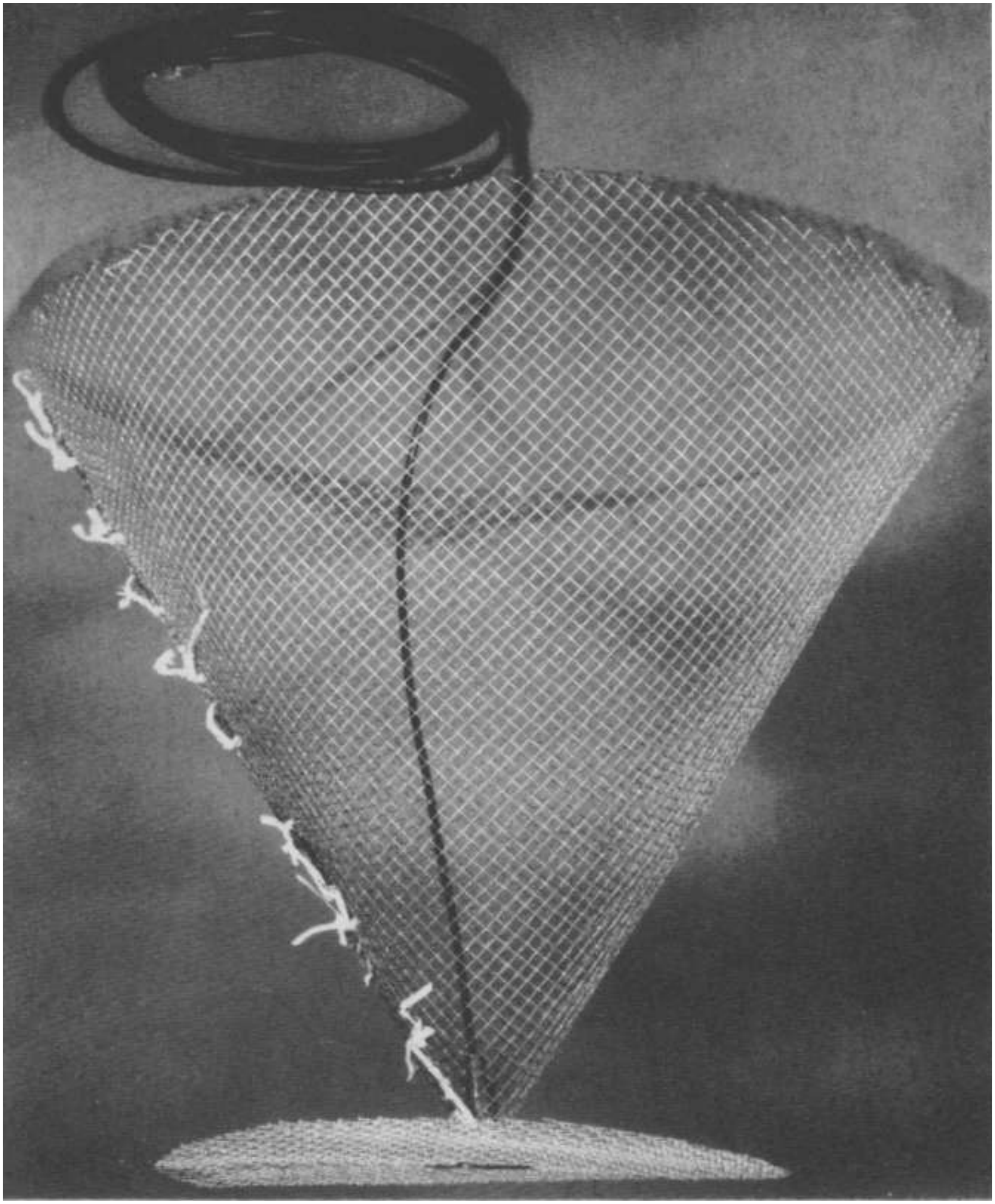
Physics 515, Electrodynamics III
Department of Physics, University of Washington
Spring quarter 2020
April 10, 2020, 11am
On-line lecture

