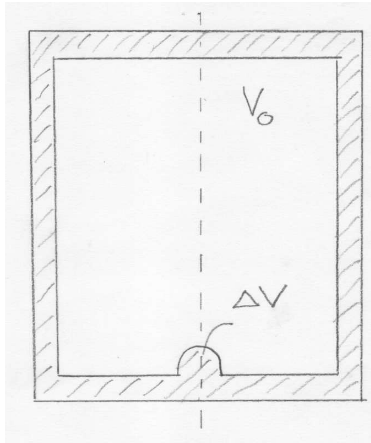
Electrodynamics III: Assignment 1

Due April 10 at 11:00 am.

1. **Scan your solutions as a single PDF file**
2. **Name your file HW1-*lastname*.pdf**
3. **Attach your file to an email…**
4. **… with subject line HW1-*lastname* …**
5. **… and send the email to ljrosenberg@phys.washington.edu**

1. The half-circle waveguide (done is several texts). A class of waveguide problems involves starting with, e.g., rectangular or circular guides, then inserting conducting planes along the axis (like Jackson problem 8.5). Consider a waveguide with cross-section a half-circle of radius R.
a. In the mode-terminology of the corresponding circular waveguide, what modes exist in the half-circle guide? You can argue this based on the boundary condition imposed by the flat section of the guide.
b. What are the cutoff angular frequencies for the modes?

2. Perturbation of boundary. A tiny volume ΔV is removed on the axis of a cylindrical resonator with vacuum inside, as shown below. The volume of the resonator is V_0. The resonator operates in the lowest TM mode. (a) Find the resulting shift of the resonant frequency. (b) Suppose the resonator operates at 1 GHz, and the amount of volume removed $\Delta V / V_0$ is 1/1000. What's the numeric value of the frequency shift? Hint: For (a) you'll again need to use the identity for $\int_0^R r\, J_0^2(kr)\, dr$. For a discussion of the perturbation expression presented in lecture, see J.C. Slater, "Microwave Electronics," *Rev. Mod. Phys.* **18** (1946) especially eqn. 11.89 (lower) on p. 482.



3. Consider the resonator above (without the missing volume) having diameter R and length d with vacuum inside. The resonator operates in the lowest TM mode. The walls have surface resistance R_s.
a. Suppose the axial electric field has magnitude E_0. Find the power dissipation. Hints: (i) Finding power lost in the side wall is easy since **H** for this mode has translational and azimuthal symmetry. (ii) The end walls are harder since **H** varies with radius: to evaluate the necessary integral you might use the identity:
$\int rJ_n^2(kr)dr = \frac{r^2}{2}\left[J_n^2(kr) - J_{n-1}(kr)J_{n+1}(kr)\right]$ and recalling the boundary condition $J_0(kR) = 0$.
N.B., this identity also applies to, e.g., Bessel functions of the 2nd kind and both sets of Hankel functions. It also applies to non-integer order. This and other identities are frequently seen in cylindrical guided-wave theory. (iii) There's a very simple relation between the effective surface currents K and H_ϕ.
b. Find the Q for this mode.

4. Consider field points for the infinitesimal electric dipole discussed in lecture at positions in the extreme near region ($\omega/c \ll 1/r$).
a. Find the form of the electric and magnetic fields in this region (the quasi-stationary fields); they should look very familiar.
b. Show that the time-average radiated power for these quasi-stationary fields vanish in every direction, they are reactive fields.