Physics 515, Electrodynamics III
Department of Physics, University of Washington
Spring quarter 2020
May 8, 2020, 11am
On-line lecture

Administrative:

1. No HW#5.

2. You should be getting your homework back; if not let me know.

Lecture:

J. Chapter 11: Covariant and relativistic electrodynamics.
J. Chapter 12: Dynamics of relativistic particles … .
J. Chapter 14: Radiation by moving charges.

1. J. C. 14.1: Liénard–Wiechert potentials I.

2. J. C. 14.3: Liénard–Wiechert fields and radiation I.

3. J. C. 11.9: Electromagnetic field-strength tensor and Maxwell's equations in covariant form.

4. J. C. 12.1-4 Lagrangian of particle in electromagnetic fields.

SUMMARY FROM LAST TIME

- RETARDED POTENTIALS

$$\Phi(\vec{r},t) = \iiint \frac{[\rho(\vec{r}',t)]_{RET}}{|\vec{r}-\vec{r}'|}\, dV'$$

$$A(\vec{r},t) = c^2 \iiint \frac{[\vec{J}(\vec{r}',t)]_{RET}}{|\vec{r}-\vec{r}'|}\, dV'$$

$$A^\mu = (\vec{A}, \Phi)$$ { JACKSON USES CONVENTION $A^\mu = (\Phi, \vec{A})$ EQN. 11.132. }

FOR A POINT CHARGE e THESE ARE INTEGRATED IN THE "PROPER" FRAME TO

$$A_0^{\ \mu} = \left(\vec{0}, \frac{e}{r_0}\right)$$

AND IN AN ARBITRARY FRAME

$$A^\mu = \left(\frac{e\vec{v}/c}{s}, \frac{e}{s}\right)$$

WITH s THE LIÉNARD-WIECHERT DENOMINATOR $s = r - \vec{r}\cdot\vec{v}/c$ WITH $\vec{r}$ THE DISPLACEMENT VECTOR FROM THE RETARDED SOURCE POSITION TO FIELD POINT.

IN COVARIANT FORM

$$A^\mu = e\frac{v^\mu}{v^\mu r_\mu}.$$

LIÉNARD - WIECHERT FIELDS.

IT IS DO-ABLE TO TAKE DERIVATIVES

$$\vec{E} = -\vec{\nabla}\Phi - \frac{1}{c}\frac{\partial}{\partial t}\vec{A}\,;\quad \vec{B} = \vec{\nabla}\times\vec{A}.$$

BUT WE ALREADY KNOW THE ANSWER. IN J. Ch. 6 WE FOUND THE RETARDED POTENTIALS (EQN 6.48). WE THEN SOLVED THE WAVE EQUATIONS FOR $\vec{E}$ AND $\vec{B}$, FINDING RETARDED SOLUTIONS (JEFIMENKO'S EQUATIONS J. EQN'S 6.55-56), THEN APPLYING A DELTA-FUNCTION SOURCE:

$$[\rho(\vec{r}\,';t)]_{RET} = e[\delta(\vec{r}\,' - \vec{r}_0(t))]_{RET}$$

$$[\vec{J}(\vec{r}\,';t)]_{RET} = \rho[\vec{v}(t)]_{RET},$$ GIVING

J. EQN'S 6.58-9. THESE ARE IN A MORE USABLE FORM IN J. EQN.'S 14.13-4, e.g.,

NO $\dot{\vec{v}}$, $\frac{1}{r^2}$

$$\vec{E}(\vec{r},t) = e\left[\frac{\hat{r} - \vec{v}/c}{\gamma^2(1-\hat{r}\cdot\vec{v}/c)^3}\frac{1}{r^2}\right]_{RET}$$

$$+ \frac{e}{c}\left[\frac{\hat{r}\times\{(\hat{r}-\vec{v}/c)\times\frac{d}{dt}\vec{v}/c\}}{(1-\hat{r}\cdot\vec{v}/c)^3}\frac{1}{r}\right]_{RET}$$

$\dot{\vec{v}}$, $\frac{1}{r}$

AND $\vec{B} = [\hat{r}\times\vec{E}]_{RET}.$

THE FIELDS SEPARATE INTO FIELDS INDEPENDENT OF $\dot{\vec{v}}$:

"VELOCITY FIELDS", "QUASI-STATIC FIELDS",

$\sim 1/r^2$

AND FIELDS LINEAR IN $\dot{\vec{v}}$

"ACCELERATION FIELDS", "RADIATION FIELDS",

$\sim 1/r$.

