

Electrodynamics III: Assignment 5.

Due May 17 at 11:00am in class or 10:45am in the instructor's mailbox.

1. Use the expression for the electric field from a point charge moving at constant velocity (homework 4, problem 1) to find the electric and magnetic fields produced by line of constant charge λ per length (not measured in the proper frame) moving at constant velocity \mathbf{v} .

2. A variation of Jackson problem 11.14.

a. Argue or demonstrate whether or not there can be an inertial frame with no \mathbf{E} field and another frame with no \mathbf{B} field. And suppose some frame has $\mathbf{E}=0$, find the resulting constraints on \mathbf{E} and \mathbf{B} . The muon g-2 experiment, on which the University of Washington plays a major role, exploits this by using muons at a “magic” momentum, thereby negating effects of laboratory electric fields in the rest frame of the muon; ask Prof. Hertzog.

b. In class and on the exam we studied invariants associated with contractions of the field tensor and its dual. Suppose there are fields threading electric and magnetic materials. Find the resulting invariants. See the short discussion of Jackson associated with equation 11.145. The machinery in Jackson only allows this to be evaluated in the rest frame.

3. Point charge e moving in a field.

a. Show that the canonical momentum (Jackson eqn 12.14) is obtained from the Lagrangian (Jackson eqn 12.12).

b. Show that the corresponding Euler-Lagrange equation reduces to the Lorentz force law $\frac{d\mathbf{p}}{dt} = -\frac{e}{c} \frac{\partial \mathbf{A}}{\partial t} - e\nabla\Phi + \frac{e}{c} \mathbf{v} \times (\nabla \times \mathbf{A})$ with \mathbf{p} the mechanical momentum.

c. And hence show that if particular path is traced out with time going forward, then the reverse path is taken on changing the sign of \mathbf{B} while keeping \mathbf{E} unchanged. (You can show this by elementary means, but consider using the equation of motion in part b.)

4. Show that the Minkowski force density $K_\mu = \frac{1}{c}F_{\mu\nu}J^\nu$ contains the Lorentz force law. What is the physical significance of the “time” component of the force density?