

Mid-Term Exam

Printed Name _____
last *first*

- If you need more space than is available to answer any part of a problem, use the **back side of the same page** to complete your answer. Scratch paper will not be graded.
- Show your work in enough detail so that the grader can follow your reasoning and your method of solution.
- This is an open-book exam; you may refer to Jackson in paper or electronic form.

POINTS ARE TOTALED ON THE BACK OF THE EXAM

I. (30 points) Relativity.

Two thin, straight, infinitely long, parallel wires are separated by a distance d . The wires are at rest in the unprimed frame. According to a stationary observer in the unprimed frame, each wire carries a linear charge density λ . An observer stationary in the primed frame moves with speed v parallel to the wires.

- a. Find the electric and magnetic fields in the unprimed frame.
- b. Find the force per unit length between the wires in the unprimed frame.
- c. Find the electric and magnetic fields in the primed frame.
- d. Using the fields and sources observed in the primed frame, find the force per unit length between the wires in the primed frame. Explain whether or not you would have expected the result of part d to be identical to the result in part b.

II. (35 points) Proca form of Maxwell equations. An infinite slab of “superconductor” having thickness $2d$ is positioned at $-d < z < +d$. The material obeys Proca electrodynamics with inverse scale length μ . On both surfaces of the slab there is a uniform magnetic field $\mathbf{B} = B_0 \hat{\mathbf{y}}$.

- a. Find the magnetic field everywhere inside the material. Hint: \mathbf{B} as usual is continuous; also see the comment below Jackson equation 12.99.
- b. Find the current density \mathbf{J} .

III. (35 points) Diffraction.

Consider an aperture in a conducting plane. Use vector diffraction theory (e.g., Jackson section 10.7) to show that the effective magnetic dipole moment associated with an aperture is given by the integral over the aperture

$$\mathbf{m} = \frac{2}{i\omega\mu} \iint (\mathbf{n} \times \mathbf{E}_{\text{tan}}) da$$

where \mathbf{n} is the normal to the aperture and \mathbf{E}_{tan} is the tangential field. You can assume the long-wavelength limit and consider field points in the Fraunhofer (far) region.