction theory.
a. Ignore the vector nature of light…so this methods speaks to amplitudes and intensities, not polarization, etc.
b. The only sources for the diffraction region are apertures, which are Huygens sources of outgoing spherical waves.
c. The waves distant from the aperture in the diffraction region are outgoing spherical waves.

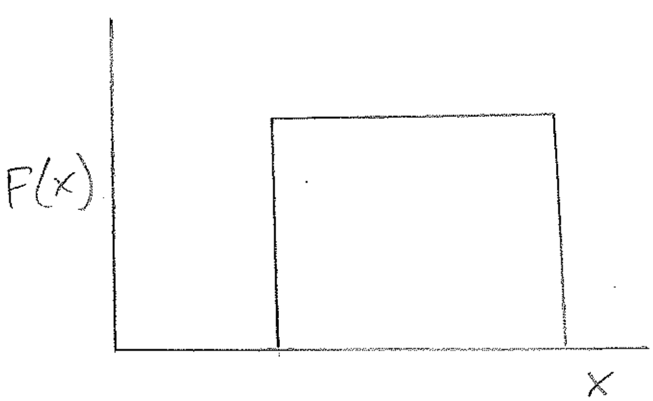
SCATTERING AND DIFFRACTION ARE CLOSELY RELATED.

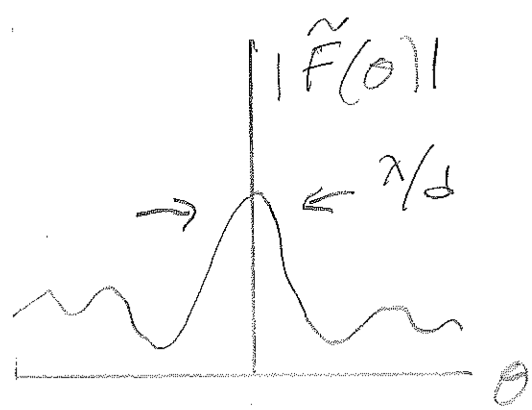
AT THE VERY LOWEST ORDER: WE HAVE "GEOMETRIC OPTICS" (THAT IS, RAY TRACING). THIS IS PRETTY GOOD IN MANY CIRCUMSTANCES. THIS WON'T DESCRIBE DIFFRACTION AROUND OBSTACLES.

A NEXT COMPLICATION IS DIFFRACTION. A TYPICAL GEOMETRY IS







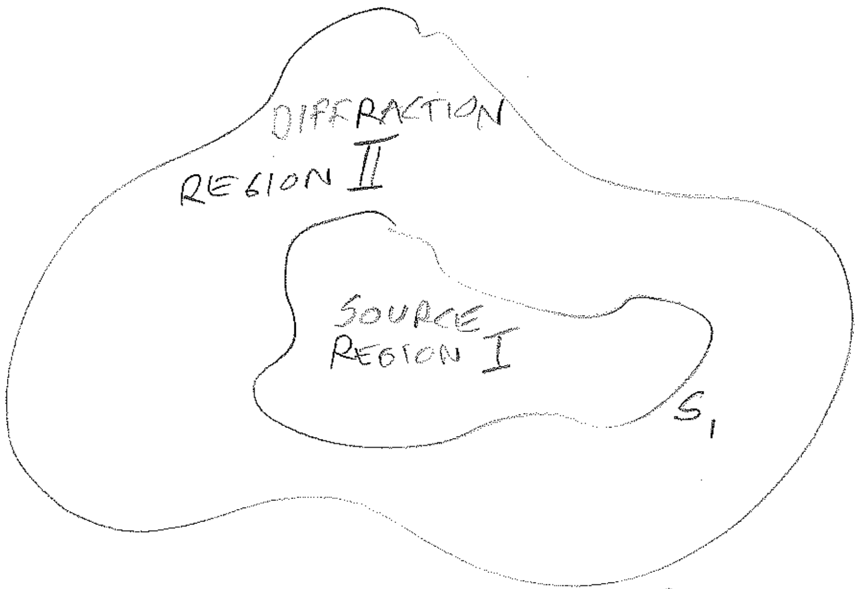


A FIRST LEVEL OF DIFFRACTION IS "SCALAR DIFFRACTION THEORY". IT SUPPOSES WAVES HAVE NO VECTOR NATURE, BUT DO POSSESS A PHASE.