Administrative

1. Homework 1 due now. (See submission details on homework assignment).
2. HW#2 posted today.

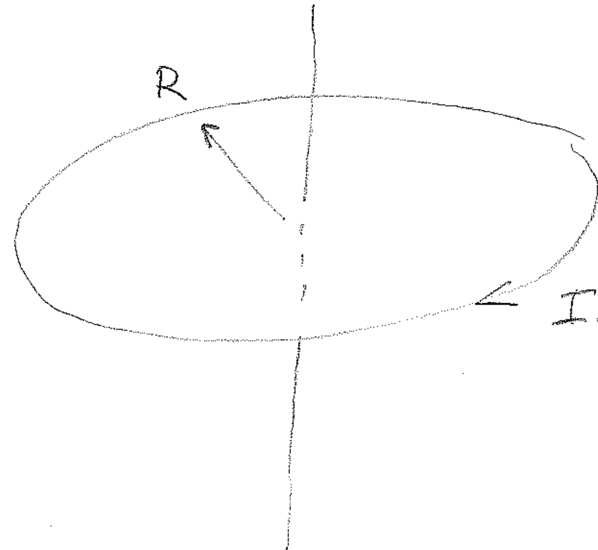
Lecture: **Close out radiating systems (antennas), start diffraction and scattering.**

1. Jackson problem 9.14. The infinitesimal magnetic dipole.
2. a. Apertures I: Excitation of waveguides and resonators. Jackson C. 9.5.
b. Recap Thompson & Rayleigh scattering: Gives the right answer, but…
c. Jackson C. 10.1 Diffraction & scattering in the long-wavelength limit. The multipole approximation.



“Discone” antenna (1978) for 420 MHz band.

CLOSE-OUT ANTENNAS WITH THE "INFINITESIMAL MAGNETIC DIPOLE"



$I(t) = I_\omega e^{i\omega t}$

INFINITESIMAL: $R \ll \lambda_0$.

WE HAVE SEVERAL TOOLS TO UNDERSTAND THIS, e.g.,

- FOLLOW THE PROCEDURE OF JACKSON §9.4 (CURRENTS SOURCE THE RETARDED POTENTIAL).
- POLARIZATION-VECTOR (SUPER POTENTIAL FORMALISM) OF JACKSON §6.13.
- DUALITY

LET'S TRY USING DUALITY.

AN INFINITESIMAL MAGNETIC DIPOLE OF LENGTH l AND CARRYING "MAGNETIC CURRENT" I_M, AND A SMALL ELECTRIC LOOP OF RADIUS R AND CURRENT I_ω ARE BOTH MAGNETIC DIPOLES.

RELATE THE TWO MAGNETIC DIPOLE MOMENTS

$$\frac{I_M}{i\omega} l = \pi R^2 I_\omega \times \mu_0$$

SINCE E & H ARE THE DUALS.

WE'RE DONE.

WE CAN, e.g., FIND THE RADIATION $\vec{E}$ AND $\vec{H}$ FIELDS.

RECALL THE RADIATION FIELDS FOR THE INFINITESIMAL ELECTRIC DIPOLE

$$E_\theta = i \frac{1}{c\epsilon_0} \frac{1}{4\pi} I \ell \frac{\omega}{c} \frac{1}{r} \sin\theta$$

$$H_\phi = i \frac{1}{4\pi} I \ell \frac{\omega}{c} \frac{1}{r} \sin\theta$$

DUALITY IS APPLIED BY

1. REPLACING I WITH I_m,
2. $\vec{H} \rightarrow -\vec{E}/Z_0$; $\vec{E} \rightarrow \vec{H} Z_0$,
3. REPLACE $I_m \ell$ WITH

$$i\omega \, \pi R^2 I_\omega \times \mu_0$$

THERE ARE RADIATION FIELDS IN E_ϕ AND H_θ.

WE CAN FIND, e.g., THE RADIATION RESISTANCE, R_{RAD}.