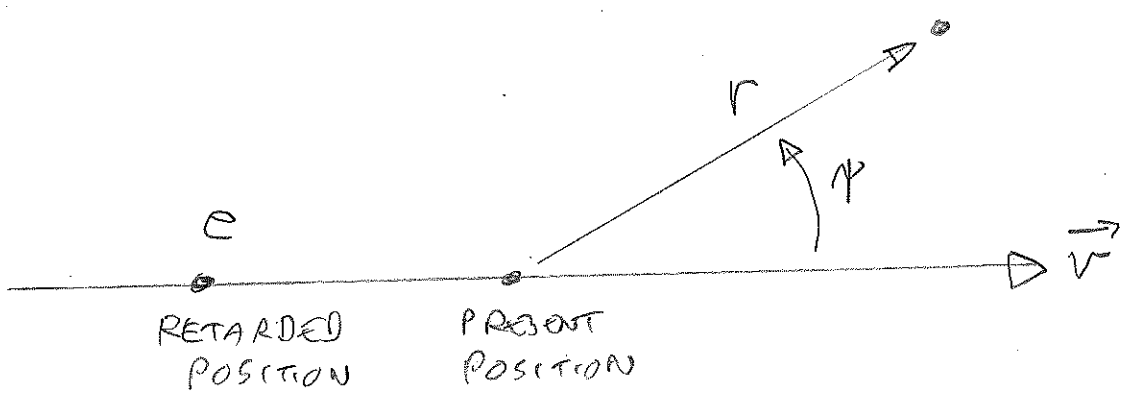
WE SAW THIS SEPARATION BEFORE IN J.6.6 JEFIMENKO EQUATIONS.

SOME COMMON APPLICATIONS OF LIÉNARD-WIECHERT POTENTIALS.

1. UNIFORM MOTION;
2. LINEAR TRAJECTORY: $\dot{\vec{v}} \sim \vec{v}$;
3. CIRCULAR TRAJECTORY: $\vec{v}$ IS CIRCULAR, $\dot{\vec{v}}$ AT RIGHT ANGLES.

AN ARBITRARY TRAJECTORY AT EACH POINT IS SOME COMBINATION OF THESE.

WE LOOKED AT CASE I: UNIFORM MOTION.



WE FOUND FIELDS

$$\vec{E} = \frac{e}{r^2}\frac{1}{\gamma^2}\frac{1}{(1-\beta^2 \sin^2\psi)^{3/2}}\hat{r}$$

REMARKABLE: $\vec{E}$ EMANATES FROM THE PRESENT POSITION; $\vec{E}$ REDUCED IN FORWARD AND BACKWARD DIRECTIONS BY γ^2; $\vec{E}$ INCREASED IN THE TRANSVERSE DIRECTION BY γ. A "PANCAKE".

ELECTROMAGNETIC FIELD-STRENGTH TENSOR $F^{\mu\nu}$: J. Eq. 11.9.

START WITH $A^{\mu} = (\vec{A}, \Phi)$

WE ARGUED THE SIMPLEST FORM OF A TENSOR INCORPORATING THE DEGREES-OF-FREEDOM IN $\vec{E}$ AND $\vec{B}$ IS

$$F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$$

{SOME THEORIES, e.g., YANG-MILLS THEORIES, ARE FORCED INTO A MORE COMPLICATED FORM OF $F^{\mu\nu}(A^{\mu}, A^{\nu})$; THE ADDITIONAL COMPLEXITY GIVES VERY INTERESTING NEW PHYSICS... ASK PROF. KARCH.}

IN A PARTICULAR FRAME WE HAVE

$\longrightarrow \nu$ (J. USES μ)

$$F_{\mu\nu} = \downarrow \mu \; (\text{J. } \nu) \begin{pmatrix} 0 & -B_z & +B_y & -E_x \\ +B_z & 0 & -B_x & -E_y \\ -B_y & +B_x & 0 & -E_z \\ +E_x & +E_y & +E_z & 0 \end{pmatrix}$$