I'M AMAZED THIS WORKS AT ALL, BUT JACKSON NOTES "... WORKS REMARKABLY WELL ...". THE MAIN TOOL IS THE KIRCHHOFF INTEGRAL (J. EQN 10.79).

THE "SOURCE REGION" AND "DIFFRACTION REGION ARE SEPARATED BY A "SCREEN"

S_I:



S_2 IS A SCREEN AT ∞

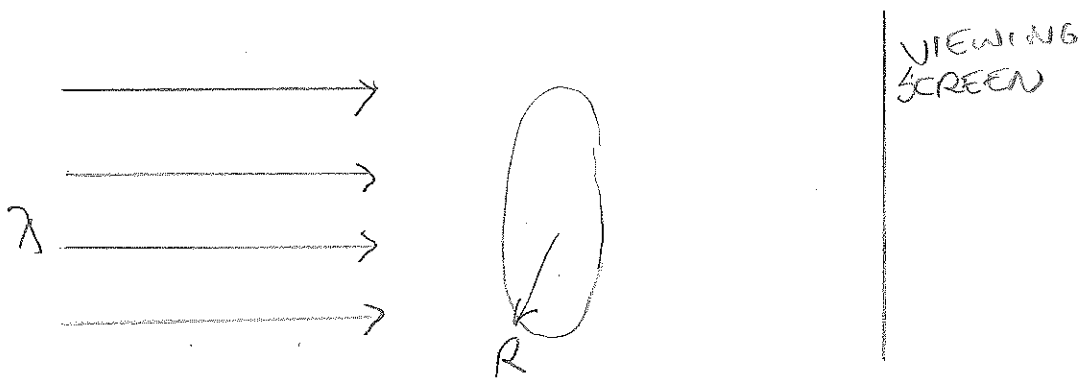
THERE ARE SOURCES IN I AND NO SOURCES IN II. WE SEEK THE FIELDS IN II GIVEN THE INTERACTIONS OF THE SOURCE-FIELDS IN I WITH S_I. THE ANGULAR DISTRIBUTION OF FIELDS IN II IS THE "DIFFRACTION PATTERN".

SCATTERING AND DIFFRACTION II:

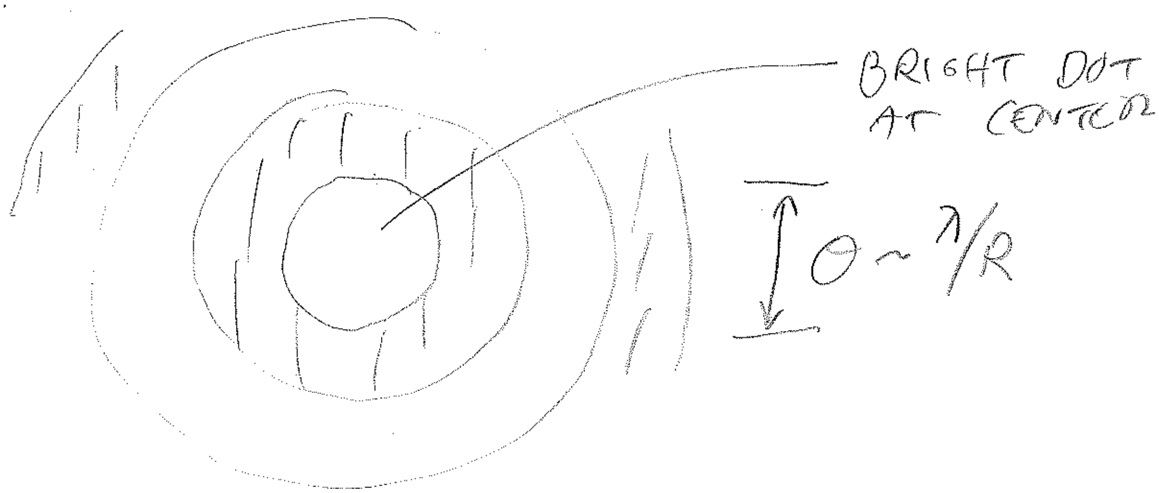
J. C. 10.5 SCALAR-DIFFRACTION THEORY

WHEN A PLANE WAVE ENCOUNTERS AN OBJECT, SOME PART OF THE WAVE "BENDS" (THAT IS, DIFFRACTS). THIS IS EASY TO OBSERVE.

