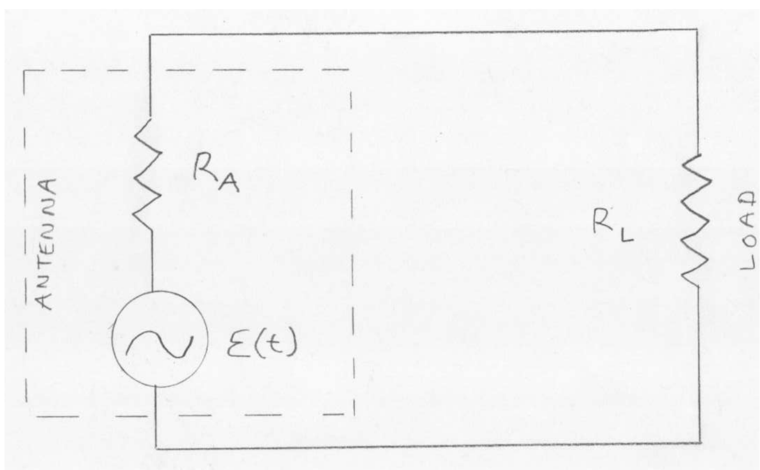
Physics 323, Spring Quarter 2016
Electrodynamics: Homework Assignment 7
(a) Turn in all problems and clearly note all constants and assumptions you use.
(1-point penalty each otherwise)
(b) Use 8½ x 11 paper & staple
(1-point penalty each otherwise)
(c) Due May 19 either 9:00 am in class or 8:45 am in the instructor's mailbox;
late homework gets 0.



1. The purpose of a receiving antenna is to extract power from incident radiation and deliver it into a load (of resistance R_L). In a simple circuit model, the antenna is a source of EMF in series with the antenna radiation resistance R_A. (See the figure.)

a. Show that maximum power is delivered to the load, generally a desirable condition, is when R_L=R_A. This is called load-matching. In terms of the EMF $\varepsilon = \varepsilon_0 e^{-i\omega t}$ and the resistances, find the time-average power delivered to the load. From now on the antenna is considered to be load-matched.

b. The “cross section” of an antenna is defined through $\langle P\rangle = \langle S\rangle A$ where $\langle P\rangle$ is the time-average power delivered to the load, $\langle S\rangle$ is the time-average Poynting flux, and *A* is the cross section. In terms of the free-space wavelength λ of the radiation and antenna parameters, find the cross section for the infinitesimal electric dipole of Griffiths section 11.1.2. You should assume the incident wave arrives in the mid-plane of the antenna with polarization that maximizes the signal. Hint: The EMF amplitude induced in the antenna is $\varepsilon_0 = E_0 L$, where E_0 is the amplitude of the incident electric field, and L is the length of the antenna.

c. Comment on the relative sizes of *A* and L^2.

d. The “directivity” of an antenna (or “gain”) G in some direction is given by the cross section of the antenna for radiation from that direction (with optimal polarization, usually) divided by the cross section of a fictional antenna that radiates power isotropically (and thus has cross section $\frac{\lambda^2}{4\pi}$). Find the gain of the antenna in part b. If your signal is coming from a small solid angle, the higher G the better, obviously. Those awful cheap “HD antennas” you see advertised on TV might have gain around 2. A very high quality rooftop TV antenna might have gain of 100; this delivers 50 times more power to the load than the cheap antenna. The downside of high gain is its very narrow reception pattern: a high-gain antenna only receives signals from a very narrow angular range.

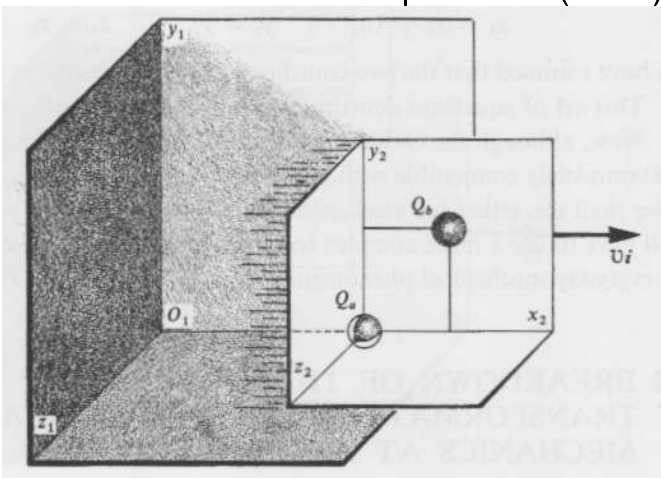
This problem also has a very deep connection to thermodynamics and the notion of thermal equilibrium. You could have derived part b this way.

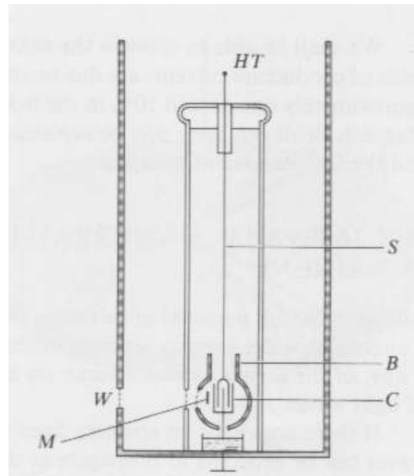
2. Suppose you have potentials **A** and V in some unspecified gauge.
a. You'd like to transform these into Lorentz gauge **A'** and V'. (Actually, another physicist Lorenz first wrote down this gauge, but the better-known physicist Lorentz got the credit: see the history of gauges Jackson & Okun 2001.) What's the differential equation for the gauge function λ?
b. There's not just one non-trivial solution to the differential equation of part a. What do these various gauge function solutions have in common? Why are there multiple non-trivial solutions? Hint: degrees of freedom.
c. At this point you by fiat set $\partial V/\partial t = 0$, giving new potentials **A''** and V''. Why am I allowed to do this? What is this new gauge?
d. Can I continue this? For instance, after section c., can I now by fiat set A_x=0 and get yet new potentials **A'''** and V'''?

3. A pulsar is a spinning neutron star where the magnetic-dipole axis of the star is misaligned by an angle θ with its spin axis. The radius of a typical neutron star is around 10 km, and its pole field is around 10^{12} Gauss.
a. The famous Crab pulsar has a period of 33 ms. Estimate how much power is it radiating?
b. The rarer magnetar has pole field or around 10^{15} Gauss and some have period around 1 ms. Estimate how much power is it radiating?

Note for another day: The spinning magnetic field of the magnetar is so high that you could envision issues of particle creation and annihilation coming into play.

4. Trouton and Noble experiment (1902).





Consider two charges at rest in frame 2: Q_a and Q_b. Trouton and Noble were interested in the electromagnetic force on Q_b. They suspended a charged parallel-plate capacitor is such a way that its plates were vertical and the capacitor could rotate about a vertical axis (see right figure). Since the Earth has a velocity relative to the Sun (treating the Sun fixed), the capacitor has that velocity relative to the Sun, and that velocity is normal to the face of the plates. One plate represents Q_a, the other Q_b.

a. Assume Galilean relativity: what forces act on Q_2? Hint: in Galalean relativity are there magnetic forces?

b. Still assuming Galilean relativity: after some time what's the orientation of the capacitor plates?

c. Trouton and Noble did not see any particular preferred orientation for the plates. What's the preferred modern explanation for this null result?