Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$; $\quad d\tau = dx\,dy\,dz$

Gradient: $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl: $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian: $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$; $\quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient: $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Curl: $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}}$$
$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian: $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$; $\quad d\tau = s\,ds\,d\phi\,dz$

Gradient: $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Curl: $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

Laplacian: $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

FUNDAMENTAL CONSTANTS

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/Nm^2}$	(permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$	(permeability of free space)
$c = 3.00 \times 10^8\ \mathrm{m}/s$	(speed of light)
$e = 1.60 \times 10^{-19}\ \mathrm{C}$	(charge of the electron)
$m = 9.11 \times 10^{-31}\ \mathrm{kg}$	(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{r}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{r}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \tan^{-1}\left(\sqrt{x^2+y^2}/z\right) \\ \phi = \tan^{-1}(y/x) \end{cases} \qquad \begin{cases} \hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \end{cases}$$

Cylindrical

$$\begin{cases} x = s\cos\phi \\ y = s\sin\phi \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{s}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{s}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\begin{cases} s = \sqrt{x^2+y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{s}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$\mathbf{F} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{r^2}\hat{r}$ Coulomb's Law $\quad \mathbf{F} = Q\mathbf{E} + Q\mathbf{v}\times\mathbf{B}$ Lorentz force

$\mathbf{E} = \dfrac{1}{4\pi\varepsilon_0}\iiint \dfrac{\rho(\mathbf{r}')}{r^2}\hat{r}\, d\tau'$ electric field from continuous charge distribution

$$\oiint \mathbf{E} \bullet d\mathbf{a} = \frac{1}{\varepsilon_0} Q_{\text{encl}}$$ Gauss' Law (integral) $\oint \mathbf{E} \bullet d\mathbf{l} = 0$ (statics)

$$V(\mathbf{r}) = -\int_{\wp}^{\mathbf{r}} \mathbf{E} \bullet d\mathbf{l}$$ Electrostatic potential, and $\mathbf{E} = -\nabla V$ (statics)

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$ Poisson's equation, and $\nabla^2 V = 0$ Laplace's equation (regions of no charge)

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$ and $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho(\mathbf{r}')}{r} d\tau'$ (setting reference point at infinity)

$$\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\varepsilon_0} \hat{\mathbf{n}}$$ and $\frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} = \frac{1}{\varepsilon_0} \sigma$ boundary conditions

$$D^{\perp}_{\text{above}} - D^{\perp}_{\text{below}} = \sigma_f$$

$$V(\mathbf{b}) - V(\mathbf{a}) = \frac{W}{Q} \quad W = \frac{1}{2} \sum_{i=1}^{n} q_i V(\mathbf{r}_i)$$ and $W = \frac{1}{2} \iiint \rho V \, d\tau$

$$Q = CV \quad W = \frac{1}{2} CV^2 \quad W = \frac{1}{2} Q^2 / C$$ capacitors

$$\nabla^2 V = -\frac{1}{\varepsilon_0} \rho$$ Poisson's equation $\nabla^2 V = 0$ Laplace's equation

$$V(x,y) = \sum_{n=0}^{\infty} (A_n e^{+kx} + B_n e^{-kx})(C_n \sin ky + D_n \cos ky)$$ solution to Laplace's equation in Cartesian coordinates in two dimensions

$$\int_0^a \sin(n\pi \frac{y}{a}) \sin(n'\pi \frac{y}{a}) dy = \begin{cases} 0 & n \neq n' \\ \frac{a}{2} & n = n' \end{cases}$$ orthogonality of sines

$$\int_0^a \sin(n\pi \frac{y}{a}) dy = \begin{cases} 0 & \text{n even} \\ \frac{2a}{n\pi} & \text{n odd} \end{cases} \qquad e^{+kx} + e^{-kx} = 2\cosh kx$$

$\int_0^{\pi} \cos^2\theta \sin\theta \, d\theta = 2/3$

$$P_0(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = (3x^2 - 1)/2$$
$$P_3(x) = (5x^3 - 3x)/2$$
$$P_4(x) = (35x^4 - 30x^2 + 3)/8$$
$$P_5(x) = (63x^5 - 70x^3 + 15x)/8$$

TABLE 3.1 Legendre Polynomials.