e.g. ABSORBING DISK

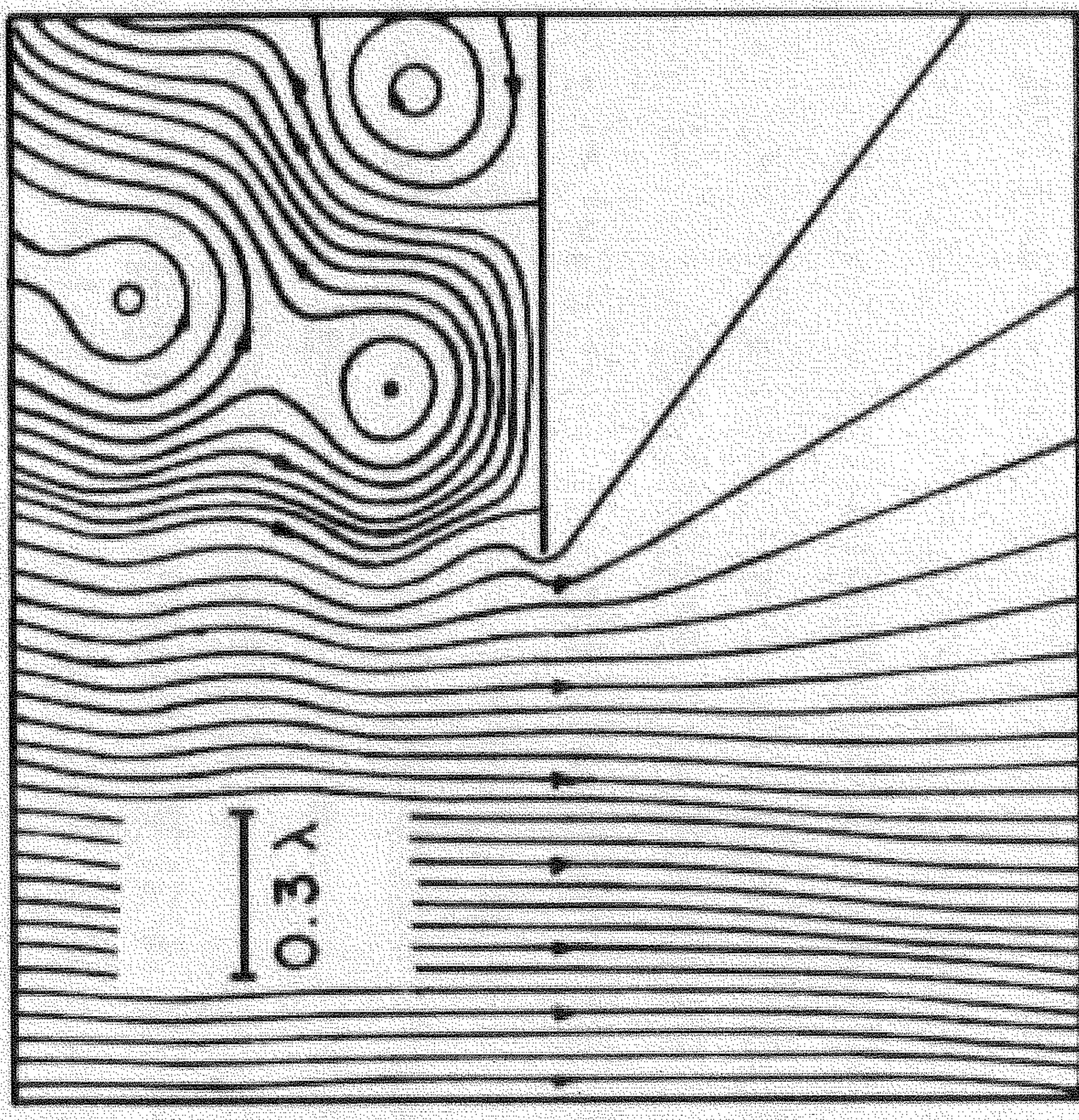


YOU SEE "FRAUNHOFER RINGS" ON THE VIEWING SCREEN.



THE "d" (APERTURE SIZE) $\gg \lambda$ IS THE REGIME OF "RAY TRACING". IT IS STILL WISE, IN THIS LIMIT, NOT TO LOOK TOO CLOSELY AT THE BOUNDARY OF A SHADOW.

FOR FUN, SEE BRAUNBEK & LAUKIEN, OPTIK 9 (1952) 174 FOR THE "FULL" CALCULATION OF A PLANE WAVE INCIDENT ON A PERFECTLY-CONDUCTING $\frac{1}{2}$- PLANE.



BRAUNBEK & LAUKIEN (1952)

WHAT'S HAPPENING IN DIFFRACTION IS THE INCIDENT WAVE INDUCES OSCILLATING CURRENTS ALONG THE EDGES OF THE SCREEN, AND THESE CURRENTS IN TURN RADIATE. THIS RE-RADIATION IN TURN INDUCES RE-RE-RADIATION, AD NAUSEUM. THIS IS A VERY HARD PROBLEM, IT'S SIMILAR IN CHARACTER TO FINDING THE CURRENT PROFILE IN AN ANTENNA.

ALL THE VARIOUS WAVES THEN INTERFERE. THIS, PLUS MAXWELL'S EQUATIONS, PLUS BOUNDARY CONDITIONS ON THE SCREEN, GIVES THE FIELDS.

EXACT SOLUTIONS OF DIFFRACTION ARE VERY DIFFICULT TO COMPUTE. I KNOW OF ONLY 3:

- CONDUCTING HALF-PLANE.
- CIRCULAR APERTURE IN CONDUCTING PLANE.
- SPHERICAL BODIES.

THAT'S THE RESULT OF 150 YEARS.

SO, WE EMPLOY APPROXIMATIONS, STARTING WITH "SCALAR DIFFRACTION THEORY". THIS IS SOMETIMES CALLED THE HUYGENS–FRESNEL PRINCIPLE: IN THE ORIGINAL FORMULATION FROM THE 1800'S, ALL POINTS IN THE "APERTURE PLANE" ARE CONSIDERED AS POINT SOURCES OF SPHERICAL WAVES, WHICH INTERFERE IN THE "DIFFRACTION REGION". THIS IS SOMETIMES CALLED "ABSORBING" OR "BLACK" SCREEN DIFFRACTION.

THIS SEEMED TO WORK WELL WHEN THE OBSERVATION POINT WAS MANY WAVELENGHTS FROM THE DIFFRACTING OBJECT.

IN THIS ORIGINAL FORMULATION,

$$dE = \frac{ik}{2\pi} E_0 \frac{e^{ikr}}{r} e^{-i\omega t} da$$

WE'LL SEE WITH NO SCREEN HOW THE FACTOR $\frac{ik}{2\pi}$ ARISES.

