Physics 515, Electrodynamics III
Department of Physics, University of Washington
Spring quarter 2020
April 1, 2020, 11am
On-line lecture

Administrative

All lectures are on Zoom.
The midterm and final are take-home exams.
We’re still working out details of submission/grading/return.
I don’t know what’s happening with the EM-MRE.
Go over syllabus.
Go over schedule.
The class performance in 514 was very good.

Lecture

Close out J.C.8: Waveguides & resonant cavities.
J.C.8.8 Power losses & cavity Q. A comment on dielectric losses.
J.C.8.6 Perturbation of cavity boundaries. (On homework.).
Slater perturbation formalism.

Start J.C.9. Radiating systems.
J.C.9.1-2 Usual treatment of the the infinitesimal electric dipole.

Physics 515, Spring Quarter 2020

Prof. Leslie J Rosenberg, Department of Physics, University of Washington

General information:
Physics 515, the third course in graduate electrodynamics

Textbook: J.D. Jackson, "Classical Electrodynamics," third edition

Because the campus is closed, the entire course is on-line throughout the quarter. Zoom lectures are Wednesdays & Fridays 11:00-12:20 Pacific time and the first lecture is Wednesday, April 1.

Join the Zoom lectures with the URL https://washington.zoom.us/j/808790643 (to join, you'll need your UWnetID credentials, then enter the Zoom "SSO" of "washington" or "washington.zoom.edu" depending on your system).

Course Instructor:
Prof. Leslie J Rosenberg
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Office: Physics & Astronomy Building, room C503
Office Hours: Will be via Zoom, Wednesdays: 12:30 or by appointment
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TAs/graders:
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Useful Information:

- Readings, Lectures and Exams
- On-Line Lectures
- Special Lectures
- Homework

- Midterm-exam information. The midterm is a take-home exam. It will be posted on Friday, May 1. You'll scan your solutions and submit them by email to the instructor lrosenberg@phys.washington.edu by 11 am Pacific time May 4. A substantial number of points will be deducted for a late submission.

- Final-exam information. The final is a take-home exam. It will be posted on Friday, June 5 You'll scan your solutions and submit them by email to the instructor lrosenberg@phys.washington.edu by 11 am Pacific time June 8. A substantial number of oints will be deducted for a late submission.

Recent course news:

- [31Mar2020 11:15] Lecture for 01April2020 will start 11:00 on Zoom from the link https://washington.zoom.us/j/808790643
- [31March2020 9:00] The first class day is Wednesday, April 1. Lectures are Wednesdays and Fridays.

Lecture Instructor's Comments

Welcome to Physics 515, the third of a three-quarter sequence of graduate classical electrodynamics. This is a wonderful topic, it's challenging and stimulating. Electrodynamics is crucial for understanding the underpinnings of the physical and biological sciences. It's also crucial for modern technology. In your career, you will need a familiarity with Jackson chapters 1-16 in order to converse sensibly with your colleagues.

Regarding the course: We will use Jackson's text "Classical Electrodynamics". You might want more details or other topics than found in Jackson, or perhaps you'd like an alternative approach. In which case you might want to look at Panofsky and Phillips "Classical Electricity and Magnetism". Two very good, very readable, books for some slightly more formal aspects of the classical field theory with fewer applications are Landau and Lifshitz "The Classical Theory of Fields" and "Electrodynamics of Continuous Media". Another nice thing about Landau and Lifshitz "Fields" is halfway through "Theory of Fields", General Relativity enters rather seamlessly. A slightly more elementary alternate text is Slater and Frank "Electromagnetism". Most homework problems, and indeed the majority of homework problems in most texts, are adapted from Smythe, "Static and Dynamic Electricity", a challenging text with an unusual notation. A more modern text is Zangwell, "Modern Electrodynamics", it has good reviews, but I haven't yet gone through it. There's no perfect text, and every text has gems scattered throughout.

Mathematical methods are interspersed throughout the course as needed, Jackson is good about introducing the mathematics background. For a math refresher, you could refer to Dennery and Krzywicki "Mathematics for Physicists".

That said, for the first and second quarters we'll follow Jackson's text somewhat closely. The third quarter will be guided by Jackson but the approach will sometimes be different.