$$V(r,\theta) = \sum_{\ell=0}^{\infty} (A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}) P_\ell(\cos\theta)$$

solution to Laplace's equation in spherical coordinates with azimuthal symmetry

$\int_0^{\pi} P_\ell(\cos\theta) P_{\ell'}(\cos\theta) \, d\cos\theta = \begin{cases} 0 & \ell \neq \ell' \\ \dfrac{2}{2\ell+1} & \ell = \ell' \end{cases}$ orthogonality of Legendre polynomials

$V(r,\theta) = -E_0 \left(r - \dfrac{R^3}{r^2} \right) \cos\theta$ potential outside a neutral conducting sphere in uniform field

Law of cosines $c^2 = a^2 + b^2 - 2ab \cos\gamma$, with γ the angle opposite to side c.

$\dfrac{1}{\Re} = \dfrac{1}{r} \sum_0^{\infty} \left(\dfrac{r'}{r} \right)^n P_n(\cos\alpha)$ 1/r expansion in Legendre polynomials

$V(r) = \dfrac{1}{4\pi\varepsilon_0} \sum_0^{\infty} \dfrac{1}{r^{n+1}} \iiint (r')^n P_n(\cos\alpha) \rho(\mathbf{r}') d\tau'$ multipole expansion

$\vec{p} = \iiint \vec{r}' \rho(\vec{r}') d\tau'$ $\quad \vec{p} = \sum_1^n q_i \vec{r}_i'$ $\quad V_{\text{dip}}(\vec{r}) = \dfrac{1}{4\pi\varepsilon_0} \dfrac{\vec{p} \bullet \hat{r}}{r^2}$ dipole moment

$$\vec{E}_{\text{dip}} = \frac{p}{4\pi\varepsilon_0 r^3} (2\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta})$$

$$\mathbf{F}_{\text{mag}} = \int I \, d\mathbf{l} \times \mathbf{B}$$

$\mathbf{K} = d\mathbf{I} / d\ell_\perp$ $\quad \mathbf{J} = d\mathbf{I} / da_\perp$ surface and volume currents

$\nabla \bullet \mathbf{J} + \dfrac{d\rho}{dt} = 0$ conserved current, continuity equation

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}' \times \hat{r}}{|\mathbf{r} - \mathbf{r}'|^2} \quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K} \times \hat{r}}{|\mathbf{r} - \mathbf{r}'|^2} da' \quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I \int \frac{\mathbf{J} \times \hat{r}}{|\mathbf{r} - \mathbf{r}'|^2} d\tau'$$

$$\oint \mathbf{B} \bullet d\mathbf{l} = \mu_0 I_{\text{enc}}$$

$\mathbf{B} = \nabla \times \mathbf{A}$ magnetic vector potential.

$\nabla \bullet \mathbf{A} = 0$ "Coulomb gauge" convention $\quad \nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$ (for Coulomb gauge)

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\tau' \quad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} da' \quad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{l}'$$

$$\oint \mathbf{A} \bullet d\mathbf{l} = \Phi_m$$

$$B^{\perp}_{\text{above}} - B^{\perp}_{\text{below}} = 0 \qquad B^{\parallel}_{\text{above}} - B^{\parallel}_{\text{below}} = \mu_0 K \qquad \mathbf{B}_{\text{above}} - \mathbf{B}_{\text{below}} = \mu_0 \mathbf{K} \times \hat{n}$$

$$\mathbf{A}_{\text{above}} - \mathbf{A}_{\text{below}} = 0 \qquad \frac{\partial \mathbf{A}_{\text{above}}}{\partial n} - \frac{\partial \mathbf{A}_{\text{below}}}{\partial n} = -\mu_0 \mathbf{K}$$

$$\mathbf{m} = I \iint \hat{n}\, da = I\mathbf{a}$$

$$\mathbf{m} = \frac{1}{2} \oint \mathbf{r} \times \mathbf{I}\, d\ell \quad \mathbf{m} = \frac{1}{2} \iint \mathbf{r} \times \mathbf{K}\, da \quad \mathbf{m} = \frac{1}{2} \iiint \mathbf{r} \times \mathbf{J}\, d\tau$$

