Physics 515, Electrodynamics III
Department of Physics, University of Washington
Spring quarter 2020
June 5, 2020, 11am
On-line lecture

Administrative:

1. The final exam will be posted on the course web site today at 3pm PDT, it's due via email Monday June 8, 11am PDT. See the course web site for more information. Let me know if you have questions or run into problems.
2. You should be getting your homework back; if not let me know. Has anyone not heard back about regrades?
3. I'm aware of the many difficulties and stresses this quarter has presented and I'll evaluate grades accordingly.
4. Feel free to contact me with questions, concerns, or to talk about physics or anything else for that matter.

Lecture:

Optional lecture.
An example of non-Maxwellian classical electrodynamics: Wheeler & Feynman, "Classical Electrodynamics in Terms of Direct Interparticle Action", Rev. Mod. Phys. 21 (1949) 425-433.

WE FOUND 3 ISSUES WITH MAXWELL ELECTRODYNAMICS:

1. BY FIAT, WE CHOOSE RETARTED, NOT ADVANCED, SOLUTIONS TO THE WAVE EQUATION EVEN THOUGH THE WAVE EQUATION IS TIME-SYMMETRIC.

2. THE CONCEPT OF ELECTROMAGNETIC MASS IS UNCLEAR; IT COULD NOT BE COMPUTED SATISFACTORIALLY. THERE'S AN EQUALLY UNSATISFACTORY NOTION OF NON-ELECTROMAGNETIC MASS. THE TWO MASSES WERE EMBEDDED INTO AN EMPIRICAL MASS. WE THEREFORE DIDN'T DELVE INTO THE NATURE OF A "POINT" ELECTRON.

3. OUR INITIAL WAY OF ELIMINATING RUNAWAY SOLUTIONS OF THE ABRAHAM-LORENTZ EQUATION WAS TO APPLY AD HOC BOUNDARY CONDITIONS.

(2)

ARE THERE ALTERNATIVES TO MAXWELL ELECTRODYNAMICS? YES, BUT THEY ARE NOT IN WIDESPREAD USE.

ONE OF MANY SUCH EXAMPLES OF NON-MAXWELLIAN ELECTRODYNAMICS:

"WHEELER-FEYNMAN ELECTRODYNAMICS"

REVIEWS OF MODERN PHYSICS VOLUME 21, NUMBER 3 JULY, 1949

Classical Electrodynamics in Terms of Direct Interparticle Action[1]

JOHN ARCHIBALD WHEELER AND RICHARD PHILLIPS FEYNMAN[2]
Princeton University, Princeton, New Jersey

". . . the energy tensor can be regarded only as a provisional means of representing matter. In reality, matter consists of electrically charged particles. . . ."[3]

INTRODUCTION AND SUMMARY

MANY of our present hopes to understand the behavior of matter and energy rely upon the notion of field. Consequently it may be appropriate to re-examine critically the origin and use of this century-old concept. This idea developed in the study of classical electromagnetism at a time when it was considered appropriate to treat electric charge as a continuous substance. It is not obvious that general acceptance in the early 1800's of the principle of the atomicity of electric charge would have led to the field concept in its present form. Is it after all essential in classical field theory to require that a particle ... quantum theories of ... hardly ...

action,[8]

$$J=-\sum_a m_a c\int(-da_\mu da^\mu)^{\frac{1}{2}}+\sum_{a<b}(e_a e_b/c)$$
$$\times\int\int\delta(ab_\mu ab^\mu)(da_\nu db^\nu)=\text{extremum.}\quad(1)$$

All of mechanics and electrodynamics is contained in this single variational principle.

However unf...

I'M CERTAINLY NOT AN EXPERT ON THIS; THIS SPECIAL LECTURE GAVE ME THE OPPORTUNITY TO GO THROUGH IT.