Syllabus The syllabus for 515 starts with chapter 8 in Jackson; we'll close out the discussion of cavities with the issue of losses. We'll then follow the text in more or less the text ordering, though we'll supplement Jackson's presentation with added material. See above for a link to the readings and lectures. Try to read the relevant text and added material before class; this will take time but there's a big payoff in understanding.

Grading 40% of your grade is assigned to the midterm exam, 40% to the final exam, 20% to the homework.

- **Midterm exam**: There will be one midterm exam and a final exam. Both are take-home exams. Exams are to be your own work; you are not permitted to collaborate with any other person.
- **Note that there are no make-up exams or make-up homework**. Students with outside professional, service, or career commitments (i.e. military service, professional conference presentation, etc.) conflicting exactly with the exam dates must contact the instructor in the first two weeks of the quarter to establish alternate procedures. Students who miss an exam or homework due to illness should contact the instructor as soon as you're reasonably able to discuss alternate procedures. Except for debilitating illness or other crisis, students who miss an exam or homework without making prior arrangements with the lecture instructor will get a zero for that score. Except for illness and circumstances noted above, a final grade of 0.0 may be assigned to any student who misses a midterm or final exam.
- **Homework**:
Lecture homework will generally be assigned and collected weekly. We're still working out the system for submitting and returning homework. The graders will consider neatness and logic of presentation, points will be deducted for lack of either. Words help in explaining your solution. Briefly, if your homework is a messy, incoherent scrawl, the graders won't evaluate your homework. I strongly encourage you to work collaboratively, but your submitted work must be your own.
- **Communication**:
For administrative issues, it's best to contact me via email. But, for physics questions, please don't use email (unless the question answer is of the "yes/no" variety). Physics is best discussed at Zoom office hours. Also, don't hesitate to make a Zoom appointment to talk with me.
- **Religious Accommodations**:
Washington state law requires that UW develop a policy for accommodation of student absences or significant hardship due to reasons of faith or conscience, or for organized religious activities. The UW’s policy, including more information about how to request an accommodation, is available at Religious Accommodations Policy (https://registrar.washington.edu/staffandfaculty/religious-accommodations-policy/). Accommodations must be requested within the first two weeks of this course using the Religious Accommodations Request form (https://registrar.washington.edu/students/religious-accommodations-request/).

Physics 515, Spring 2020, University of Washington
Graduate Classical Electrodynamics: Quarter 3 of 3

Text: John David Jackson, "Classical Electrodynamics," 3th ed.

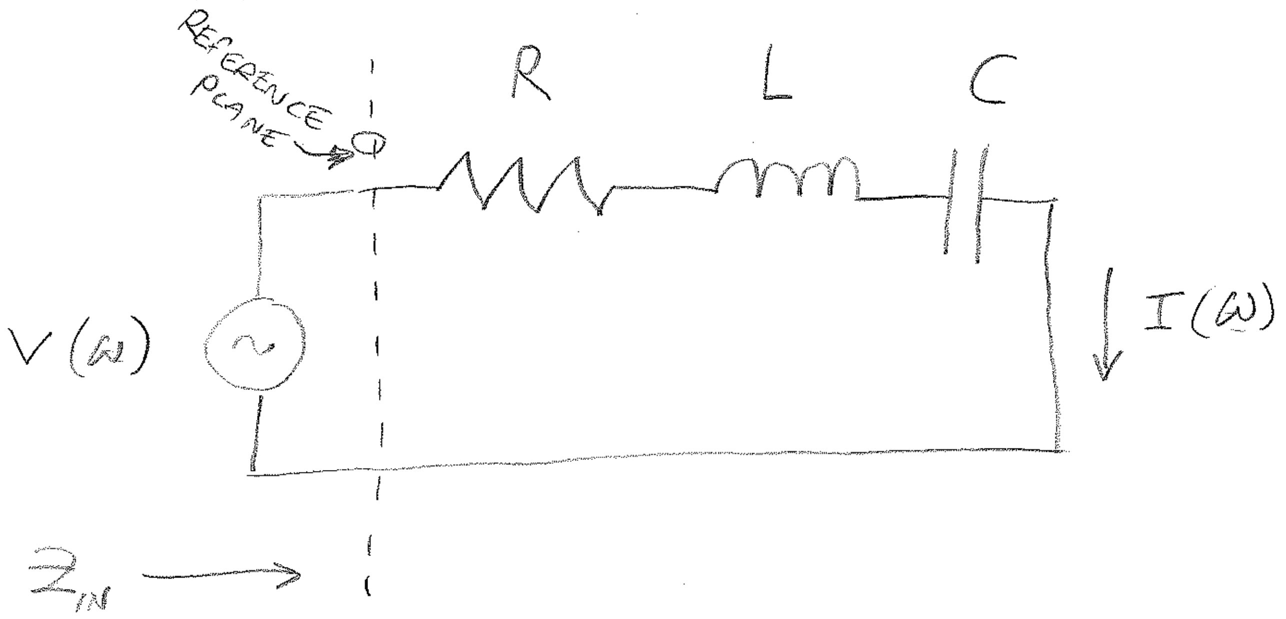
(syllabus ver. 31March2020 08:00)