$$\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{r}}{r^2}$$

$$\mathbf{N} = \mathbf{m} \times \mathbf{B} \qquad \mathbf{F} = \nabla(\mathbf{m} \bullet \mathbf{B}) \qquad U = -\mathbf{m} \bullet \mathbf{B}$$

$$\mathbf{J}_b = \nabla \times \mathbf{M} \qquad \mathbf{K}_b = \mathbf{M} \times \hat{n}$$

$$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M} \qquad \nabla \times \mathbf{H} = \mathbf{J}_f \qquad \oint \mathbf{H} \bullet d\mathbf{l} = I_{\text{f,encl}}$$

$$H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} = -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}}) \qquad \mathbf{H}^{\parallel}_{\text{above}} - \mathbf{H}^{\parallel}_{\text{below}} = \mathbf{K}_f \times \hat{n}$$

$$\mathbf{M} = \chi_m \mathbf{H} \qquad \mu = \mu_0 (1 + \chi_m) \qquad \mathbf{B} = \mu \mathbf{H}$$

$$\mathbf{J} = \sigma \mathbf{E} \qquad V = IR \quad P = VI = I^2 R$$

$$\mathcal{E} = \oint \mathbf{E} \bullet d\ell \quad \mathcal{E} = -\frac{d\Phi_M}{dt}$$

$$\Phi_2 = M\, I_1 \quad \Phi = LI \quad \mathcal{E}_2 = -M \frac{dI_1}{dt} \quad \mathcal{E} = -L \frac{dI}{dt} \quad W = \frac{1}{2} L I^2$$

$$\nabla \bullet \left(\frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \right) + \frac{\partial}{\partial t} \left(\frac{\varepsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2 \right) = -\mathbf{E} \bullet \mathbf{J}$$ Poynting's theorem (differential)

$$\frac{dW}{dt} = -\frac{d}{dt} \iiint \left(\frac{\varepsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2 \right) d\tau - \oiint \left(\frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \right) \bullet \hat{n}\, da$$ Poynting's thm (integral)

$$\frac{\partial^2 f}{\partial z^2} - \frac{1}{V^2}\frac{\partial^2 f}{\partial t^2} = 0 \quad \lambda = 2\pi / k \quad T = 2\pi / kV \quad \nu = 1/T \quad \omega = 2\pi\nu = kV$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

vacuum $\nabla^2 \mathbf{E} - \mu_0\varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad \nabla^2 \mathbf{B} - \mu_0\varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0 \quad c = 1/\sqrt{\mu_0\varepsilon_0}$

$\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i(kz-\omega t)} \quad \tilde{\mathbf{B}}(z,t) = \tilde{\mathbf{B}}_0 e^{i(kz-\omega t)} \quad E_0 = cB_0$ (vacuum)

$\langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2 \quad \langle \mathbf{S} \rangle = \frac{1}{2} c\varepsilon_0 E_0^2 \hat{k}$ (vacuum) $\quad I = \langle S \rangle \quad \langle \mathbf{p} \rangle = \frac{1}{2c}\varepsilon_0 E_0^2 \hat{k}$

lossless $\nabla^2 \mathbf{E} - \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad \nabla^2 \mathbf{B} - \mu\varepsilon \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0 \quad v = 1/\sqrt{\mu\varepsilon} = c/n \quad n = \sqrt{\mu\varepsilon / \mu_0\varepsilon_0}$

$$u = \frac{1}{2}\left(\varepsilon E^2 + \frac{1}{\mu}B^2\right) \quad \mathbf{S} = \frac{1}{\mu}\mathbf{E}\times\mathbf{B} \quad I = \frac{1}{2}\varepsilon v E_0^2$$

$$\tan\theta_B \cong n_2 / n_1 \quad \sin\theta_c = n_2 / n_1$$

$\tilde{E}_{0R} = \frac{1-\beta}{1+\beta}\tilde{E}_{0I} \quad \tilde{E}_{0T} = \frac{2}{1+\beta}\tilde{E}_{0I} \quad \beta = \frac{\mu_1 v_1}{\mu_2 v_2}$ normal incidence

lossy $\nabla^2 \mathbf{E} - \mu\sigma \frac{\partial \mathbf{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad \nabla^2 \mathbf{B} - \mu\sigma \frac{\partial \mathbf{B}}{\partial t} - \mu\varepsilon \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$