RECALL FOR THE INFINITESIMAL ELECTRIC DIPOLE

$$R_{RAD} \sim Z_0 \, \ell^2 .$$

TO HAVE R_{RAD} WITH UNITS OF OHMS, WE HAVE

$$R_{RAD} \sim Z_0 \left(\ell / \lambda_0 \right)^2 .$$

FOR THE MAGNETIC DIPOLE,

$$\ell \rightarrow R^2 \ (\text{RADIUS}^2),$$ SO

$$R_{RAD} \sim Z_0 \, R^4 .$$

TO HAVE R_{RAD} WITH UNITS OF OHMS, WE HAVE

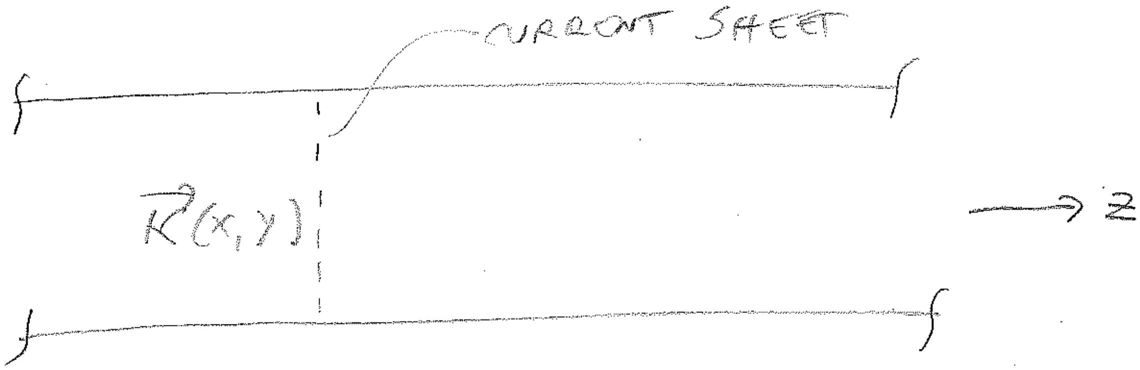
$$R_{RAD} \sim Z_0 \left(R / \lambda_0 \right)^4$$

NOTICE THE 4th POWER HERE.

APERTURES I.

HOW TO COUPLE WAVEGUIDES AND CAVITIES?

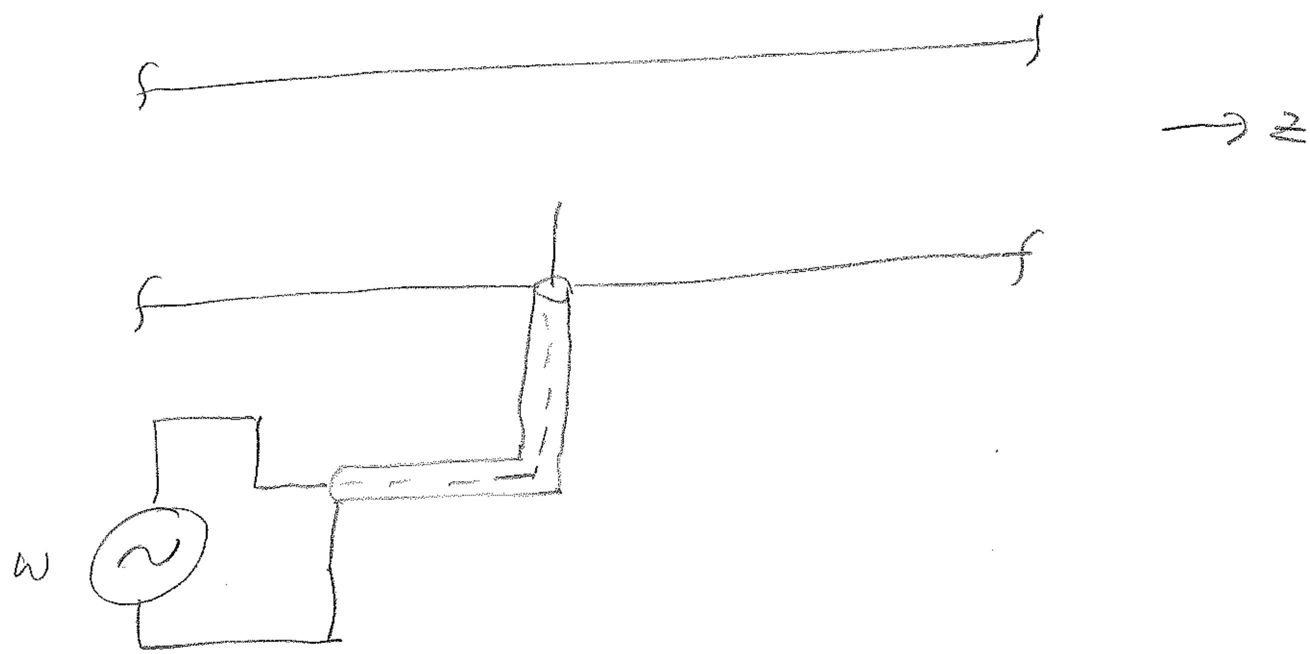
THE USUAL INTRODUCTION TO THIS TOPIC IS TO IMAGINE A WAVEGUIDE IN CROSS SECTION!



