Physics 515, Spring Quarter 2018 Electrodynamics: Homework Assignment 4 Due April 27, either 11:00am in class or 10:45am in the instructor's mailbox.

1. Consider a very long line carrying linear charge density ρ, and an inertial frame travelling at velocity v parallel to the line. Find the current observed in the moving frame. Via Lorentz transformations demonstrate you recover the result of Ampere's law Jackson eqn 5.6. This suggests eqn 5.6 is relativistically exact for any boost velocity.

2. Point charge e moving in a field.
a. Show that the canonical momentum (Jackson eqn 12.14) is obtained from the Langrangian (Jackson eqn 12.12).
b. Show that the Euler-Lagrange equation reduces to the Lorentz force law $\frac{d\mathbf{p}}{dt} = -\frac{e}{c}\frac{\partial \mathrm{A}}{\partial t} - e\nabla\Phi + \frac{e}{c}\mathbf{v}\times(\nabla\times\mathrm{A})$ with **p** the mechanical momentum.
c. And hence show that if particular path is traced out with time going forward, then the reverse path is taken on changing the sign of **B** while keeping **E** unchanged. (You can show this by elementary means, but you might consider using the equation of motion in part b.)

3 Slight variant of Jackson problem 11.14.
a. Argue or demonstrate whether or not there can be an inertial frame with no **E** field and another frame with no **B** field. And suppose some frame has **E**=0, find the resulting constraints on **E** & **B**. The muon g-2 experiment, on which the University of Washington plays a major role, exploits this by using muons at a "magic" momentum, thereby negating effects of laboratory electric fields in the rest frame of the muon.
b. In class we wrote down the invariants associated with contractions of the field tensor and its dual. Suppose there are fields in the presence of electric and magnetic materials. Find the resulting invariants. See the short discussion of Jackson associated with

equation 11.145. The machinery in Jackson only allows this to be evaluated in the rest frame.

4. In class we found the Liénard-Wiechert potentials by means of basic Lorentz symmetries, but we did not differentiate them to find the fields. However, we can find the fields in other ways. Consider a point charge at rest in some inertial frame. Via Lorentz transformations, show that the **E** field in another inertial frame is

$$\mathbf{E}' = q\frac{(1-\beta^2)\mathbf{r}'}{r'^3(1-\beta^2 sin^2\theta)^{3/2}}$$

where r' is the vector from the *present* position of the charge to the observation point, and θ is the angle between the velocity and **r**'.