week	date	lecture topic	readings*
1	1-Apr	Power losses in a cavity; Q of a cavity. Perturbation of boundaries	8.8, 8.6
	3-Apr	Radiation from a localized source, antennas 1. Dipole radiation.	9.1-4
2	8-Apr	spherical waves. The scalar wave equation.	9.6
	10-Apr	Poynting formalism 2; multipole fields. Angular momentum 2.	9.8-9
3	15-Apr	Scattering & Diffraction 1: Long-wavelength cross-section	10.1-2
	17-Apr	Scattering & Diffraction 2: Scalar & vector diffraction.	10.5-7
4	22-Apr	Scattering & Diffraction 3: Complementary screens, aperatures, optical theorem	10.8-11
	24-Apr	Relativity 1.	11.1-6
5	29-Apr	Relativity 2.	11.7-12
	1-May	Lagrangian Formalism 1.	12.1-3
	1-May	**Midterm exam posted**	
6	4-May	**11:00 am PST: Midterm due**	
	6-May	Lagrangian Formalism 2.	12.4-7
	8-May	Lagrangian Formalism 3.	12.8-11
7	13-May	Collision & energy loss 1	13.1-3
	15-May	Collision & energy loss 2	13.4-7
8	20-May	Radiation by moving charges 1	14.1-3
	22-May	Radiation by moving charges 2	14.4-5
9	27-May	Radiation damping 1.	16.1-3
	29-May	Radiation damping 2.	16.4-6
10	3-Jun	Radiation damping 3.	16.7-8
	5-Jun	Review	
	5-Jun	**Final exam posted**	
11	8-Jun	**11:00 PST: Final exam due**	

* The pace of the class, and therefore the readings, will likely vary from this syllabus.
Also, there will be special topics discussed in lecture.

POWER LOSSES IN CAVITIES

YOU'VE SEEN THIS BEFORE IN CIRCUIT THEORY IN, e.g., A SERIES LRC CIRCUIT.



OHM'S LAW FOR THIS CIRCUIT IS

$$V(t) = I(t) Z_{IN} ,$$ WITH

$$Z_{IN} = R + i\omega L + \frac{-i}{\omega C}$$

THE TIME-AVERAGE POWER SUPPLIED BY THE SOURCE IS

$$\langle P_{IN} \rangle = \frac{1}{2} \mathrm{Re}\{ V I^* \} = \frac{1}{2} \mathrm{Re}\{ Z_{IN} I I^* \}$$

THE TIME-AVERAGE POWER DISSIPATED IN THE RESISTOR IS

$$\langle P_R \rangle = \frac{1}{2} \mathrm{Re}\{ R I I^* \}$$

THE MAGNETIC ENERGY IN THE INDUCTOR IS

$$\langle W_L \rangle = \frac{1}{2} Re\left\{\frac{1}{2} L I I^*\right\}$$

THE ELECTRIC ENERGY IN THE CAPACITOR IS

$$\langle W_E \rangle = \frac{1}{2} Re\left\{\frac{1}{2} C V_c V_c^*\right\}$$

$$= \frac{1}{2} Re\left\{\frac{1}{2} C \frac{-i}{\omega C} I \frac{+i}{\omega C} I^*\right\}$$

$$= \frac{1}{2} Re\left\{\frac{1}{2} \frac{I I^*}{\omega^2 C}\right\}$$

IN $\langle P_{IN} \rangle$, REPLACE L & C BURIED IN Z_{IN} WITH L & C FROM $\langle W_L \rangle$ AND $\langle W_C \rangle$:

$$\langle P_{IN} \rangle = \frac{1}{2} Re\left\{\left(R + i\omega L + \frac{-i}{\omega C}\right) I I^*\right\}$$

$$= \langle P_R \rangle + 2i\omega\left(\langle W_L \rangle - \langle W_C \rangle\right)$$