HERE'S THE JACKSON FORMULATION

COMPONENTS OF $\vec{E}$ AND $\vec{B}$ ARE NOT TREATED; YOU WORK WITH AN AMPLITUDE ψ,

$\psi(\vec{r}, t)$ WITH TIME DEPENDENCE OF $e^{i\omega t}$.

ψ SATISFIES A HELMHOLTZ EQUATION

$$(\nabla^2 + k^2)\psi = 0.$$

THEN EMPLOY GREEN'S FUNCTION TECHNIQUES TO FIND ψ IN TERMS OF $\psi|_s$ AND $\frac{\partial\psi}{\partial n}\Big|_s$.

$\psi\psi^*$ IS THE FIELD INTENSITY.

RECALL THE DEFINITION OF $G(\vec{r},\vec{r}\,')$:

$$(\nabla^2 + k^2)G(\vec{r}-\vec{r}\,') = -\delta(\vec{r}-\vec{r}\,')$$

WITH

$$\psi(\vec{r}) = \oint\!\!\oint_S \Big\{ \psi(\vec{r}\,')\, \hat{n}' \cdot \vec{\nabla}' G(\vec{r},\vec{r}\,') - G(\vec{r},\vec{r}\,')\, \hat{n}' \cdot \vec{\nabla}' \psi(\vec{r}\,') \Big\}\, da'$$

WITH $\hat{n}'$ DIRECTED "INWARD"
(IGNORE SOURCES IN THE DIFFRACTION REGION.)

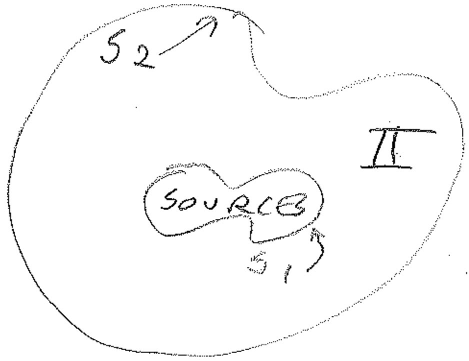
FROM HUYGENS-FRESNEL ASSUMPTIONS, THE WAVES FROM EACH BIT OF AREA ARE OUTGOING SPHERICAL WAVES:

$$G(\vec{r},\vec{r}\,') = \frac{1}{4\pi}\,\frac{e^{ik|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}$$

(LET'S CALL $\vec{R} = \vec{r} - \vec{r}\,'$.) SO,

$$\psi(\vec{r}) = -\frac{1}{4\pi}\oint\!\!\oint_S \frac{e^{ikR}}{R}\, \hat{n}' \cdot \Big\{ \vec{\nabla}'\psi(\vec{r}\,') + ik\Big(1 + \frac{i}{kR}\Big)\frac{\vec{R}}{R} \Big\}\, da'$$

FINALLY, WE CONSIDER THE "FIELD" IN REGION II



THE SURFACE OVER WHICH WE'LL EVALUATE THE SURFACE INTEGRAL IS $S_1 + S_2$. THE KEY POINT IS FIELDS FROM WITHIN S_1 (REGION I) "ESCAPE" INTO REGION II; THE PRESUMPTION IS, NEAR S_2, THE FIELDS ARE OUTGOING WAVES SATISFYING THE "RADIATION CONDITION"

$$\psi(\vec{r}) \sim \frac{e^{ikr}}{r}\,; \quad \frac{\partial\psi(\vec{r})}{\partial r} \sim \psi(\vec{r})\left\{ik - \frac{1}{r}\right\}$$

(J. EQNS. 10.78).

NOW WE CAN EVALUATE THE CONTRIBUTION OF S_2 TO $\oiint \ldots da$.

WE MIGHT AS WELL CHOOSE S_2 TO BE A SPHERE. PLUG IN THE "RADIATION CONDITION" $\psi(r)$ AND $d\psi(r)/dr$ INTO $\iint_{S_2} \ldots da$, WITH RESULT

- ik TERMS CANCEL.
- OTHER INTEGRAND TERMS $\sim 1/r^3$.