$F^{\mu\nu} = \{ F_{\mu\nu}$ WITH $\vec{E} \to -\vec{E}$; J. EQN. 11.138 $\}$

WE FOUND $\partial_\mu F^{\mu\nu} = 4\pi J^\nu$ GIVES

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho$$

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial}{\partial t}\vec{E}.$$

BUT EXPRESSIONS LIKE $\vec{\nabla}\cdot\vec{B}$ AND $\vec{\nabla}\times\vec{E}$ HAVE OPPOSITE PARITY TO $\vec{\nabla}\cdot\vec{E}$ AND $\vec{\nabla}\times\vec{B}$; SO $\vec{\nabla}\cdot\vec{B}$ AND $\vec{\nabla}\times\vec{E}$ CAN'T COME FROM $\partial_\mu F^{\mu\nu}$

THE DUAL OF $F^{\mu\nu}$: $\tilde{F}^{\mu\nu}$, $\mathcal{F}^{\mu\nu}$, $G^{\mu\nu}$

(JACKSON USES $\mathcal{F}^{\mu\nu}$ EQN 14.140)

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

$$\tilde{F}^{\mu\nu} = \left\{ F^{\mu\nu} \text{ WITH } \vec{E}\to\vec{B},\ \vec{B}\to-\vec{E},\ \text{SEE COMMENT BELOW EQN 11.140.} \right\}$$

ONWARDS TO THE OTHER 2 MAXWELL EQUATIONS: $\vec{\nabla}\cdot\vec{B}$ AND $\vec{\nabla}\times\vec{E}$.

FORMULATING THEM IS MORE COMPLICATED.

THE USUAL PATH FORWARD IS TO START WITH THE "BIANCHI IDENTITY" FOR ELECTRODYNAMICS:

$$\partial_\gamma F_{\alpha\beta} + \partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} = 0.$$

THIS REPRESENTS A RANK 3 TENSOR ($4\times4\times4$ EQUATIONS).

EXERCISE: SHOW THAT THE LEFT SIDE VANISHES UNLESS $\alpha \neq \beta \neq \gamma$ (NO TWO INDICES ALIKE).

E.g., $\alpha = \beta = 1$; $\gamma = 4$.

$$\underbrace{\partial_4 F_{11}}_{\substack{\to 0 \\ \text{ANTI-SYMMETRY}}} + \underbrace{\partial_1 F_{14} + \partial_1 F_{41}}_{\substack{\to 0 \\ \text{ANTI-SYMMETRY}}} \to 0$$

NOTICE PERMUTATION OF INDICES GIVES THE SAME EQUATION. HENCE ONLY 4 OF THE 64 EQUATIONS ARE INDEPENDENT AND NON-TRIVIAL:

$$\begin{aligned} \alpha &= 1,\ 2,\ 3,\ 4 \\ \beta &= 2,\ 3,\ 4,\ 1 \\ \gamma &= 3,\ 4,\ 1,\ 2 \end{aligned}$$

WE MAY BE ON TO SOMETHING:
CONSIDER ONE OF THE NON-TRIVIAL EQUATIONS:

$$\partial_3 F_{12} + \partial_1 F_{23} + \partial_2 F_{31} = 0.$$

THE SET OF F's ABOVE ARE THE MAGNETIC FIELD COMPONENTS IN $F^{\mu\nu}$.

$$\frac{\partial B_3}{\partial z} + \frac{\partial B_1}{\partial x} + \frac{\partial B_2}{\partial y} = 0 ; \quad \vec{\nabla} \cdot \vec{B} = 0.$$

SIMILARLY, SET $\alpha = 1$, $\beta = 2$; $\gamma = 4$

$$\partial_4 F_{12} + \partial_1 F_{24} + \partial_2 F_{41} = 0$$

$$\frac{\partial B_z}{\partial ct} + \frac{\partial E_y}{\partial x} + \frac{E_x}{\partial y} = 0$$

$$\left[\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial}{\partial t}\vec{B} \right]_x = 0$$

THE THREE EQUATIONS YIELD

$$\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial}{\partial t} = 0.$$

HOW DOES THE DUAL TENSOR $\tilde{F}^{\mu\nu}$ COME IN?