WE HAVE $P_{IN} = P_R + 2i\omega\left(\langle W_L \rangle - \langle W_C \rangle\right)$

OR $\langle P_{IN} \rangle = \langle P_R \rangle$ AS EXPECTED

COMMENTS:

- AT RESONANCE

$\therefore \quad \langle W_L \rangle = \langle W_C \rangle$

$\therefore \quad Z_{IN} = R$

$\therefore \quad \omega_0^2 = \frac{1}{LC}$

THE QUALITY FACTOR Q IS

$$Q \equiv 2\pi \frac{\text{AVERAGE STORED ENERGY}}{\text{ENERGY LOST PER CYCLE}}$$

$$= \omega_0 \frac{\text{AVERAGE STORED ENERGY}}{\text{POWER LOST PER CYCLE}}$$

(JACKSON EQN 8.86)

Q CHARACTERIZES THE LOSSES IN A RESONATOR:

$Q \to \infty$ IS LOSSLESS.

EXPRESS Q AS

$$Q \sim \frac{\langle W_L \rangle + \langle W_C \rangle}{\langle P_{IN} \rangle},$$

- FOR LATER, AT RESONANCE

$\langle W_L + W_C \rangle = 2 \langle W_L \rangle = 2 \langle W_C \rangle$, SO

$Q = \omega_0 \frac{L}{R} = \frac{1}{\omega_0 CR} \quad (\omega = \omega_0).$

THESE HAVE SENSIBLE LIMITS!

FOR $R \to 0$, $Q \to \infty$ (NO LOSSES).

NOW, STUDY THIS OSCILLATOR NEAR RESONANCE.

$$Z_{IN} = R + i\omega L\left(1 - \frac{1}{\omega^2 LC}\right)$$

$$= R + i\omega L\left(1 - \frac{\omega_0^2}{\omega^2}\right)$$

$$= R + i\omega L\left(\frac{\omega^2 - \omega_0^2}{\omega^2}\right)$$

NEAR RESONANCE, $\omega \approx \omega_0$, SO

$$\omega_0^2 - \omega^2 = (\omega - \omega_0)(\omega + \omega_0)$$

$$= \Delta\omega\,(2\omega - \Delta\omega)$$

WITH $\Delta\omega = \omega - \omega_0$.

$$\approx 2\Delta\omega \cdot \omega \quad (\Delta\omega \ll \omega, \omega_0)$$

HENCE

$$Z_{IN} \approx R + i2L\,\Delta\omega$$

IF THE RESONATOR IS LOSSLESS,
$R \to 0$ AND $Z_{IN} = i2L\,\Delta\omega$.