HERE'S ONE WRONG WAY TO TRY TO UNDERSTAND RADIATION REACTION:

- POSIT AN ELECTRON FIELD CAN'T ACT ON ITSELF;
- IMAGINE A UNIVERSE CONSISTING OF JUST TWO ELECTRONS: ACCELERATE ONE, THIS CAUSES ACCELERATIONS OF THE SECOND, WHICH IN TURN CAUSES A "BACK-REACTION" ACCELERATION ON THE FIRST.

IS THIS RADIATION REACTION?

NO; IT GIVES THE WRONG ANSWER (OBVIOUSLY).

Q: WHY IS IT OBVIOUSLY WRONG?

A: WHAT WE'VE DESCRIBED IS REFLECTION OF WAVES FROM A MIRROR.

HERE'S ANOTHER WRONG APPROACH:

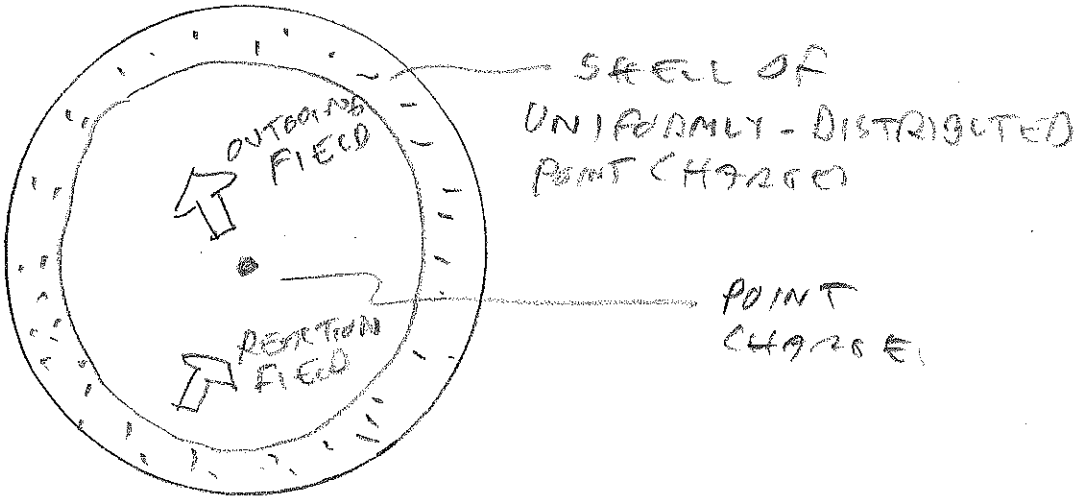
- SURROUND A SINGLE CHARGE WITH MANY SURROUNDING CHARGES IN A SHELL, UNIFORMLY-DISTRIBUTED. THIS SHELL OF CHARGE IS THE "ABSORBER".

- ACCELERATE THE SINGLE CHARGE; THE $1/r^2$ INTENSITY FALL-OFF WOULD BE COMPENSATED BY THE r^2 IN A SHELL VOLUME ELEMENT, SO THE BACK-REACTION WOULD BE PROPORTIONAL TO THE THICKNESS OF THE SHELL.

  THIS DOESN'T GIVE THE RIGHT RADIATION-REACTION EITHER.

- THE BACK-REACTION FROM THE SHELL IS A MIX OF ADVANCED AND RETARDED FIELDS.

THIS SEEMS WILD, BUT...



SOME DETAILS:

- THE ADVANCED REACTION FIELDS (AMY, TBD) GET THE RADIATION-REACTION BACK AT THE RIGHT TIME: THIS IS DIFFERENT THAN A MIRROR.

- THE ABSORBER CONTAINS LOTS OF ELECTRONS, IT HAS AN INDEX-OF-REFRACTION n. HENCE
  - RETARDED FIELDS FROM THE SOURCE HAVE DIFFERENT WAVELENGTH IN THE ABSORBER.
  - ADVANCED FIELDS FROM THE ABSORBER HAVE UNCHANGED WAVELENGTH.
  - THERE'S A PHASE SHIFT BETWEEN SOURCE AND RETURN WAVES. (THIS IDEA COMES BACK...)
  - THE SUPERPOSITIONS ONLY HAVE APPRECIABLE CONTRIBUTION FROM A THIN SHELL OF THICKNESS $\frac{\lambda}{n-1}$ (THE "FIRST FRESNEL ZONE" IN OPTICS); THE ABSORBER OUTSIDE THIS DON'T MUCH CONTRIBUTE.

