University of Washington **Spring Quarter 2020**
Physics 515 **June 5-8, 2020**
Graduate Electrodynamics III

Final Exam

- The exam is due via email Monday, June 8, at 11 am PDT. Points will be deducted for a late submission.

- Use a separate sheet of paper for each problem solution; you will therefore have a minimum of 3 pages in your submitted PDF file. Assemble pages in problem-order (1, 2, 3).

- This is an open-book exam; you may refer to the Jackson textbook and lecture notes.

- Show your work in enough detail so the grader can follow your reasoning and your method of solution.

- The exam should take about an hour and 50 minutes to complete.

- The equations are in cgs (Gaussian) units.

- Feel free to contact me with questions.

- Email-submission instructions:
  1. Scan your solutions as a single PDF file
  2. Name your file `final-`*lastname*`.pdf`
  3. Attach your file to an email ...
  4. ... with subject line `final-`*lastname*
  5. And send your email to `ljrosenberg@phys.washington.edu`

I. (40 points) Radiation scattering in a thermal bath.

The Larmor formula gives the radiated power from an accelerated electron in vacuum. However, there's an additional contribution to the radiated power when the electron's in a thermal bath of temperature *T*.

a. Consider an observer attached to an electron. Find the observer's non-relativistic additional radiated power due to scattering off a thermal bath of temperature *T*. Hint: The radiated power is related to the scattering of the thermal radiation near the electron. You may need to recall from quantum mechanics the energy spectral density of thermal radiation is $\frac{dU}{d\nu} = \frac{8\pi}{c^3}\frac{h\nu^3}{e^{h\nu/kT}-1}$ where k is Boltzmann's constant.

b. Hawking suggested an observer outside a black hole sees a bath of thermal radiation at temperature $T = \frac{\hbar g}{2\pi ck}$ where g is the acceleration at the observer. There's an idea in gravitational theory called the Principle of Equivalence: An accelerated observer in a gravity-free region has the same physics as an observer stationary in a gravitational field, so this Hawking bath temperature is equally expressed as $T = \frac{\hbar a}{2\pi ck}$ where a is the acceleration of the (instantaneous) stationary observer. In terms of the acceleration *a* of the electron in the frame of the observer attached to the electron, what's observer's non-relativistic additional radiated power due to scattering off the thermal bath?

c. In quantum electrodynamics there's a "Schwinger field" E_s with $E_s = \frac{m^2c^3}{e\hbar}$ (about 10^{16} V/cm, or 10^{13} Gauss) where "hard" quantum effects become dominant in certain contexts. Suppose an observer attached to the electron sees an electric field *E* near the electron. In terms of the Schwinger field E_s, find that magnitude of the electric field *E* for the case where the Hawking power from part (b) equals the non-relativistic Larmor power (if you use the fine structure constant $\alpha = \frac{e^2}{\hbar c}$, this electric field magnitude *E* has a simple form); what's the corresponding acceleration in units of the acceleration at the surface of the Earth?

II. (30 points) Magnetar surface magnetic field.

A new class of pulsar was fairly recently discovered called a "magnetar". Magnetars are essentially an unusual form of isolated neutron star observed through their periodic x-ray pulses. The stars have a magnetic dipole moment pinned to the matter in the star, and the pulses are radiated electromagnetic waves generated by the star's rotating magnetic dipole moment. The character of the pulses is that they have a period τ of 10 seconds, but the period is slowing at the usually fast rate $\dot{\tau}$ of 10^{-10}. The magnetar mass and radius are those of typical neutron stars of 1.4 solar masses and 10 km.

a. Assume the slowing of the period is due to electromagnetic radiation. Assume the star's mass is uniform and the matter spins in uniform rotation. Assume the star's spin axis is at right angles to the magnetic dipole moment of the star. What's the peak magnetic field at the surface of the star?

b. How does this peak field compare the Schwinger field (introduced in problem 1)?

III. (30 points) Effective mass.

Consider a spherically-symmetric charge distribution (in its rest frame) moving at a uniform non-relativistic velocity in the x-direction.

a. What's the form of the Minkowski stress tensor in the charge's rest frame S_0?

b. What's the form of the Minkowski stress tensor in a frame S where the charge is seen to move as stated above?

c. Show from part (b) therefore that the momentum density of the fields in frame S describes an effective mass of $\frac{4}{3}\frac{W_0}{c^2}$ where W_0 is the field energy.

The non-relativistic condition allows you to work to lowest order in β, saving effort. (The result in part (c) also holds when the non-relativistic condition is relaxed.)