SO S_2 DOESN'T CONTRIBUTE TO $\oiint \ldots da$

- WE'RE LEFT WITH THE CONTRIBUTION OF S_1 TO $\oiint \ldots da$.

$$\psi(\vec{r}) = -\frac{1}{4\pi} \iint_{S_1} \frac{e^{ikR}}{R} \times \hat{n}' \cdot \left\{ \vec{\nabla}' \psi + ik\left(1 + \frac{i}{kR}\right) \frac{\vec{R}}{R} \psi \right\} da'$$

THIS IS KIRCHHOFF'S INTEGRAL FORMULA — NOTICE ONLY S_1, THE DIFFRACTING SCREEN CONTRIBUTES. (JACKSON EQN. 10.79.)

EVALUATING KIRCHHOFF'S INTEGRAL IS CHALLENGING. YOU NEED $\psi|_{S_I}$ OR $\partial\psi/\partial n|_{S_I}$, WHICH YOU GENERALLY DON'T KNOW.

HERE'S KIRCHHOFF'S SIMPLIFYING ASSUMPTIONS

- $\psi|_{S_I}$ $\left(\frac{\partial\psi}{\partial n}\Big|_{S_I}\right)$ VANISHES EVERYWHERE ON S_I EXCEPT AT THE APERTURES
- $\psi|_{S_I}$ $\left(\frac{\partial\psi}{\partial n}\Big|_{S_I}\right)$ ARE THOSE OF THE INCIDENT WAVE WITHOUT THE SCREEN.

Q: IN REGION II, IS THERE ANY DIFFERENCE BETWEEN AN ABSORBING AND REFLECTING SCREEN?

A: NO.

THIS FORMULATION IS FRAUGHT WITH CONCEPTUAL PROBLEMS: SEE J. P. 480 DISCUSSION.

MY OWN FEELING IS NONE OF THIS MAKES SENSE. BUT ALL CLASSICAL DIFFRACTIVE OPTICS IS BASED ON THIS, IT SEEMS. AND IT SEEMS TO WORK ON A BROAD CLASS OF PROBLEMS.

ONWARDS TO THE GREEN'S FUNCTION SOLUTIONS

START WITH DIRICHLET BOUNDARY CONDITIONS:

$\psi \big|_{S_I}$ IS SPECIFIED.

WE THEN SEEK $G(\vec{r}, \vec{r}\,')$ SATISFYING

$G(\vec{r}, \vec{r}\,')\big|_{S_I} = 0$ RESULTING IN

$$\psi(\vec{r}) = \iint_{S_I} \psi(\vec{r}\,') \frac{\partial}{\partial n} G(\vec{r}, \vec{r}\,')\, da'.$$

IN OUR APPROXIMATION $\psi\big|_{S_I} = 0$ EXCEPT AT APERTURES, AND ψ IS THAT OF THE INCIDENT WAVE AT APERTURES.

FOR NEUMANN BOUNDARY CONDITIONS

$\frac{\partial}{\partial n}\psi\Big|_{S_I}$ IS SPECIFIED.

WE THEN SEEK $G(\vec{r},\vec{r}')$ SATISFYING

$\frac{\partial}{\partial n}G(\vec{r},\vec{r}')\Big|_{S_I}=0$ RESULTING IN

$$\psi(\vec{r}')=\iint_{S_I} G(\vec{r},\vec{r}')\frac{\partial}{\partial n}\psi(\vec{r}')\,da'.$$

IN OUR APPROXIMATION $\frac{\partial}{\partial n}\psi\Big|_{S_I}=0$

EXCEPT AT APERTURES, AND

$\frac{\partial}{\partial n}\psi\Big|_{S_I}$ IS THAT OF THE INCIDENT

WAVE AT THE APERTURES.