HOW MUCH DOES THE STRUCTURE OF THE ABSORBER (SHELL) AFFECT THE THEORY?

DENSITY: IN THE SHELL, THE FEWER ELECTRONS IN IT, THE LESS EACH CONTRIBUTES, BUT THE THICKER THE SHELL.

CHARGE: THE GREATER THE CHARGES OF OBJECTS IN THE SHELL, THE MORE EACH OBJECT CONTRIBUTES, BUT THE THINNER THE SHELL.

AMBIGUITY: THE BACK REACTION IS INDEPENDENT OF THE STRUCTURE AND NATURE OF CHARGES IN THE SHELL.

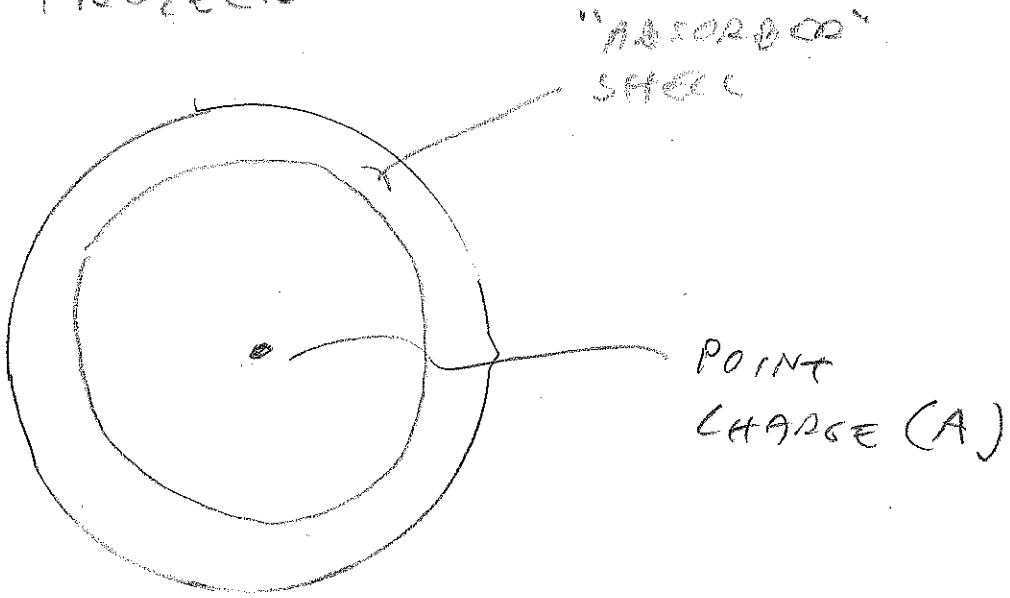
POSTULATES OF WHEELER-FEYNMAN ELECTRODYNAMICS. II

- THE ACCELERATION OF A POINT CHARGE IS DUE TO ITS INTERACTIONS WITH OTHER CHARGED PARTICLES (AND POSSIBLY WITH NON ELECTRO-MAGNETIC FORCES). A CHARGE DOES NOT ACT ON ITSELF.
- ELECTROMAGNETIC FORCES ARE CONTAINED IN THE USUAL COULOMB FORCE LAW. ELECTROMAGNETIC FIELDS ARE THOSE OF MAXWELL'S EQUATIONS.
- THE FUNDAMENTAL MICROSCOPIC PHENOMENA ARE TIME-SYMMETRIC.
- THE PROPAGATION SPEED IS c.

SIMPLE COROLLARY: A UNIVERSE CONSISTING OF A SINGLE CHARGE HAS NO RADIATION. FOR A TIME-SYMMETRIC THEORY.

THIS, IT TURNS OUT, GIVES FINITE RESULTS FOR A TRUE POINT CHARGE.