LOOK AT $\partial^{\nu} \tilde{F}_{\mu\nu} = 0$

LET'S CHOOSE $\mu = 1$:

$$\partial^1 F_{11} + \partial^2 F_{12} + \partial^3 F_{13} + \partial^4 F_{14} = 0$$

$$0 \quad -\frac{\partial E_z}{\partial y} + \frac{\partial E_y}{\partial z} + \frac{\partial B_x}{\partial ct} = 0$$

$$\left[\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial}{\partial t}\vec{B}\right]_x = 0$$

$$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial}{\partial t}\vec{B} = 0.$$

SO $\partial^{\nu} \tilde{F}_{\mu\nu} = 0$ CONTAINS THE SAME EQUATIONS AS

$$\partial_\gamma F_{\alpha\beta} + \partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} = 0,$$

OUR HOMOGENEOUS MAXWELL EQUATIONS ARE EMBEDDED IN THE FORM OF

$$\partial^{\nu} \tilde{F}_{\mu\nu} = 0.$$

THIS IS NOT HUGELY SURPRISING.
WE SAW $\partial^\nu F_{\mu\nu} = 4\pi J_{E\,\mu}$

EMBEDDED MAXWELL EQUATIONS

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho_E$$

$$\vec{\nabla}\times\vec{B} - \frac{1}{c}\frac{\partial}{\partial t}\vec{E} = \frac{4\pi}{c}\vec{J}_E.$$

IF WE APPLY DUALITY TO IT,

$$\partial^\nu \tilde{F}_{\mu\nu} = 4\pi J_{M\,\mu}$$

EMBEDS MAXWELL EQUATIONS

$$\vec{\nabla}\cdot\vec{B} = 4\pi\rho_M \quad \{=0\}$$

$$\vec{\nabla}\times\vec{E} + \frac{1}{c}\frac{\partial}{\partial t}\vec{B} = \frac{4\pi}{c}\vec{J}_M \quad \{=0\}.$$

THE USUAL COVARIANT FORMULATION OF MAXWELL'S EQUATIONS ARE

$$\partial^\nu F_{\mu\nu} = 4\pi J_\mu;$$

$$\partial^\nu \tilde{F}_{\mu\nu} = 0.$$

WHERE WE CHOOSE $J_M{}^\mu = 0$ IN CHOSING THE FIELD OF A STATIC e^- TO BE PURELY ELECTRIC,

INVARIANTS

$$F_{\mu\nu} F^{\mu\nu} = -2\left(E^2 - B^2\right) \quad (\text{CGS})$$

DUALITY GIVES

$$\tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} = -2\left(B^2 - E^2\right) \quad (\text{CGS})$$

$$F_{\mu\nu} \tilde{F}^{\mu\nu} = -4\left(\vec{E} \cdot \vec{B}\right) \quad (\text{CGS})$$

TRANSFORMATION OF THE FIELDS VIA "BRUTE FORCE"

$$F'^{\mu\nu} = \Lambda^{\alpha}_{\mu}(\gamma)\Lambda^{\beta}_{\nu}(\gamma)F^{\alpha\beta}$$

LEADS TO

$$E'_{\parallel} = E_{\parallel}; \qquad B'_{\parallel} = B_{\parallel}$$

Q: IS THIS SENSIBLE?

$$\vec{E}'_{\perp} = \gamma(\vec{E}_{\perp} + \vec{\beta}\times\vec{B}_{\perp})$$
$$\vec{B}'_{\perp} = \gamma(\vec{B}_{\perp} - \vec{\beta}\times\vec{E}_{\perp})$$

(JACKSON EQ.N.S 11.149 ARE THESE FIELDS IN AN UNUSUAL FORM; IT'S MORE USEFUL TO USE JACKSON EQ.N.S 11.148.)

EXAMPLE: LORENTZ TRANSFORM STATIC CHARGE TO MOVING FRAME, JACKSON PP. 559–561

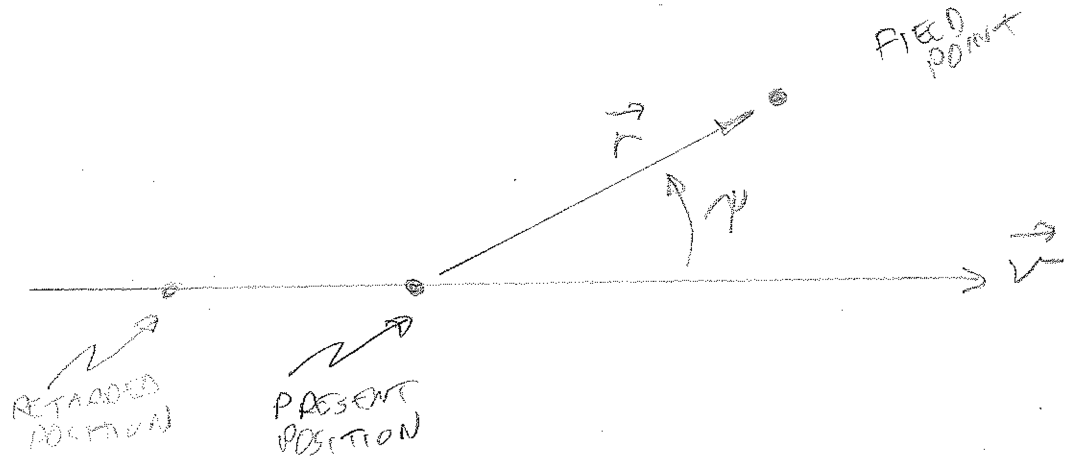
WE'LL OBVIOUSLY RECOVER THE LIÉNARD-WIECHERT FIELDS FOR A CHARGE IN UNIFORM MOTION.