NOW, SUPPOSE WE MAKE THE SUBSTITUTION

$$\omega_0 \longrightarrow \omega_0 \left(1 + \frac{i}{2Q}\right).$$

NOTE THIS DOESN'T CONTAIN ANY CIRCUIT-THEORY PARAMETERS; IT'S A GENERIC SUBSTITUTION FOR ANY OSCILLATOR. HENCE

$$Z_{IN} = i2L\left(\omega - \omega_0\left\{1 + \frac{i}{2Q}\right\}\right)$$

$$= \frac{\omega_0 L}{Q} + i2L(\omega - \omega_0)$$

$$= R + i2L\,\Delta\omega$$

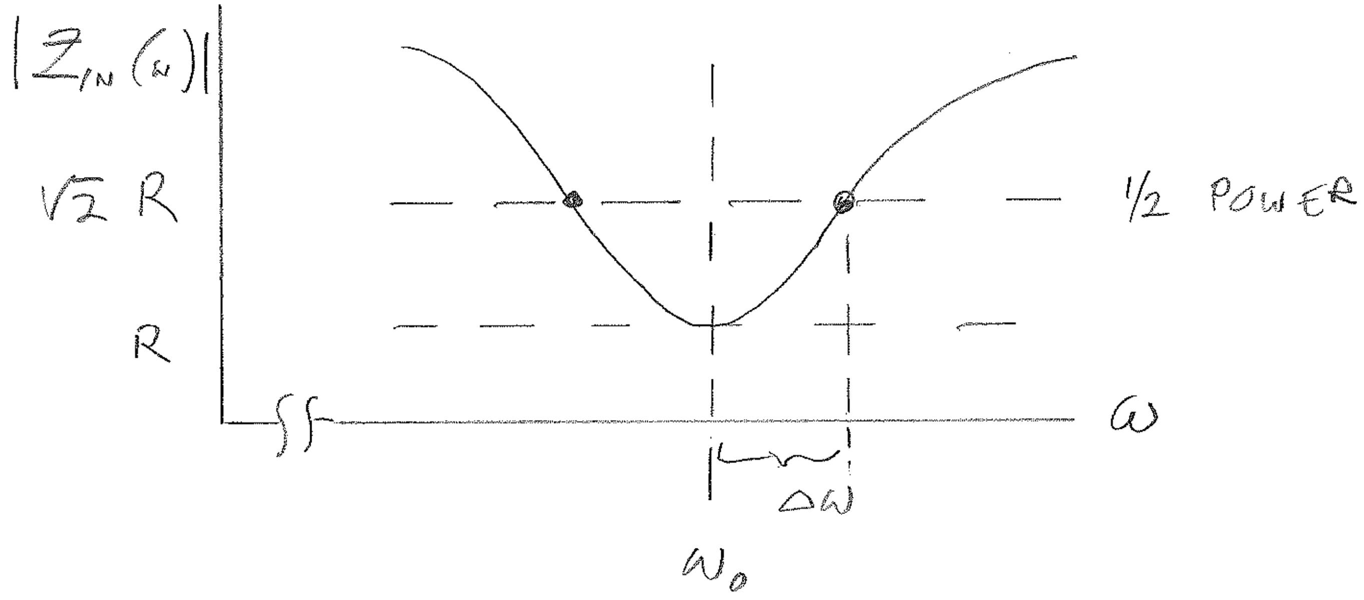
WE HAVE RECOVERED THE "LOSSY" IMPEDANCE BY ADDING AN IMAGINARY PART TO THE FREQUENCY. SEE JACKSON EQN 8.99.

NOW, BACK TO REAL POWER LOSS

$$\langle P_R \rangle = \frac{1}{2} \mathrm{Re}\{R I I^*\}.$$

RECALL AT RESONANCE $Z_{IN} = R$.

AT FREQUENCIES (NOT RESONANT) CORRESPONDING TO $\frac{1}{2}$ OF THE REAL, RESONANT, POWER LOSS, THE MAGNITUDE OF I IS REDUCED BY $1/\sqrt{2}$ AND $|Z_{IN}|$ INCREASES BY $\sqrt{2}$ TO $\sqrt{2}\,R$.



THAT IS, $\frac{\text{"BW"}}{2} = \frac{\Delta\omega}{\omega_0}$

WITH "BW" THE FRACTIONAL "3dB" BANDWIDTH.

RECALL

$$Z_{IN} = R + i2RQ\frac{\Delta\omega}{\omega_0}.$$

AT THE "$\frac{1}{2}$ - POWER" ω

$$Z_{IN}Z_{IN}^* = R^2 + R^2Q^2(BW)^2$$

$$(= 2R^2)$$

HENCE $BW = \frac{1}{Q}$.

THIS IS GENERIC FOR ANY OSCILLATOR.

SEE JACKSON EQN 8.91

(WHERE JACKSON'S Γ IS THE FREQUENCY FROM ω_0 TO THE "1/2 - POWER" FREQUENCY).

COMMENT SPECIFIC TO DIELECTRIC LOSSES.

WE HAVE A METHOD TO DEAL WITH THIS. GIVE AN IMAGINARY PART TO ε. THIS IS ESPECIALLY USEFUL WHEN THE RESONATOR OR WAVEGUIDE IS COMPLETELY FILLED WITH DIELECTRIC.

$$-K^2 = K_c^2 - \varepsilon\mu(1 + i \tan\delta)$$

WITH $\tan\delta$ THE "LOSS TANGENT" OF THE DIELECTRIC; YOU ASK AN ENGINEER FOR THIS.