$$\tilde{k}^2 = \mu\varepsilon\omega^2 + i\mu\sigma\omega \quad d = 1/\operatorname{Im}\tilde{k}$$

$$\operatorname{Re}\tilde{k} = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2} + 1\right]^{1/2} \quad \operatorname{Im}\tilde{k} = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2} - 1\right]^{1/2}$$

poor conductor $\operatorname{Re}\tilde{k} = \sqrt{\mu\varepsilon}\,\omega \quad \operatorname{Im}\tilde{k} = \sqrt{\frac{\mu}{\varepsilon}}\frac{\sigma}{2}$

good conductor $\operatorname{Re}\tilde{k} = \operatorname{Im}\tilde{k} = \sqrt{\frac{\omega\sigma\mu}{2}}$

$$\ddot{y}+\gamma\dot{y}+\omega_0^2 y=\frac{e}{m}E(t) \qquad E(t)=E_0 e^{i\omega t} \qquad y(t)=y_0 e^{i\omega t} \qquad y_0=\frac{e/m}{\left(\omega_0^2-\omega^2\right)-i\gamma\omega}E_0$$

$$\varepsilon=\varepsilon_0+\frac{Nfe^2}{m}\frac{1}{\left(\omega_0^2-\omega^2\right)}-i\gamma\omega$$

dilute $n=\frac{c}{\omega}\operatorname{Re}\tilde{k}=1+\frac{1}{2\varepsilon_0}\frac{Nfe^2}{m}\frac{\left(\omega_0^2-\omega^2\right)}{\left(\omega_0^2-\omega^2\right)^2+\gamma^2\omega^2}$

$$\alpha=2\operatorname{Im}\tilde{k}=1+\frac{\omega}{c\varepsilon_0}\frac{Nfe^2}{m}\frac{\gamma\omega}{\left(\omega_0^2-\omega^2\right)^2+\gamma^2\omega^2}$$

plasma $\sigma=\frac{Nfe^2}{m}\frac{1}{\gamma-i\omega}$ dilute plasma $\sigma=\frac{Nfe^2}{m}\frac{1}{-i\omega}$ $\omega_p=\sqrt{\frac{Nfe^2}{\varepsilon_0 m}}$

dilute plasma $\tilde{k}^2=\frac{1}{c^2}\left\{\omega^2-\omega_p^2\right\}$

waveguide transverse fields

$$E_x=\frac{i}{\left(\omega/c\right)^2-k^2}\left(k\frac{\partial E_z}{\partial x}-\omega\frac{\partial B_z}{\partial y}\right)$$

$$E_y=\frac{i}{\left(\omega/c\right)^2-k^2}\left(k\frac{\partial E_z}{\partial y}-\omega\frac{\partial B_z}{\partial x}\right)$$

$$B_x=\frac{i}{\left(\omega/c\right)^2-k^2}\left(k\frac{\partial B_z}{\partial x}-\frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)$$

$$B_y=\frac{i}{\left(\omega/c\right)^2-k^2}\left(k\frac{\partial B_z}{\partial y}-\frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right)$$

waveguide longitudinal fields

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\left(\omega/c^2\right)-k^2\right]E_z=0$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\left(\omega/c^2\right)-k^2\right]B_z=0$$

"waveguide equation" $\frac{1}{\lambda_g^2}+\frac{1}{\lambda_c^2}=\frac{1}{\lambda_0^2}$

TE & TM guided wavenumber $k^2 = \left(\frac{\omega}{c}\right)^2 - \pi^2\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]$

TE & TM cutoff angular frequency $\omega_{mn} = c\pi\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

phase velocity $v = \omega / k$ group velocity $\frac{1}{dk / d\omega}$

dynamical potentials $\mathbf{E} = -\nabla V - \partial\mathbf{A}/\partial t$, $\mathbf{B} = \nabla\times\mathbf{A}$

potential equations $\nabla^2 V + \frac{\partial}{\partial t}\nabla\bullet\mathbf{A} = -\rho/\varepsilon_0$ $\left(\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}\right) - \nabla\left(\nabla\bullet\mathbf{A} + \frac{1}{c^2}\frac{\partial V}{\partial t}\right) = -\mu_0\mathbf{J}$

gauge transformations $A' = A + \nabla\lambda$ $V' = V - \partial\lambda/\partial t$