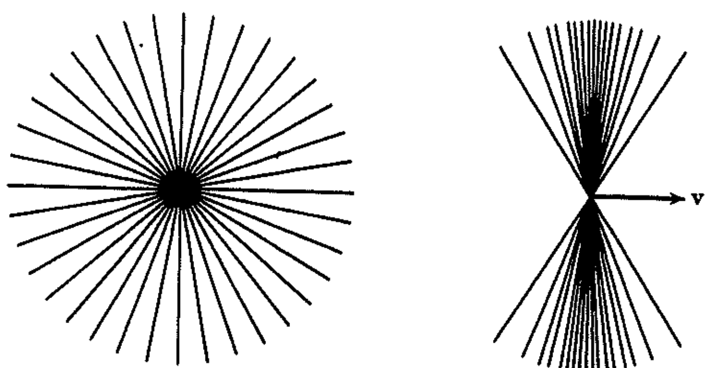
ET VOILA:

$$\vec{E} = \frac{e}{r^2} \frac{1}{\gamma^2 \left(1-\beta^2 \sin^2 \psi\right)^{3/2}} \hat{r}$$

(JACKSON EQN 11.154)

WITH FAMILIAR GEOMETRY





(*b*)

Figure 11.9 Fields of a uniformly moving charged particle. (*a*) Fields at the observation point *P* in Fig. 11.8 as a function of time. (*b*) Lines of electric force for a particle at rest and in motion ($\gamma = 3$). The field lines emanate from the *present* position of the charge.

ACTUALLY, NONE OF THIS IS SURPRISING. E.G., FROM OTHER CONSIDERATIONS:

- A PARTICLE MOVING IN ELECTRIC AND MAGNETIC FIELDS FEELS AN EFFECTIVE ELECTRIC FIELD $\vec{E}_{EFF}$

$$E_{EFF} = \vec{E} + \frac{1}{c}\vec{v} \times \vec{B}$$

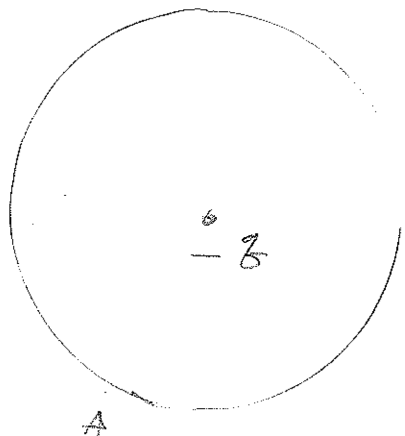
- A WIRE LOOP MOVING THROUGH AN ELECTRIC FIELD FEELS AN EFFECTIVE MAGNETIC FIELD

$$\vec{B}_{EFF} = \vec{B} + \frac{1}{c}\vec{V} \times \vec{E}$$

IN EXPERIMENTAL PHYSICS, IT'S SOMETIMES DESIREABLE TO TRANSFORM AWAY ELECTRIC OR MAGNETIC FIELDS, BUT IT'S NOT ALWAYS POSSIBLE.

CLASSIC PROBLEM: SUPPOSE $\vec{E}$ AND $\vec{B}$ ARE PERPENDICULAR IN SOME FRAME. THERE EXISTS A FRAME WHERE $E=0$ OR $B=0$, DEPENDING ON WHICH IS SMALLER. (IF $|\vec{E}|=|\vec{B}|$ THERE IS NO SUCH FRAME) THIS IS VERY IMPORTANT TO THE "$g-2$" EXPERIMENT.