NOW PUT A CURRENT SHEET $\vec{K}(x,y)$ AT SOME z IN THE GUIDE. THIS IS ALMOST EQUIVALENT TO SPECIFYING $\vec{H}_{\parallel}$ AT THE SURFACE OF THE SHEET. AN APPROPRIATE CHOICE OF $\vec{K}$ CAN "LAUNCH" A MODE HAVING THAT $\vec{H}_{\parallel}$.

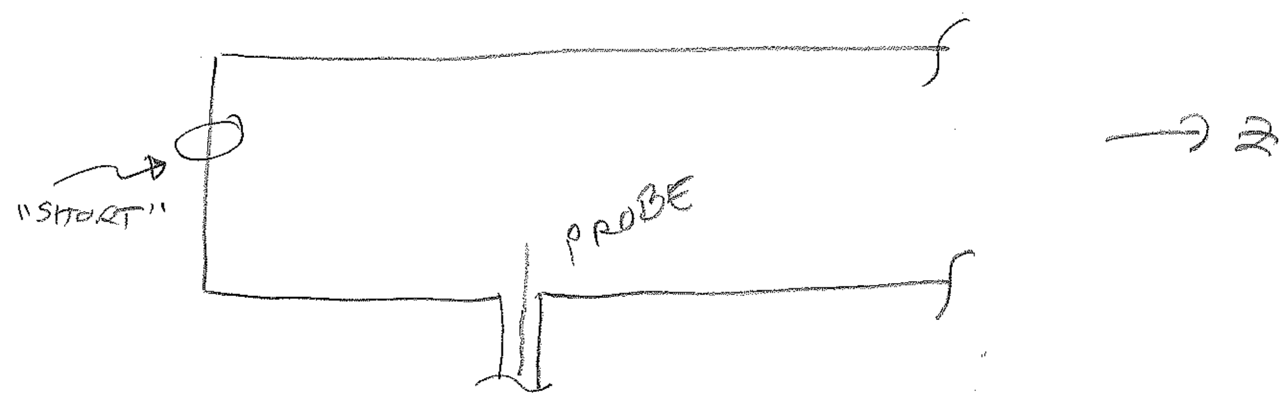
BUT IT'S NEARLY IMPOSSIBLE TO PRODUCE SUCH CURRENTS IN THE SHEET.

WE INSTEAD HAVE TO MAKE DO WITH CURRENTS THAT COUPLE TO LOTS OF MODES. e.g.,



IN PRACTICE, ω OF THE SOURCE IS BELOW THE WAVEGUIDE CUTOFF FOR ALL BUT THE LOWEST MODE. THE OTHER MODES ARE EVANASCENT MODES AND RETURN POWER BACK TO THE SOURCE. THESE OTHER MODES ARE A REACTANCE.

TO CANCEL THIS MODE-REACTANCE AS "SEEN" BY THE SOURCE (JACKSON PROBLEM 8.19)