Coulomb gauge $\nabla\bullet\mathbf{A} = 0$, Lorentz gauge $\nabla\bullet\mathbf{A} + \frac{1}{c^2}\partial V/\partial t = 0$

retarded time $t_r = t - r/c$

retarded potentials $V(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\iiint\frac{\rho(\mathbf{r}',t_r)}{r}d\tau'$ $\mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\iiint\frac{\mathbf{J}(\mathbf{r}',t_r)}{r}d\tau'$

Jefimenko's equations

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\iiint\left[\frac{\rho(\mathbf{r}',t_r)}{r^2}\hat{r} + \frac{\dot{\rho}(\mathbf{r}',t_r)}{cr}\hat{r} - \frac{\dot{\mathbf{J}}(\mathbf{r}',t_r)}{c^2 r}\right]d\tau'$$

$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\iiint\left[\frac{\mathbf{J}(\mathbf{r}',t_r)}{r^2} + \frac{\dot{\mathbf{J}}(\mathbf{r}',t_r)}{cr}\hat{r}\right]\times\hat{r}\,d\tau'$$

Liénard-Wiechert potentials

$$V(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\frac{qc}{(rc - \mathbf{r}\bullet\mathbf{v})}$$

$$\mathbf{A}(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\frac{qc\mathbf{v}}{(rc - \mathbf{r}\bullet\mathbf{v})} = \frac{\mathbf{v}}{c^2}V(\mathbf{r},t)$$

Point charges $\mathbf{E}(\mathbf{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{r}{(\mathbf{r}\bullet u)^3}\left[(c^2 - v^2)\mathbf{u} + \mathbf{r}\times\mathbf{u}\times\mathbf{a}\right]$, $\mathbf{u} = c\hat{\mathbf{r}} - \mathbf{v}$

radiation fields $\mathbf{E}_{\text{rad}}(\mathbf{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{r}{(\mathbf{r}\bullet u)^3}[\mathbf{r}\times\mathbf{u}\times\mathbf{a}]$, $\mathbf{S}_{\text{rad}}(\mathbf{r},t) = \frac{1}{\mu_0 c}E_{\text{rad}}^2\hat{\mathbf{r}}$

charge at rest at time t_r $\mathbf{E}_{\text{rad}}(\mathbf{r},t)=\frac{\mu_0 q}{4\pi r}\left[(\hat{\mathbf{r}}\bullet\mathbf{a})\hat{\mathbf{r}}-\mathbf{a}\right]$ $\mathbf{S}_{\text{rad}}(\mathbf{r},t)=\frac{\mu_0 q^2 a^2}{16\pi^2 c}\frac{\sin^2\theta}{r^2}\hat{\mathbf{r}}$

magnetic dipole radiation $\langle P\rangle=\frac{\mu_0 m_0^2\omega^4}{12\pi c^3}$ electric dipole radiation $\langle P\rangle=\frac{\mu_0 p_0^2\omega^4}{12\pi c}$

Larmor formula $P=\frac{\mu_0 q^2 a^2}{6\pi c}$

generalization of Larmor formula $P=\frac{\mu_0 q^2\gamma^6}{6\pi c}\left(a^2-\left|\frac{\mathbf{v}\times\mathbf{a}}{c}\right|^2\right)$, $\gamma^2=\frac{1}{1-(v/c)^2}$

Abraham-Lorentz radiation-reaction force $\mathbf{F}=\frac{\mu_0 q^2}{6\pi c}\dot{\mathbf{a}}$

relativistic velocity addition $v_{AC}=\frac{v_{AB}+v_{BC}}{1+v_{AB}v_{BC}/c^2}$

velocity factor $\beta=v/c$ boost factor $\gamma=\frac{1}{\sqrt{1-v^2/c^2}}$

Lorentz transformation
$$\begin{aligned}x'&=\gamma(x-vt)\\ y'&=y\\ z'&=z\\ t'&=\gamma\left(t-\frac{v}{c^2}x\right)\end{aligned}\qquad\text{or}\qquad\begin{aligned}x^{0\,\prime}&=\gamma\left(x^0-\beta x^1\right)\\ x^{1\,\prime}&=\gamma\left(x^1-\beta x^0\right)\\ x^{2\,\prime}&=x^2\\ x^{3\,\prime}&=x^3\end{aligned}$$