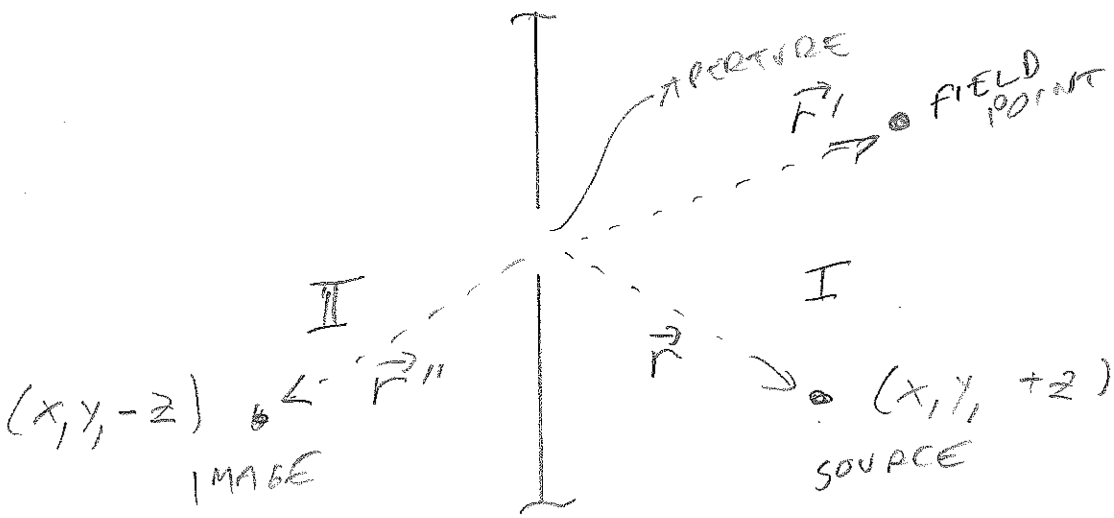
JACKSON ASSERTS (P. 480 #2) THAT THE GREEN'S FUNCTION AND THE APPROPRIATE GREEN'S FUNCTION INTEGRAL, PLUS ASSUMPTIONS

$$\psi \Big|_{S_I} = 0 \quad \text{EXCEPT AT APERTURES}$$

$$\left(\frac{\partial}{\partial n} \psi \Big|_{S_I} = 0 \quad \text{EXCEPT AT APERTURES} \right)$$

IS A SELF-CONSISTENT THEORY; THAT IS, THE PROBLEMS OF P. 480 ARE REMOVED.

EXAMPLE OF FINDING $G(\vec{r}, \vec{r}')$ FOR A CONDUCTING PLANE AT $z = 0$ WITH DIRICHLET BOUNDARY CONDITIONS



THIS IS A FAMILIAR IMAGE-CHARGE PROBLEM

$$G(\vec{r}, \vec{r}') = \frac{1}{4\pi} \frac{e^{ikR}}{R} - \frac{1}{4\pi} \frac{e^{ikR'}}{R'}$$

$$|\vec{R}| = |\vec{r} - \vec{r}'| = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{1/2}$$

$$|\vec{R}'| = |\vec{r} - \vec{r}''| = \left[(x-x')^2 + (y-y')^2 + (z+z')^2\right]^{1/2}$$

WITH $\psi(\vec{r}) = \iint_{S_I} \psi(\vec{r}') \frac{\partial}{\partial n} G(\vec{r}, \vec{r}')\, da$

DIRECTLY EVALUATED.