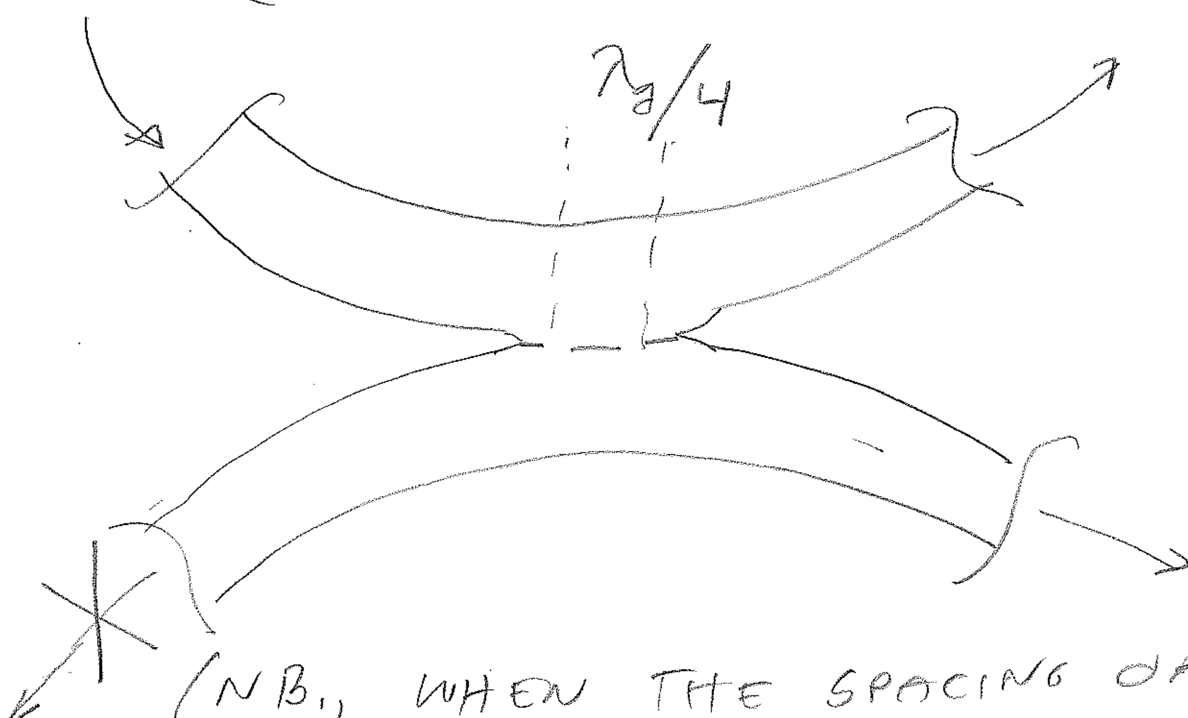


A "SHORT" (GROUND PLANE) IS PLACED AT A CERTAIN DISTANCE FROM THE PROBE. IN PRACTICE THE POSITION IS DETERMINED EMPIRICALLY.

ANOTHER WAY TO COUPLE TO WAVEGUIDES AND CAVITIES

APERTURES II JACKSON §9.5.

WAVEGUIDES, e.g., CAN BE COUPLED THROUGH SMALL APERTURES, e.g. THE "DIRECTIONAL COUPLER" ("SCHWINGER COUPLER").

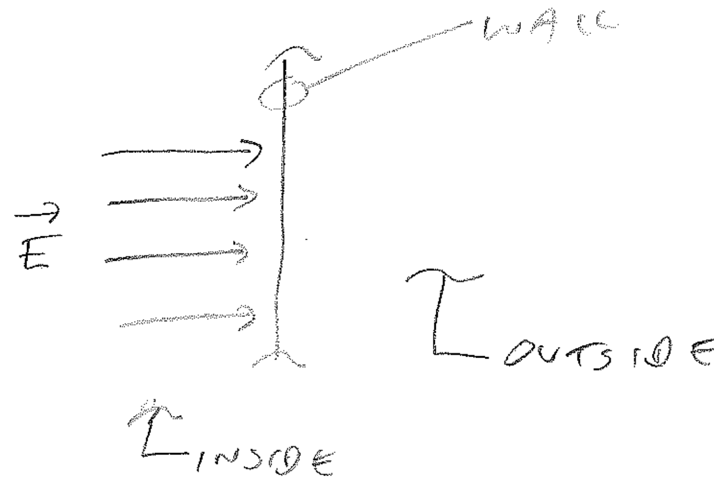


(NB., WHEN THE SPACING OF THE TWO HOLES IS $\lambda_g/4$, THIS HAS WONDERFUL PROPERTIES.)

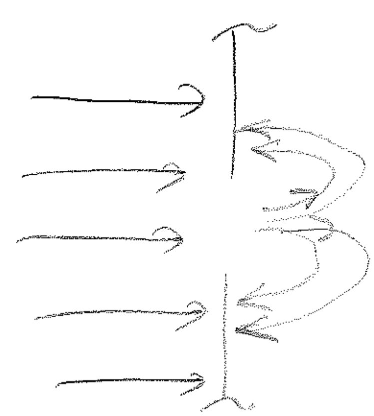
APERTURES IN CONDUCTING SHEETS ARE A KIND OF ANTENNA.

