

Final Exam

Printed Name _____
last *first*

- Print out the first and last pages of this exam. Write your name in the space above and staple this cover page to the front of your answer sheets. Staple the “points addition” page (last page) to the back of your answer sheets.
- This is a take-home exam. The exam is “open book”, you may refer to any text.
- Please spend no more than 1 hour and 50 minutes on this exam.
- The exam is due Friday, June 7 at 11 am in class.
- Your answers must be your own work. Show sufficient detail in your answers so the grader can follow your derivation and logic, don’t just write the answer. Sloppy or incomplete answers will be downgraded.

I. (30 points) Liénard-Wiechert potentials and the wave equation.

Consider a charge e moving at constant velocity \mathbf{v} . Starting from the wave equations for the potentials $\left\{ \nabla^2 - \frac{1}{c} \frac{\partial^2}{\partial t^2} \right\} \begin{Bmatrix} \Phi \\ \mathbf{A} \end{Bmatrix} = -4\pi \begin{Bmatrix} \rho \\ \mathbf{J}/c \end{Bmatrix}$,

- a.** Show the wave equations can be written in terms of a spatial wave equation consisting of spatial derivatives and the velocity.
- b.** Show that solutions to these spatial wave equations are the Liénard-Wiechert potentials. (This is another path to the Liénard-Wiechert potentials.)

II. (35 points) Classical Rutherford scattering. Recall Rutherford scattering: A particle of charge e passes through matter of atomic number Z . On occasion the particle, in the Coulomb field of the nucleus, will scatter elastically and its path will be deflected. You should feel free to refer to E. Rutherford, *Phil. Mag.* **21** (1922) 672; note in his Fig. 1 the non-standard notation where his θ is the angle of the orbit asymptotes and his p is the impact parameter.

a. Show what we had asserted in class, namely the Lenz vector $\boldsymbol{\xi}$ (in terms of the particle momentum \mathbf{p} and angular momentum \mathbf{L}) is a constant of the motion. Assume the nucleus is infinitely heavy, so it doesn't move.

b. Thereby show that the particle radius r from the nucleus is a hyperbola for $|\boldsymbol{\xi}| > 1$ where $|\boldsymbol{\xi}|$ is interpreted as the orbit eccentricity.

c. And thereby show the probability of a deflection into impact parameter between b and $b + db$ is proportional to $1/\sin^2(\theta/2)$, where $\theta=0$ is forward scattering. (This then leads to the Rutherford result in a similar path to that of Bohr's ionization calculation.)

III. (35 points) Lorentz force and the stress tensor. Show that the Lorentz force density f_μ is related to the stress tensor $T_{\mu\nu}$ through the tensor divergence $f_\mu = \partial^\nu T_{\mu\nu}$. You might want to make use of the Bianchi identity Jackson eqn. 11.143 as well as tensor-symmetry properties.

POINTS

1. _____/30

2. _____/35

3. _____/35

Total _____/100