

## Physics 323, Spring Quarter 2016

### Electrodynamics: Homework Assignment 2

**(a) Turn in all problems and clearly note all constants and assumptions you use.**

**(1-point penalty each otherwise)**

**(b) Use 8½ x 11 paper & staple**

**(1-point penalty each otherwise)**

**(c) Due April 14 either 9:00 am in class or 8:45 am in the instructor's mailbox; late homework gets 0.**

1. Consider electromagnetic waves in a medium containing completely free charges; there's no restoring force and no damping, only inertia. The non-magnetic medium consists of  $N$  free particles in a volume  $V$ , each charge of mass  $m$  and charge  $+Q$ . To keep the system neutral, there's  $N$  rigidly fixed charges each of charge  $-Q$ . The fixed charges play no dynamical role.

a. Each particle feels an external field  $\mathbf{E}$ . From Newton's law, write the equation of motion of a charge.

b. Suppose  $\mathbf{E} = \mathbf{E}_0 e^{-i\omega t} \hat{x}$ . Find the displacement of a charge.

c. What's the phase relation between  $\mathbf{E}$  and the particle displacement?

d. Find the dipole moment  $d$  of a charge and find the dipole moment per unit volume  $P$  of the free charges.

e. Find the electric susceptibility  $\chi_E$  and the relative dielectric constant  $\epsilon_R$  of the medium. You can assume a linear medium.

2. A plane wave travels in the above medium. The complex wave number  $\mathbf{k}$  is in the z-direction,  $\mathbf{E}_0$  is along the x-direction, and  $\mathbf{B}_0$  is along the y-direction. You can assume the total free charge is zero and the total free current is zero.

- a. Taking the free current as zero seems wrong since there are free charges that can move in response to  $\mathbf{E}$  fields. Why can we assume this?
- b. Find the wave number  $k$  in terms of  $\omega$  and  $\epsilon_R$ .
- c. Find the relations between  $\mathbf{E}$  and  $\mathbf{B}$ .

Now assume there's a boundary (the x-y plane) between vacuum and the medium.

- d. In terms of  $\omega$  and  $k$ , find the ratio  $r$  between the amplitudes of the reflected and incident waves.
- e. Find the reflection coefficient (sometimes called the reflectance) of the medium using  $k$  from part b and  $\epsilon_R$  from Q1 part e. Hint  $R=rr^*$ .

3. In the infrared, silicon has a relative dielectric constant of 11.6. Consider the interface between vacuum and silicon. The following require numerical answers.

- a. What's the refractive index?
- b. What's the normal-incidence reflection coefficient? What's the phase change between incident and reflected wave?
- c. What's Brewster's angle?
- d. For incident waves at Brewster's angle, what's the ratio  $r$  (defined in Q2 part d) for the two polarizations?(One of them is zero, of course.)

4. Consider a hollow conducting cylindrical tube of inner radius  $R$  possibly containing solely transverse (relative to the tube axis) electric and magnetic fields.

- a. What is the "parallel" (tangential) component of  $\mathbf{E}$  at the inside surface of the tube?
- b. Find the components of  $\mathbf{E}$  and  $\mathbf{B}$  everywhere inside the tube. Explain.
- c. Discuss how your result in part b would change if there were a small-diameter conductor coaxial with the tube.
- d. You can obviously shine light down the tube axis. How do you reconcile this observation with your result of part b?

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