HENCE, $\psi(\vec{r}) = \iint_{S_I} \psi(\vec{r}\,') \frac{\partial}{\partial n} G(\vec{r}, \vec{r}\,')\, da'$

WITH $\frac{\partial}{\partial n} \to \hat{n} \cdot \vec{\nabla}$ BECOMES

$$\psi(\vec{r}) = \frac{ik}{2\pi} \iint_{S_I} \frac{e^{ikR}}{R} \left(1 + \frac{i}{kR}\right)$$

$$\times\ \hat{n}' \cdot \frac{\vec{R}}{r}\, \psi(\vec{r}\,')\, da'$$

(JACKSON EQN. 10.85)

AND IN THE FAR ZONE

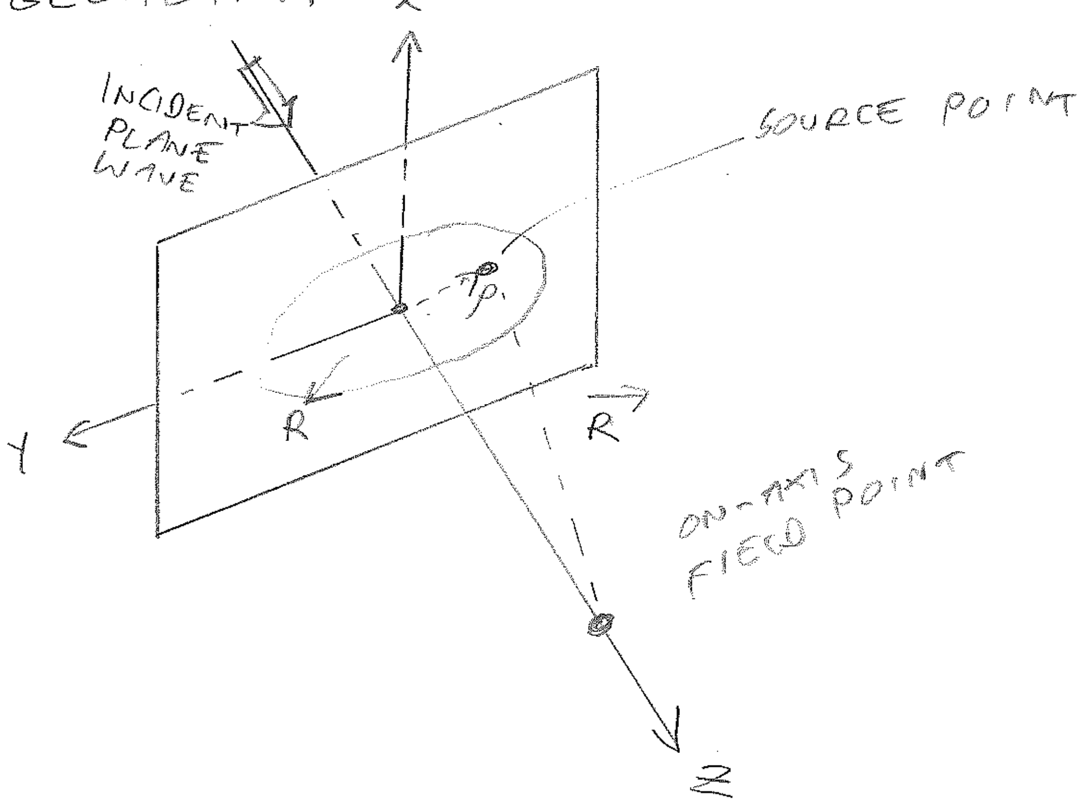
$$\left(1 + \frac{i}{kR}\right) \to 1$$

ASIDE FROM THOSE VERY FEW EXACT SOLUTIONS, IT'S SURPRISINGLY HARD TO IMPROVE THIS APPROXIMATION.

THIS IS THE ORIGINAL HUYGENS - FRESNEL FORMULATION.

EXAMPLE: CIRCULAR APERTURE IN AN ABSORBING ("BLACK") SCREEN.

GEOMETRY:



LOOK AT RADIATION-ZONE (FAR) FIELDS

$$E(z) = \frac{ik}{2\pi} \iint_{S_I} \frac{e^{ikR}}{R} \, \hat{n}' \cdot \frac{\vec{R}}{R} E(\vec{r}') \, da$$