LET'S TRY TO UNDERSTAND THE RADIATION OUT OF A SMALL APERTURE IN A WAVEGUIDE.

1. LET'S START BY LOOKING AT THE ELECTRIC FIELD NEAR THE WAVEGUIDE WALL (CALLED A "SCREEN").

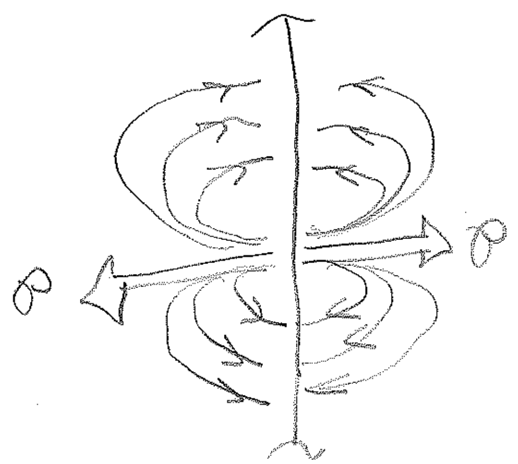


2. NOW INTRODUCE A SMALL APERTURE IN THE "SCREEN"



NOTICE THE FIELD LINES "FRINGE" IN AND AROUND THE APERTURE.

3. NOW REMOVE THE APERTURE AND CONSIDER THE ELECTRIC FIELD FROM TWO ELECTRIC DIPOLES



THE SIMILARITY OF (2) APERTURE AND (3) TWO DIPOLES SUGGESTS AN APERTURE EXCITED BY A "NORMAL" ELECTRIC FIELD CAN BE UNDERSTOOD AS NEARLY TWO ANTI-PARALLEL INFINITESIMAL ELECTRIC RADIATING DIPOLES NORMAL TO THE CONDUCTING WALL. THERE ARE DIPOLE EFFECTS INSIDE AND OUTSIDE.

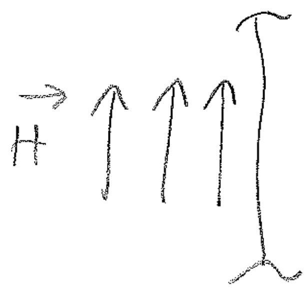
THE STRENGTH OF THE DIPOLE $\wp$ IS PROPORTIONAL TO THE NORMAL ELECTRIC FIELD $\vec{E}$ WITH

$$\wp = \varepsilon_0 \alpha_E E \, \delta(x-x_0)\, \delta(y-y_0);$$

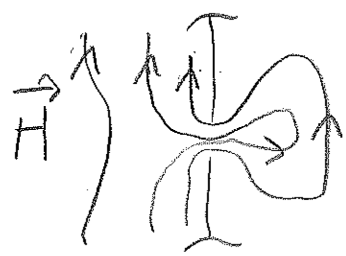
α_E THE "ELECTRIC POLARIZABILITY" OF THE APERTURE.

WE CAN DO THE SAME FOR $\vec{H}$.

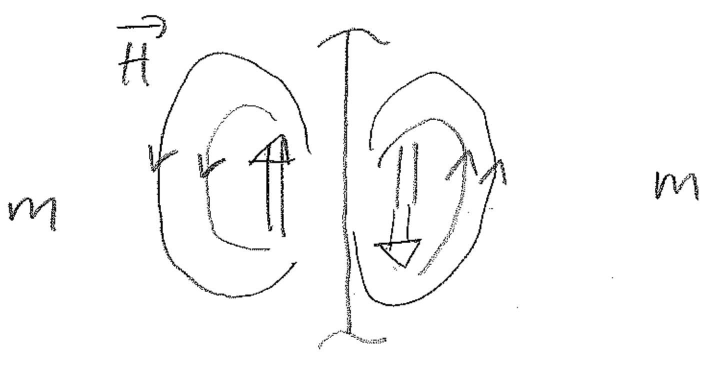
1. A PARALLEL $\vec{H}$ AT THE WALL WITH NO APERTURE



2. NOW INTRODUCE A SMALL APERTURE. $\vec{H}$ "FRINGES"



3. COMPARE (2) TO THE TWO MAGNETIC DIPOLES WITHOUT AN APERTURE



HERE, THE APERTURE CAN BE REPLACED BY TWO ANTI-PARALLEL MAGNETIC DIPOLES.

$$m = -\alpha_M H \delta(x-x_0)\delta(y-y_0).$$

THE STRENGTH OF THE DIPOLE m IS PROPORTIONAL TO THE PARALLEL $\vec{H}$ WITH

α_M THE MAGNETIC POLARIZABILITY OF THE APERTURE.