A CASE WHERE SPECIAL RELATIVITY FAILS



OUTSIDE
$E = B = 0.$

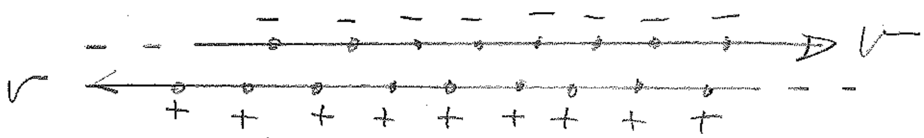
SHELL OF UNIFORM SURFACE-CHARGE DENSITY. TOTAL SURFACE CHARGE IS $+q$.

NOW, ROTATE THE SHELL:
OUTSIDE $B \neq 0$.
THIS ISN'T EMBEDDED IN THE LORENTZ TRANSFORMATION OF $F^{\mu\nu}$

Q: WHY IS THIS OBVIOUS?

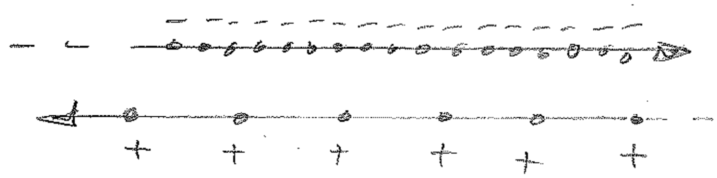
THOUGHT PROBLEM

- CASE 1: COUNTER-PROPAGATING LINE CHARGES:



E OUTSIDE = 0.

- CASE 2: BOOST CASE I TO THE REST FRAME OF ONE OF THE LINE CHARGES:



E OUTSIDE ≠ 0.

- CASE 3: USUAL CURRENT-CARRYING CONDUCTING WIRE: e^- ARE MOBILE, POSITIVE IONS AT REST.

WHY IS $E_{OUTSIDE} = 0$?

LAGRANGIAN FORMULATION OF ELECTRODYNAMICS I

J. C. 12.

THIS HAS ROOTS IN FERMAT'S PRINCIPLE OF LEAST TIME ($\sim$ 1600's).

LAGRANGIAN FORMULATION FOR A PARTICLE:

T = KINETIC ENERGY (NON-REL.)
$= \frac{1}{2} m [\dot{x}(t)]^2$;

V = POTENTIAL ENERGY = $V(x(t))$;

L = LAGRANGIAN = $T - V$

A = ACTION = $\int_0^T L\,dt$ (SOME CALL IT S)

HAMILTON'S PRINCIPLE OF LEAST ACTION

$$\frac{\delta A}{\delta x(t)} \rightarrow \frac{\delta L}{\delta x(t)} - \frac{d}{dt}\frac{\delta L}{\delta \dot{x}(t)} = 0$$

(EULER - LAGRANGE EQUATION)

THE LAGRANGIAN IS RELATED TO THE LAGRANGIAN DENSITY $\mathcal{L}$ IN 1-DIM BY

$$L = \int_x \mathcal{L}\, dx \rightarrow A = \iint_{x\,t} \mathcal{L}\, dt\, dx$$

AS YOU KNOW, THE COORDINATO CAN BE "GENERALIZED": $L = L(q_i, \dot{q}_i)$ WITH

$$\frac{dL}{dt} = \sum_i \frac{\partial L}{\partial q_i}\dot{q}_i + \sum_i \frac{\partial L}{\partial \dot{q}_i}\ddot{q}_i$$

WITH "CANONICAL MOMENTUM"

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

AND $\frac{dL}{dt}$ CAN BE WRITTEN (USING THE EULER-LAGRANGE EQUATIONS)

$$\frac{d}{dt}\left(p_i \dot{q}_i - L\right) = 0$$

EXAMPLE: RELATIVISTIC FREE PARTICLE

COMMENTS: WE'LL INCLUDE A POTENTIAL LATER.

: THE ACTION IS AN INVARIANT. SEE, e.g., LANDAU & LIFSCHITZ, "MECHANICS": A LORENTZ TRANSFORMATION ON L ADDS A TOTAL DERIVATIVE, WHICH ADDS A CONSTANT TO THE ACTION. .... etc.