IN PRINCIPLE, YOU MEASURE $\tan\delta$ BY COMPLETELY FILLING A RESONATOR WITH DIELECTRIC, THEN MEASURING Q!

$$\frac{1}{Q} = \frac{1}{Q_{WALL}} + \frac{1}{Q_{DIELECTRIC}}$$

WITH Q_{WALL} MEASURED WITHOUT DIELECTRIC.

CLOSE OUT J. C. 8.

TWO LOOSE ENDS.

- SEE JACKSON EQN 8.96

$$Q = \frac{\mu}{\mu_c}\left(\frac{V}{S\delta}\right) \times (\text{GEOMETRIC FACTOR}).$$

HENCE THE SUSPICION A SPHERICAL RESONATOR HAS THE HIGHEST Q.

- SEE JACKSON §8.6 PERTURBATION OF BOUNDARIES. I FOUND THAT PRESENTATION CONFUSING.

RECALL AT RESONANCE

$$\langle w_H \rangle = \langle w_E \rangle .$$

WITH SOME ALGEBRA

$$\frac{\Delta\omega}{\omega_0} = \frac{\Delta\langle w_H\rangle - \Delta\langle w_E\rangle}{\langle w_H\rangle + \langle w_E\rangle}.$$

(THIS IS "SLATER'S PERTURBATION FORMULA.)

RADIATION II   J.C. 9.

ACCELERATED CHARGES RADIATE.

IF YOU NOTICE, JEFIMENKO'S EQUATIONS (JACKSON EQN.'S 6.58-9) HAVE TERMS $\frac{\partial}{\partial t}\left(\frac{V}{R}\right)$ FOR RETARDED SOLUTIONS OF $\vec{E}$ & $\vec{B}$. THESE ARE THE ACCELERATIONS.

WE HAVE DYNAMICAL POTENTIALS

$$\vec{E} = -\vec{\nabla}\Phi - \frac{\partial}{\partial t}\vec{A}\,; \quad \vec{B} = \vec{\nabla}\times\vec{A},$$

IN LORENTZ GAUGE

$$\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial}{\partial t}\Phi = 0.$$

THESE GIVE FAMILIAR WAVE EQUATIONS

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Phi = -\rho/\varepsilon_0,$$

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{A} = -\mu_0 J,$$

THIS CHAPTER CONCERNS FINDING EXPLICIT SOLUTIONS TO THE RESULTING RETARDED POTENTIALS

$$\Phi(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{[\rho(\vec{r}',t)]_{RET}}{|\vec{r}-\vec{r}'|}\,dV'$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\iiint \frac{[\vec{J}(\vec{r}',t)]_{RET}}{|\vec{r}-\vec{r}'|}\,dV'$$

FIRST, JACKSON LOOKS AT SIMPLE ANTENNAS. HE THEN LOOKS AT RADIATION FROM A SINGLE CHARGE.

ANTENNAS: QUESTIONS YOU MIGHT WANT TO ASK.

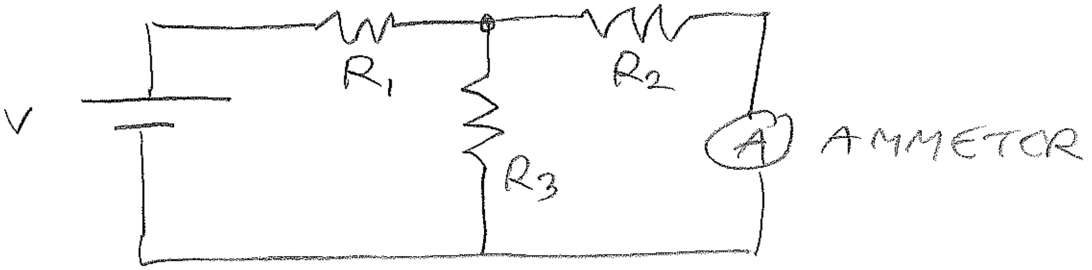
- RESISTIVE LOSS (WILL IT, e.g., MELT?) VS. RADIATED POWER.
- RADIATED POWER. TOTAL POWER (INTEGRATED OVER ALL SOLID ANGLE, OR IN A PARTICULAR DIRECTION θ, ϕ ("BEAM PATTERN").
- Z_{IN} "LOOKING INTO" THE ANTENNA.
  - THE ANTENNA IS ALMOST ALWAYS "FED" BY A TRANSMISSION LINE: HOW MUCH POWER REFLECTS BACK AT THE ANTENNA?
  - AT RESONANCE, WHAT'S R_{IN}?
- HOW TO COMPARE ANTENNA PERFORMANCE IN TRANSMITTING VS. RECEIVING MODE?