LET'S PROCEED



- THE SOURCE IS A POINT CHARGE (A). ITS RADIATION IS CORRECTLY-DESCRIBED BY RETARD FIELDS $F^A_{RET.}$. THIS IS IN ACCORD WITH OUR NOTION OF CAUSALITY; THIS IS SUBTLE AS WE NEED SOMEHOW TO ESTABLISH THE "ARROW OF TIME". THIS IS EASY FOR A COMPLEX SYSTEM, BUT HARD FOR SIMPLE SYSTEMS. BUT WE PROCEED ...

- THE RADIATION-REACTION FORCE IS REPLACED BY THE ABSORBER FIELDS. BUT WE INCLUDE ADVANCED FIELDS THUS:

$$F_{REACT} = \frac{1}{2}\left(F_{RET} - F_{ADV}\right).$$

- RECALL THE LORENTZ DIRECT CALCULATION OF THE RADIATION-REACTION FORCE OF THE ELECTRON. IT HAD EXPANSIONS LIKE

$$\vec{v}(t') = \vec{v}(t) - \frac{r}{c}\dot{\vec{v}}(t) + \cdots$$

$$\vec{v}(t') = \vec{v}(t) - \frac{r}{c}\dot{\vec{v}}(t) + \cdots$$

$$\frac{1}{s^3} = \frac{1}{r^3}\left\{1 - \frac{3\dot{\vec{v}}(t)\cdot\vec{r}}{c} + \cdots\right.$$

BUT IF DEMAND $F_{REACT} = \frac{1}{2}\left(F_{RET} - F_{ADV}\right)$, TERMS ODD IN r VANISH.

- WHEN YOU THEN EVALUATE

$$\vec{F}_{REACT} = \frac{2}{3}\frac{e^2}{c^3}|\ddot{\vec{r}}(t)|^2 - m_{em}\dot{\vec{v}}(t),$$

WITH $m_{em} = \frac{4}{3}\frac{1}{c^2}\iint_{r_0}\frac{dcde'}{r}$,

THE TERM m_{em} IS OOO IN r AND VANISHES: THIS IS THE TERM DIVERGENT IN r_0, WITHOUT IT THE REACTION FORCE

$\vec{F}_{REACT} = \frac{2}{3}\frac{e^2}{c^3}|\ddot{\vec{r}}(t)|^2$ IS FINITE EVEN FOR THE POINT ELECTRON. THIS DEALS WITH ONE OF THE ISSUES.

HOW IS IT THAT $F_{REACT} = \frac{1}{2}(F_{RET} - F_{ADV})$ IS FINITE AT THE POINT-CHARGE SOURCE?

RECALL GREEN'S FUNCTION SOLUTIONS FOR WAVE-NUMBER k:

$$G_{RET} = \frac{1}{4\pi r} e^{+ikr}$$

$$G_{ADV} = \frac{1}{4\pi r} e^{-ikr}$$

(YOU'LL NEED TO GO BACK TO OUR DERIVATION OF THE RETARDED POTENTIAL.)

USE $F_{REACT} = \frac{1}{2}(F_{RET} - F_{ADV})$:

$$\frac{G_{RET} - G_{ADV}}{2} = \frac{1}{4\pi r} \frac{e^{+ikr} - e^{-ikr}}{2}$$

$$= \frac{1}{4\pi} \frac{\sin kr}{r}$$

$$\xrightarrow[r=0]{} \frac{1}{4\pi} k \quad \text{(FINITE)}.$$

THE UNDERLYING MICROPHYSICS IN THIS MODEL IS TIME-SYMMETRIC (AS IS THE WAVE EQUATION). BUT THE MACROSCOPIC SOLUTIONS SEEM RETARDED AND CAUSAL.

THAT IS, WE NEED TO SOMEHOW GET F^A_{RET} FROM $\frac{1}{2}(F_{RET} + F_{ADV})$ WHEN WE SUPERIMPOSE FIELDS OF (A) WITH THOSE OF THE DISTANT RADIATOR.