WE HAVE

$$A = \int_{t_1}^{t_2} L(dt) = \int_{\tau_1}^{\tau_2} L(\gamma d\tau)$$

SINCE $d\tau$ IS AN INVARIANT, IT'S γL THAT'S LORENTZ INVARIANT. HENCE $L \sim 1/\gamma \sim m_0c^2/\gamma$ SINCE m_0c^2 IS THE ONLY ENERGY-RELATED INVARIANT. (ACTUALLY, IF YOU'D LIKE L TO REDUCE TO THE NON-RELATIVISTIC RESULT, $L = -m_0c^2/\gamma$.)

EXERCISE: SHOW THE FREE-PARTICLE L IS SENSIBLE IN THE NON-RELATIVISTIC LIMIT.

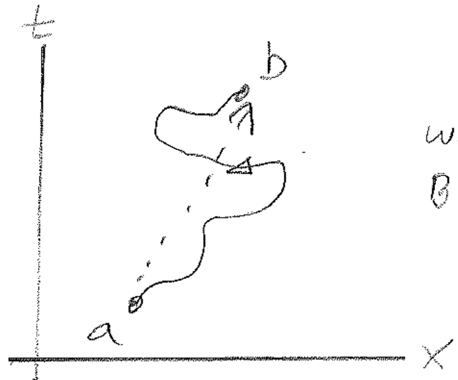
$$A = \int_{\tau_1}^{\tau_2} -m_0 c^2 \, d\tau = -m_0 c^2 \int_a^b \frac{ds}{c}$$

WITH $ds^2 = -d\vec{x}\cdot d\vec{x} + c^2 dt^2$,

BY LEAST-ACTION $\delta A = 0$, SO

$$\delta \int_a^b ds = 0 \, ,$$

HERE'S A PICTURE OF WHAT'S GOING ON



WHICH PATH HAS BIGGER $\int_a^b ds$?

THE EXPECTED TRAJECTORY IS OF COURSE A STRAIGHT LINE; SHOW $\int_a^b ds$ IS AN EXTREMUM FOR THE STRAIGHT LINE (HINT: START WITH A PARTICLE AT REST).

NOW TO INCLUDE POTENTIAL ENERGY.

RECALL $p^\mu = (c\vec{p}, E)$ $\{$OR $(\vec{p}, E)$: $c=1\}$

AND $A^\mu = (c\vec{A}, \Phi)$ $\{$OR $(\vec{A}, \Phi)\}$

EVALUATE THE ELECTRODYNAMIC POTENTIAL V.

ASSUME V IS LINEAR IN A^α (INSPIRED BY LAST QUARTER: $e\Phi$ AND $\vec{J}\cdot\vec{A}$). LET'S SEEK $V\,dt$ THAT'S A LORENTZ INVARIANT.

TRY $\quad V\,dt = -eA_\mu\, dx^\mu.$

DOES THIS REDUCE TO THE FAMILIAR FORMS OF $\vec{A}$ AND Φ? WE HAVE

$$A_\mu = (c\vec{A}, \Phi);\quad dx^\mu = (d\vec{x}, c\,dt)$$
$$= (\vec{v}, c)\,dt$$

SO

$$-eA_\mu dx^\mu = -e(\Phi - \vec{v}\cdot\vec{A})\,dt$$

AS REQUIRED.

WE NOW HAVE THE SINGLE-PARTICLE ACTION

"–" CHOSEN TO GIVE CORRECT LOW-VELOCITY LIMIT

$$A = \int_a^b \left[-m_0 \gamma c\, ds - e A_\mu\, dx^\mu \right]$$

$$= \int_{t_1}^{t_2} \underbrace{\left[\left(-\frac{1}{\gamma} m c^2 + e \vec{v} \cdot \vec{A} - e \Phi \right] \right.}_{L \text{ FOUND IN e.g., LANDAU AND LIFSCHITZ.}} dt$$

THE CANONICAL MOMENTUM IS

$$\vec{p} = \frac{\partial L}{\partial \vec{v}} = \gamma m_0 \vec{v} + e\vec{A}$$

WE'RE NOT DONE. WE'VE FOUND THE LAGRANGIAN FOR A PARTICLE $e; m_0$ IN AN EXTERNAL ELECTRO-MAGNETIC POTENTIAL A^μ. WE'LL NOW NEED TO INCORPORATE THE LAGRANGIAN FOR THE ELECTRO-MAGNETIC FIELD. THE FIELD IS CONTINUOUS, SO WE NEED TO FORMULATE A LAGRANGIAN IN THE CONTINUUM LIMIT.