ATTACK THE LAST QUESTION FIRST.

RECEIVING VS. TRANSMITTING I.

RECIPROCITY THEOREM (IN WORDS):
THE CURRENT IN A DETECTOR DIVIDED BY THE VOLTAGE AT THE SOURCE REMAINS THE SAME SO LONG AS ω AND THE IMPEDANCES ARE UNCHANGED.

A SIMPLE EXAMPLE:

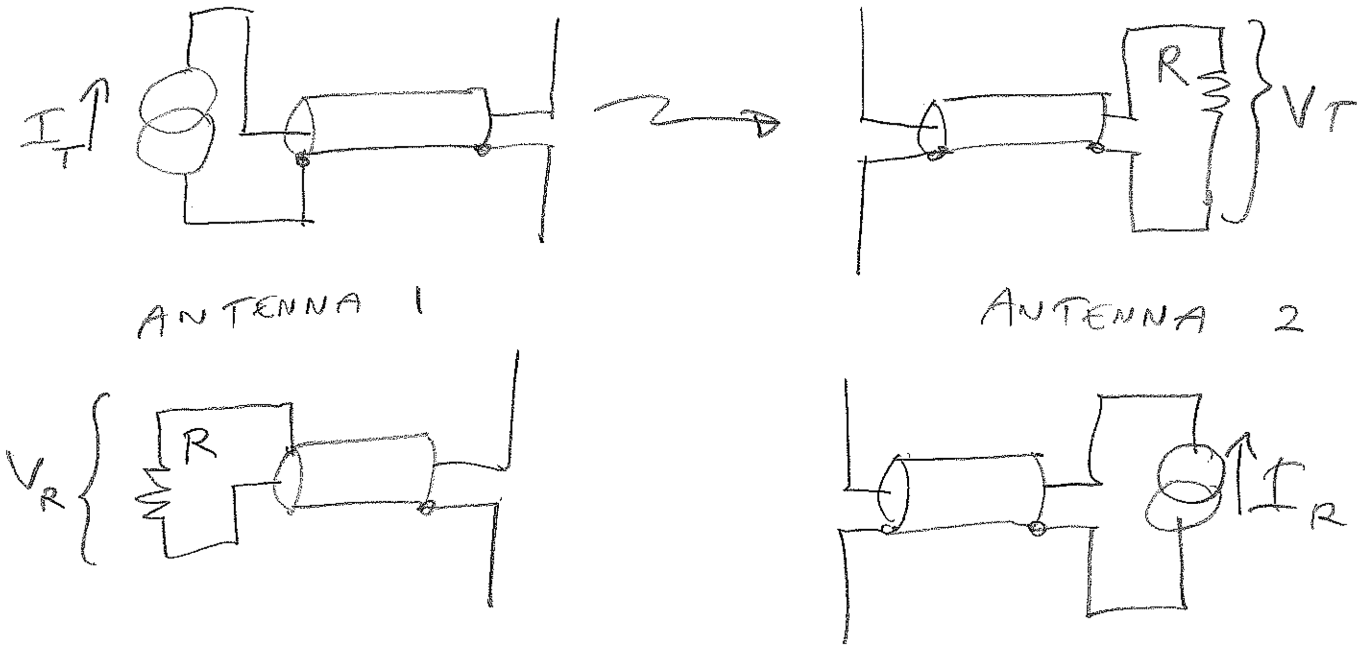


SWAP V AND A AND THE RATIO IS UNCHANGED.

BUT THE RECIPROCITY THEOREM IS A STATEMENT OF FIELD THEORY, SO IT AS WELL APPLIES TO TRANSMITTING AND RECEIVING ANTENNA PAIRS.

RECEIVING VS. TRANSMITTING II.