INTERESTINGLY, IF YOU COULD FIGURE α_E AND α_M, YOU COULD FIGURE OUT HOW MUCH POWER IS ESCAPING THROUGH THE HOLES IN YOUR MICROWAVE DOOR

WE DO KNOW α_E AND α_M.

FOR ROUND HOLES, YOU KNOW α_E AND α_M. BACK IN, e.g., JACKSON 3.183 YOU SAW IT FOR STATIC $\vec{p}$, JACKSON 5.131 YOU SAW IT FOR STATIC $\vec{m}$.

FOR ROUND HOLES

$$\alpha_E = \frac{2}{3} r_0^3, \quad \alpha_M = \frac{4}{3} r_0^3$$

(N.B., THE RESULTS EQUATIONS 3.183 AND 5.131 ARE TRICKY: AN EXAMPLE OF SOLVING LAPLACE'S EQUATION WITH MIXED BOUNDARY CONDITIONS).

(HUMMM.. JACKSON EQN 9.76 GIVES

$$\alpha_E = \frac{4}{3} r_0^3, \quad \alpha_M = \frac{8}{3} r_0^3 \quad \ldots)$$

FINAL COMMENT (FOR NOW):

THERE IS A CLOSE RELATION BETWEEN AN APERTURE IN A CONDUCTING SCREEN AND ITS "COMPLEMENT": THE COMPLEMENT IS A CONDUCTING SHEET REPLACING THE APERTURE AND AN OPENING REPLACING THE SCREEN.

THIS IS A VARIANT OF BABINET'S PRINCIPLE FROM OPTICS:

"WHEN THE FIELD BEHIND A SCREEN WITH AN OPENING IS ADDED TO ITS COMPLEMENT, THEIR SUM IS EQUAL TO THE FIELDS WITHOUT A SCREEN.

(CAVEAT: ASSUMES ABSORBING SCREEN.)

WE WILL RETURN TO APERTURES IN THE NEXT CHAPTER J.C. 9 SCATTERING AND DIFFRACTION.

SCATTERING AND DIFFRACTION I.

BEFORE J.C. 10, RECALL THOMSON SCATTERING FROM LAST QUARTER VIA "ELECTRON THEORY".

AN INCIDENT $\vec{E}$-FIELD POLARIZED ALONG $\hat{y}$ ON ELECTRON INDUCES MOTION

$$\ddot{y} + \gamma \dot{y} + \omega_0^2 y = \frac{eE}{m}$$

$$\text{WITH } y_0 = \frac{eE_0/m}{(\omega_0^2 - \omega^2) - i\gamma\omega}$$

THE EFFECT CAUSED POWER TO BE REMOVED FROM THE INCIDENT EM FIELDS AND BECOME RE-RADIATED AS DIPOLE RADIATION.

FOR FREE ELECTRONS (THOMSON SCATTERING), THE RATIO OF THE INCIDENT INTENSITY TO THE SCATTERED POWER LED TO

$$\sigma_T = \frac{8}{3}\pi (r_0)^2 ; \quad r_0 = \frac{e^2}{4\pi \varepsilon_0 m c^2}.$$

FOR BOUND ELECTRONS (RAYLEIGH SCATTERING

$$\sigma_R = \sigma_T \frac{\omega^4}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}$$

AND IN THE LONG-WAVELENGTH LIMIT $\lambda \gg \lambda_0$

$$\sigma_R \rightarrow \sigma_T \left(\frac{\omega}{\omega_0}\right)^4$$

THE $\sigma_R \sim \omega^4$ WAS FIRST STUDIED BY RAYLEIGH IN HIS INVESTIGATION OF THE BLUENESS OF THE SKY.

