

Electrodynamics III: Assignment 1.

Due April 12 at 11:00am in class or 10:45am in the instructor's mailbox.

1. Show that the sum over outgoing polarizations indeed leads to Jackson equation 10.10 $\frac{d\sigma}{d\Omega} = k^4 a^6 \left| \frac{\epsilon_r - 1}{\epsilon_r + 2} \right|^2 \frac{1}{2} (1 + \cos^2 \theta)$.

2. Consider the example of long-wavelength scattering off a conducting sphere. Find the equivalent of Jackson equations 10.15 where the polarization basis is circular polarization. (The unpolarized cross section remains Jackson equation 10.16 regardless of the polarization basis.) This is posed in several texts.

3. Consider a plane wave of angular frequency ω incident on a weak dielectric characterized by permittivity ϵ .

a. Explain why the Born approximation (Jackson section 10.2.B) is appropriate.

b. Find the currents in the dielectric. (Knowing the currents allows you to find the scattering cross section by directly calculating the retarded potentials from the currents. This approach to the cross section is not covered in Jackson.)

4. When a plane wave is incident on a scatterer, it may be some incident power is absorbed instead of being radiated. Suppose a plane wave is incident on an Ohmic sphere, where some heating arises due to $\mathbf{J} \cdot \mathbf{E}$ effects within the sphere. A sensible definition of the “absorption cross section”, the cross section area of the incident wave that goes into heating, is

$\sigma_{\text{abs}} = \iiint \mathbf{J} \cdot \mathbf{E} dV / |\langle \mathbf{S} \rangle|$, where $|\langle \mathbf{S} \rangle|$ is the magnitude of the time-average incident Poynting vector.

a. Suppose the sphere has poor, but finite, conductivity. Find the approximate induced electric dipole moment of the sphere. (Because the sphere is a poor conductor the skin depth is large. This is similar to the situation in Jackson figure 4.8.

b. Find the resulting absorption cross section of the sphere. This is done in several texts.

[ver 05apr19 13:45]