TRANSMITTING



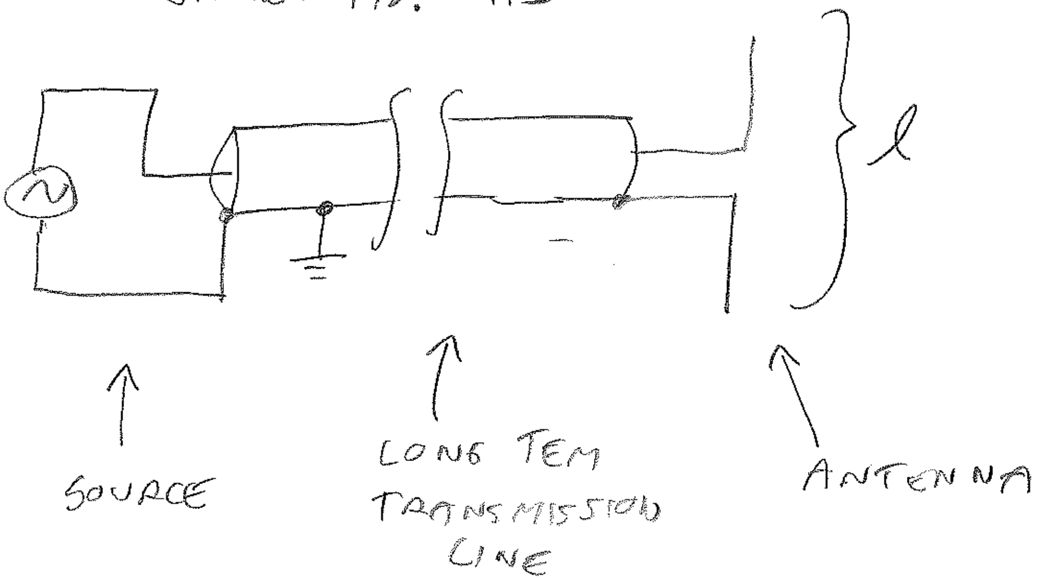
FROM RECIPROCITY

$$\frac{I_T}{V_T} = \frac{I_R}{V_R}$$

AND THE BEAM PATTERNS ARE THE SAME FOR AN ANTENNA: YOU DON'T NEED TO FIND TRANSMIT AND RECIEVE PROPERTIES FOR A SYSTEM.

THE INFINITESIMAL ELECTRIC DIPOLE, JACKSON § 9.4

JACKSON FIG. 9.3



(NOTE: THIS ANTENNA WON'T WORK AS SHOWN. CAN YOU FIGURE OUT WHY?)

INFINITESIMAL: $\ell \ll \lambda_0$, WITH λ_0 THE FREE-SPACE WAVELENGTH.

- THIS IS THE OLD "RABBIT EARS" TV ANTENNA.

- EVEN THIS SIMPLE GEOMETRY IS A VERY COMPLICATED PROBLEM!

∴ CURRENT CONSERVATION : CHARGE BUILDS UP. THERE ARE SEVERAL SOURCES OF RADIATION, INCLUDING THE RESULTING ELECTRIC DIPOLE MOMENT AND THE CURRENTS.

∴ WE'LL FOCUS ON THE CURRENTS, BUT IT'S NOT OBVIOUS OR EASY TO FIND THE CURRENT ALONG THE ANTENNA.

WE'LL ASSUME I IS UNIFORM ALONG THE ANTENNA AND IGNORE THE CHARGE BUILD-UP.

WE TALKED ABOUT "LOADING" THE DIPOLE TO ENHANCE THE ELECTRIC-DIPOLE CONTRIBUTION.

- THE TRANSMISSION LINE HAS A WAVE IMPEDANCE $H_t/E_t = Z$. IF THE IMPEDANCE "LOOKING INTO" THE ANTENNA ISN'T Z, SOME SIGNAL BOUNCES BACK TO THE SOURCE.

- DRIVE THE ANTENNA WITH A CURRENT SOURCE $I(t) = I_\omega e^{i\omega t}$.

  THEN $V(t)$ IS WHATEVER IT TAKES FOR THE SOURCE TO DRIVE $I(t)$ INTO THE TRANSMISSION LINE.

  IT'S EASIER TO DRIVE $I(t)$ THAN $V(t)$; THE SAME CURRENT $I(t)$ IS ALONG THE ANTENNA.