THERE ARE SUBTLETIES IN RAYLEIGH'S DERIVATION, IN PARTICULAR HIS (CORRECT) ASSUMPTION AIR MOLECULES SCATTER RANDOMLY, LEADING TO RANDOM SCATTERED INTENSITIES SIMPLY ADDING. THIS LEADS TO CONCEPTUAL DIFFICULTIES; WE'LL COME BACK TO IT. WE WANT TO KNOW WHAT'S REALLY GOING ON.

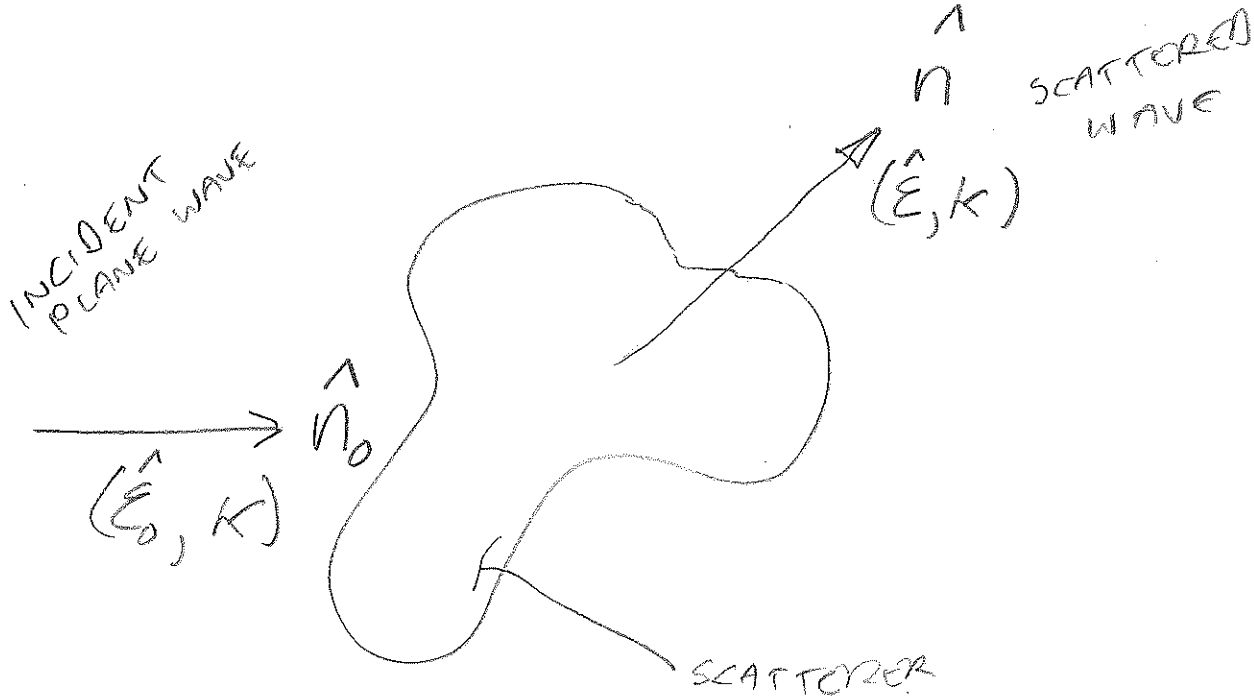
J. C. 10.1

DIFFRACTION IN THE LONG-WAVELENGTH LIMIT.

SIZE OF SCATTERER $\ll \lambda$.

BECAUSE THE SIZE OF THE SCATTER $\ll \lambda$, THE IDEA OF THIS SECTION IS TO THINK OF THE INCIDENT WAVE INDUCING ELECTRIC AND MAGNETIC MOMENTS THAT ARE COHERENT OVER THE SCATTERER; IN THE $\ll \lambda$ LIMIT, WE CAN USE THE TECHNIQUES OF STATICS TO EVALUATE THE MOMENTS.

HERE'S THE SCATTERING SITUATION



THE INCIDENT POLARIZED WAVE

$$\left.\begin{aligned}\vec{E}_0 &= \hat{\varepsilon}_0 E_0 e^{ik\hat{n}_0 \cdot \hat{r}} \\ \vec{H}_0 &= \frac{1}{Z_0} \hat{n}_0 \times \vec{E}\end{aligned}\right\} \times e^{i\omega t} \text{ AND } k = \omega/c$$