WE ALSO NEED TO UNDERSTAND WHERE A TIME-SYMMETRIC SOLUTION

$$F_{SYM} = \frac{1}{2}(F_{RET} + F_{ADV})$$

COMES FROM.

THE DISTANT RADIATION IS CRUCIAL IN THIS: AN ISOLATED CHARGE WITH $F_{SYM} = \frac{1}{2}(F_{RET} + F_{ADV})$ WOULD NOT RADIATE ENERGY.

THE IDEAS:

- ISOLATED CHARGE RADIATION GIVEN BY F_{RET}^{A};
- REACTION FIELDS FROM THE ABSORBER $F_{REACT} = \frac{1}{2}(F_{RET} - F_{ADV})$;
- TIME-SYMMETRIC THEORY #95 RADIATION $F_{SYM} = \frac{1}{2}(F_{RET} + F_{ADV})$.

THESE IDEAS APPEAR CONTRADICTORY; WE'LL EXAMINE THIS THROUGH A SIMPLE MODEL.

SIMPLE MODEL:

POINT CHARGE (A) IS SURROUNDED BY A LARGE ABSORBER CONTAINING N-1 CHARGES.

THE ACTUAL FIELDS F_{ACT} ARE TIME SYMMETRIC: $F_{ACT} = \frac{1}{2}(F_{RET} + F_{ADV})$.

THE ABSORBER DOES INDEED ABSORB ACC RADIATION, SO. OUTSIDE THE SHELL

$$\sum_{i=1}^{n} \frac{1}{2}(F_{i\,RET} + F_{i\,ADV}) = 0 \quad \text{(OUTSIDE)}.$$

BUT, ADVANCED FIELDS ARE REPRESENTED BY INCOMING WAVES, AND RETARDED FIELDS ARE REPRESENTED BY OUTGOING WAVES. OUTSIDE THE SHELL, THEY CAN'T INTERFERE DESTRUCTIVELY EVERYWHERE, SO THE SEPARATE SUMS VANISH EVERYWHERE OUTSIDE THE SHELL.

IT'S THEREFORE THE CASE THAT OUTSIDE THE SPHERE WE ALSO HAVE

$$\sum_{i=1}^{n} \frac{1}{2}\left(f_{i\,ADV} - f_{i\,RET}\right) = 0 \qquad \text{(OUTSIDE)}.$$

RECALL THE COMBINATION

$F_{ADV} - F_{RET}$ IS FINITE EVEN AT THE POSITION OF A POINT CHARGE

NON-SINGULAR, PLUS $\sum_{i=1}^{n} \frac{1}{2}\left(f_{i\,ADV} - f_{i\,RET}\right)$ VANISHING EVERYWHERE ON A CLOSED SURFACE IMPLIES

$\sum_{i=1}^{n} \frac{1}{2}\left(f_{i\,ADV} - f_{i\,RET}\right)$ VANISHES EVERYWHERE, EVEN INSIDE.

WE SEPARATE OUT THE CONTRIBUTION OF THE SOURCE (A):

$$0 = \sum_{i=1}^{n} \tfrac{1}{2}(F_{i\,ADV} - F_{i\,RET})$$

$$= \sum_{i \neq A} \left\{ \tfrac{1}{2}(F_{i\,ADV} + F_{i\,RET}) - F_{i\,RET} \right\} + \tfrac{1}{2}\left(F^{A}_{ADV} - F^{A}_{RET}\right).$$

