University of Washington **Spring Quarter 2019**
Physics 515 **May 10, 2019**
Graduate Electrodynamics III

Mid-Term Exam

Printed Name ____
last *first*

- If you need more space than is available to answer any part of a problem, use the back side of the same page to complete your answer. Scratch paper will not be graded.
- Show your work in enough detail so that the grader can follow your reasoning and your method of solution.
- This is an open-book exam; you may refer to Jackson in paper or electronic form.

POINTS ARE TOTALED ON THE BACK OF THE EXAM

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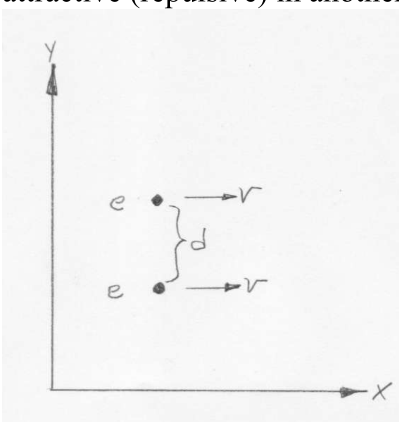
I. (35 points) Relativity I.

In the frame shown, two equal charges *e*, separated by a distance *d*, move in parallel paths with constant speed *v* in the x-direction.

a. Use Lorentz transformations from the charges rest frame to find the forces on the particles in the frame shown.

b. Verify you recover Coulomb's law in the non-relativistic limit.

c. Thereby argue whether or not a force that is attractive (repulsive) must always be attractive (repulsive) in another inertial frame.



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II. (30 points) Relativity II.

a. From direct transformations of **E** and **B** fields, show that $\mathbf{E}^2$-$\mathbf{B}^2$ is an invariant;

b. From direct transformations of **E** and **B** fields, show that $\mathbf{E}\cdot\mathbf{B}$ is an invariant;

c. What must be added to the square of the Poynting vector $\mathbf{S}^2$ to make an invariant?

d. Thereby argue whether or not a plane wave is always a plane wave in another inertial frame.

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III. (35 points) Scattering.

A plane wave propagating in the z-direction encounters a scatterer (absorbing, reflecting, or both) at the origin. Electromagnetic energy in the near-forward direction ($\theta \ll 1$) then encounters an absorbing disk of radius R whose center is along the z-axis at a distance along $+z$ much greater than R, and $R^2 \gg z/k$. Recall the spatial part of the electric fields in the Fraunhofer (far) zone have the form $E(\vec{r}) = E_0\left(e^{ikz} + \frac{e^{ikr}}{r} f(\theta,\phi)\right)$ where the amplitude $f(\theta,\phi)$ has dimensions of length.

a. Via direct integration over the disk, show that the (normalized) energy absorbed over the surface of the disk

$\frac{1}{E_0^2}\iint |E|^2 da$ is given by $\frac{1}{E_0^2}\iint |E|^2 da \approx \pi R^2 - \frac{4\pi}{k}\operatorname{Im} f(0)$

where $f(0)$ is the scattering amplitude in the forward direction.

Hint: The radial integral over the disk for $R^2 \gg z/k$ is

$$\int_0^R e^{i\frac{k\rho^2}{2z}} \rho\, d\rho = -\frac{z}{ik}$$

b. Argue this is what would be expected from the optical theorem.

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POINTS TOTALS

1. ______________________/35

2. ______________________/30

3. ______________________/35

Total ______________________/100