**University of Washington** **Spring Quarter 2018**
**Physics 515** **May 29, 2018**
**Graduate Electrodynamics III**

**Final Exam Ver1.1**

**Printed Name** ______________________________________________________
*last* *first*

- Print out the first and last pages of this file. Write your name in the space above and staple the cover page (this page) to the front of your answers. Staple the "points addition" page (last page) to the back of your answers.

- This is a take-home exam. The exam is "open book", you may refer to any text.

- The exam is due Friday, June 1, at 5pm. Slide your completed exam under my office door (Physics and Astronomy Building, room C503); do not put the exam in my mailbox or anywhere but under my door.

- Your answers must be your own work. Show sufficient detail in your answers so the grader can follow your derivation and logic, don't just write the answer. Sloppy or incomplete answers will be downgraded.

**I. (40 points) Radiation reaction from the retardation condition (the Lorentz derivation of radiation reaction).**

In class we talked about Lorentz's early "brute force" derivation of radiation reaction obtained from applying retardation to fields in a simple model of the electron. Summarizing the introduction to this class discussion, the Lorentz derivation assumes the following conditions:

i. The electron is modeled by a thin, spherically-symmetric shell of total charge *e* with radius $r_0$.

ii. You might as well work in the frame where any surface element of the charge shell is instantaneously at rest with any other surface element of the charge shell.

iii. The values of $v$, $\dot{v}$, $\ddot{v}$ (velocity, acceleration, etc.) don't change much during the light-travel-time across the electron.

iv. Terms containing an explicit dependence on $r_0$ can be ignored.

**a.** Why is condition iii (see above) required?

**b.** In terms of the retarded time *t'*, what is the electric field due to a small piece of charge *de'* (the source point) at another piece of charge *de* (the field point)?

**c.** In terms of the time *t* when the signal arrives at *de*, what is the electric field due to a small piece of charge *de*' at a field point located at another piece of charge *de*? Hint: Objects containing the retarded time *t'* (e.g., velocities, accelerations, etc., including the Liénard-Wiechert denominator) can be expanded in a power series in $\frac{r}{c}\frac{\partial}{\partial t}$. Explain why this is a reasonable expansion. Hint: Anticipating the result in (d), you need only work up to terms of order $(r/c)^3$.

**d.** Integrate over source and field points to find the reaction force. Note that if you had kept higher-order terms in *r/c*, their contribution to (d) would likewise be terms higher-order in *r/c*. These terms contain the model-dependence and are ignored (hence condition iv). We discussed in class why, even though the electromagnetic mass depends on $r_0$, written in terms of the electrostatic energy the explicit dependence on $r_0$ removes this model dependence.

**II. (30 points) Cerenkov Radiation.** In a medium without dispersion, the duration of the Cerenkov light pulse is infinitely short. However, any medium is, at some level, dispersive.

**a.** A fast-moving charged particle moves at speed $\beta$ in a straight line through a medium. An observer views the resulting Cerenkov light pulse at a distance $r_0$ from the particle axis. This observer can detect a range of angular frequencies from $\omega_1$ to $\omega_2$, with corresponding Cerenkov angles $\theta_1$ to $\theta_2$. In terms of $r_0$, $\theta_1$ and $\theta_2$, find the length of the light pulse. (This is not too difficult.)

**b.** Suppose the observer is 10 cm from the particle axis, the particle is an ultra-relativistic electron, the observer can detect optical frequencies, and the medium is water. Estimate the resulting duration of the observer's light pulse? Be explicit in how you find or determine the dispersion.

**III. (30 points) Collapse of the atom.** The classical atom consists of non-relativistic electrons in circular orbits about a spatially-fixed point-nucleus of charge *Ze*. In class we found the instantaneous radiated atomic-electron power is small, but over many periods eventually the electron falls into the nucleus and the atom collapses. (Quantum mechanics, of course, offers an explanation as to why this collapse does not occur.) Suppose an electron starts off at radius $r_0$.

**a.** What is the classical orbit period?

**b.** What is the power radiated by the electron?

**c.** What fraction of the electron's orbit energy is lost in one period? Argue therefore why an approximation that the electron's motion is circular for each period is sensible.

**d.** Find the resulting collapse lifetime of the atom. Notice that the very close-in orbits have relativistic speeds, so why was it nonetheless sensible to use a non-relativistic formula in b?

POINTS

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| 1. | ____________________ | /40 |
| 2. | ____________________ | /30 |
| 3. | ____________________ | /30 |
| Total | ____________________ | /100 |