WE APPLY THIS AT THE POSITION (A) OF THE SOURCE CHARGE, RECALLING $F_{ACT} = \tfrac{1}{2}(F_{RET} + F_{ADV})$ AND

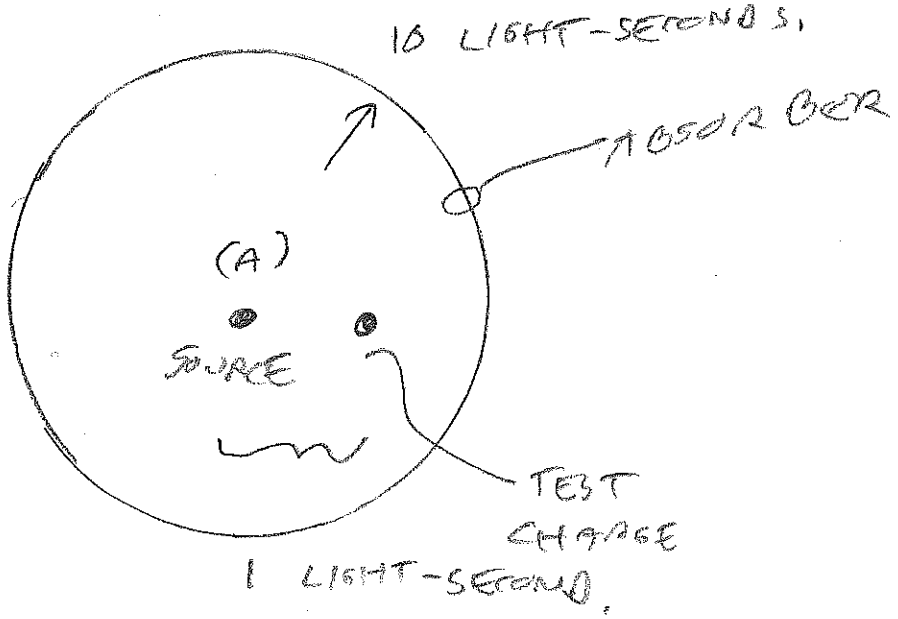
$$F_{REACT} = \tfrac{1}{2}(F_{RET} - F_{ADV}):$$

$$\sum_{i \neq A} F_{i\,ACT} = \sum_{i \neq A} F_{i\,RET} + F^{A}_{REACT}.$$

THAT IS: THE FORCE PER UNIT CHARGE ON (A) WITH THE TIME-SYMMETRIC FORMALISM IS THE SAME AS THE FIELDS FROM THE ABSORBER IN THE USUAL RETARDED FORMALISM PLUS A NON-SINGULAR RADIATION REACTION TERM $F^{A}_{REACT} = \tfrac{1}{2}(F_{RET} - F_{ADV})$.

THAT THE RADIATION FIELDS FROM THE ABSORBER ARE RETARDED MEANS OUR SENSE OF RETARDED EFFECTS IS MAINTAINED. THIS RELIEVES THE VARIOUS CONTRADICTIONS

EXAMPLE:



- THE SOURCE ACTING AT $t = 0$ THEN ACTS ON THE ABSORBER AT $t = +10$ SECONDS.
- THE ADVANCED FIELDS FROM THE ABSORBER CAN REACH THE TEST CHARGE AS EARLY AS $t = -1$ SECOND; THIS IS ALSO THE ARRIVAL TIME OF THE ADVANCED FIELDS FROM THE SOURCE TO REACH THE TEST CHARGE. IT "HAPPENS" THE TWO FIELDS ARE EQUAL AND OPPOSITE AND THEREFORE CANCEL.

- SOMEWHAT LATER THE RETARDED FIELD FROM THE SOURCE REACHES THE TEST CHARGE AT $t = +1$ SEC; LIKEWISE PART OF THE ADVANCED FIELDS FROM THE ABSORBER ARRIVE AT THE TEST CHARGE AT $t = +1$ SEC. IT "HAPPENS" THIS TIME THE TWO FIELDS HAVE THE SAME SIGN, THEREBY CONVERTING WHAT IS A HALF-RETARDED WAVE FROM THE SOURCE TO FULL RETARDED STRENGTH.

⇨ IN ASSUMING ALL ELECTROMAGNETIC FIELDS ARE HALF-ADVANCED AND HALF-RETARDED, AND IN ASSUMING THE UNIVERSE IS AN ABSORBER, YOU RESOLVE THE ISSUES WITH MAXWELL ELECTRODYNAMICS. BUT, THERE ARE NEW PROBLEMS ... (FOR ANOTHER DAY).