Lorentz transformation
$$\begin{pmatrix}x^{0\,\prime}\\ x^{1\,\prime}\\ x^{2\,\prime}\\ x^{3\,\prime}\end{pmatrix}=\begin{pmatrix}\gamma & -\gamma\beta & 0 & 0\\ -\gamma\beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x^0\\ x^1\\ x^2\\ x^3\end{pmatrix}\quad\text{or}\quad x^{\mu'}=\Lambda^{\mu}_{\nu}x^{\nu}$$

scalar product $a^\mu a_\mu=a^\mu g_{\mu\nu}a^\nu$ with $g_{\mu\nu}=\begin{pmatrix}-1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$ (note summation notation)

spacelike $a^\mu a_\mu>0$ timelike $a^\mu a_\mu<0$ lightlike $a^\mu a_\mu=0$

invariant interval $(\Delta x)^\mu(\Delta x)_\mu=-c^2t^2+d^2$

proper time $d\tau = dt/\gamma$ 4-velocity $\eta = \frac{dx^\mu}{d\tau}$

4-momentum $p^\mu = m\eta^\mu$ $p^\mu = \left(E/c, \vec{p}\right)$ $p^\mu p_\mu = -m^2c^2$ $E^2 - p^2c^2 = m^2c^4$

Compton wavelength $\lambda_C = h/mc$

relativistic force $\mathbf{F} = \frac{d\mathbf{P}}{dt}$ with $\mathbf{P}$ the relativistic momentum

transformation of force for particle at rest in unprimed frame $\mathbf{F}_\perp{}' = \mathbf{F}_\perp/\gamma$ $F_\parallel{}' = F_\parallel$

Minkowski force $K^\mu = \frac{dp^\mu}{d\tau}$ center of energy $\mathbf{R}_E = \frac{1}{M}\sum_i m_i \mathbf{r}_i$ $\mathbf{P}_{\text{tot}} = \frac{E}{c^2}\frac{d\mathbf{R}_E}{d\tau}$

transformation of fields

$$E'_x = E_x \quad E'_y = \gamma\left(E_y - vB_z\right) \quad E'_z = \gamma\left(E_z + vB_y\right)$$

$$B'_x = B_x \quad B'_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right) \quad B'_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right)$$

field strength tensor $$\mathbf{F}^{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix}$$

dual of field strength tensor $$\tilde{\mathbf{F}}^{\mu\nu} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z/c & E_y/c \\ -B_y & E_z/c & 0 & -E_x/c \\ -B_z & -E_y/c & E_x/c & 0 \end{pmatrix}$$

Maxwell's equations $\frac{\partial \mathbf{F}^{\mu\nu}}{\partial x^\nu} = \mu_0 J^\mu$ $\frac{\partial \tilde{\mathbf{F}}^{\mu\nu}}{\partial x^\nu} = 0$

Potential formulation $A^\mu = \left(V/c, \vec{A}\right)$ $\mathbf{F}^{\mu\nu} = \frac{\partial A^\nu}{\partial x_\mu} - \frac{\partial A^\mu}{\partial x_\nu}$

Maxwell's Equations

In general:

$$\begin{cases} \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{cases}$$

In matter:

$$\begin{cases} \nabla \cdot \mathbf{D} = \rho_f \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \end{cases}$$

Auxiliary Fields

Definitions:

$$\begin{cases} \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \end{cases}$$

Linear media:

$$\begin{cases} \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}, \quad \mathbf{D} = \epsilon \mathbf{E} \\ \mathbf{M} = \chi_m \mathbf{H}, \quad \mathbf{H} = \frac{1}{\mu}\mathbf{B} \end{cases}$$

Potentials

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

Lorentz force law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Energy, Momentum, and Power

Energy: $U = \frac{1}{2}\int \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) d\tau$

Momentum: $\mathbf{P} = \epsilon_0 \int (\mathbf{E} \times \mathbf{B})\, d\tau$

Poynting vector: $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$