# APPENDIX I

## UNITS AND DIMENSIONS IN ELECTROMAGNETIC THEORY

Classical mechanics is characterized by the fact that its mathematical formulation does not contain any fundamental constants inherent in the theory. Hence all physical laws in classical mechanics "scale" perfectly for any change in parameters over any arbitrary range of magnitudes. It is customary, although not at all mandatory, to formulate classical mechanics in terms of three-dimensional entities: mass ($M$), length ($L$), and time ($T$). The number could be increased, for instance, by choosing the constant $K$ in the equation Volume $= K$ (Length)$^3$ to be different from unity and to have dimensions. The dimension $L^3$ is customarily identified with volume by choice, not by necessity. Similarly, the number of fundamental units can be decreased by arbitrarily defining certain constants to be unity and dimensionless. The convention $c = \hbar = 1$ frequently used in quantum-mechanical calculations is such an example. In these units $L = T = M^{-1}$ arbitrarily.

We mention these examples only to indicate that the number of independent dimensions is arbitrary even in classical mechanics, although convenience suggests a specific choice. In general, the greater the number of dimensional entities chosen, the more independent units can be chosen to suit the orders of magnitude convenient for a particular purpose. It should be remembered, however, that changing units or even numbers of dimensions does not affect the physical content of any equation if it is correctly interpreted.

In classical mechanics the $MLT$ system is used conventionally, and hence the issues discussed above are usually not of interest. For electromagnetic theory, the conventions are of more recent origin and appear more controversial.

Electromagnetic theory differs from classical mechanics by the fact that one constant, $c$, the velocity of light *in vacuo,* appears as a fundamental constant of the theory. Physical laws thus "scale" correctly over arbitrary magnitudes only if ratios of length and time are held constant. In this property electromagnetic theory exhibits a feature which special relativity extends to all laws of physics.

If the $MLT$ system is used in the mechanical quantities in electromagnetic theory, the constant $c$ having dimensions $LT^{-1}$ will appear explicitly. Whether any additional dimensional units are introduced is entirely a matter of convention. As an example, if in Coulomb's law in the form $F = Kq_1q_2/r^2$ the constant $K$ is chosen arbitrarily to be dimensionless, then the charge $q$ automatically acquires the dimensions $L^{3/2}M^{1/2}T^{-1}$ and no basic units beyond those of $M$, $L$, and $T$ need be specified. The justification for this procedure is analogous to that for setting $K$ in the equation $V = KL^3$ equal to a dimensionless constant and thus giving a volume the dimension $L^3$.

If we do not choose a dimensionless constant in one of the equations relating mechanical and electromagnetic quantities, then we retain the freedom of choosing one of the electrical units arbitrarily and assigning to it a dimension. This has been done in what is called the mks system.

The particular set of units employed in this text is the mks system now in fairly general use. The principal convenience of this system is that it incorporates the common technical units—volt, ampere, coulomb, etc.—and is thus particularly suitable for treating applications that involve both "lumped" circuit parameters and fields. Since these technical units imply that the unit of time is the second and also define the power simply in watts, the natural choice of mechanical units are the meter, kilogram, and second. With this choice of electrical and mechanical units the constants $\epsilon_0$ and $\mu_0$ in the equations

$$\mathbf{F} = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{\mathbf{r}_{12}}{r_{12}^3}, \tag{1}$$

$$\mathbf{F} = \frac{\mu_0}{4\pi} \iint \frac{\mathbf{j}_1 \times (\mathbf{j}_2 \times \mathbf{r}_{12})}{r_{12}^3} \, dv_1 \, dv_2 \tag{2}$$

can be determined once we have selected a basic electrical unit. We shall postpone their numerical determination until we have seen how the electrical unit was chosen, but we note that two constants, $\epsilon_0$ and $\mu_0$, are carried in the equations when they are written in mks units, although only one constant, $c$, is fundamental to the theory.

Historically, a set of units (esu) was defined by using unity in place of $4\pi\epsilon_0$ in Coulomb's law, Eq. (1), and cgs units for mechanical quantities. This defines the electrostatic unit of charge with mechanical dimensions indicated above, and from this the units of potential, electric field, etc., are defined. On the other hand, if we set $\mu_0 = 4\pi$ in Eq. (2), and if cgs mechanical units are used, the equation defines a unit of current (with dimensions $M^{1/2}L^{1/2}T^{-1}$) called the electromagnetic unit (emu) or abampere. Units for other electrical quantities can be derived from the abampere and the cgs relations. The charge densities and current densities thus defined obey the relation

$$\mathbf{j}_{\text{emu}} = \rho_{\text{esu}} \frac{\mathbf{u}}{c}, \tag{3}$$

where $c$ appears here as the measured ratio of the units. This ratio was first determined by Weber and Kohlrausch by measuring the discharge of a capacitor whose electrostatic capacity was known. A consistent set of units (Gaussian units) is obtained by using the electrical quantities derived from $\rho_{\text{esu}}$ and the magnetic quantities derived from $\mathbf{j}_{\text{emu}}$ and carrying $c$ in the equations as the only constant.

As indicated in Eqs. (1) and (2), the mks system is commonly used in its rationalized form; the cgs units have been quoted in their unrationalized form. A rational system of units contains the factor $4\pi$ in Coulomb's and Ampère's laws of force so as to eliminate it in the Maxwell field equations which involve sources. General vector relations do not contain such factors. The appearance of the geometrical factor is indicated mathematically by the form of the Green's function. If a source (such as a point source) defines a problem in spherical symmetry, then in rational units one obtains $4\pi$ explicitly in the resulting solution; if the source (such as a line source) defines a field structure of circular symmetry, then $2\pi$ appears. A system of units analogous to the Gaussian system but in rationalized form is known as the Heaviside-Lorentz system.

Historically, the ampere was taken to be exactly $\frac{1}{10}$ of the abampere, or emu, of current. This fact enables us to determine the magnitude of $\mu_0$ in Eq. (2), since we have seen that the abampere is defined by setting $\mu_0/4\pi = 1$. It is customary to use the coulomb, not the ampere, as the basic electrical quantity, and we need only transform the defining equation of the electromagnetic system with all quantities of unit size into mks units. Explicitly, in the electromagnetic cgs system,

$$1 \text{ dyne} = 1 \text{ (abampere)}^2$$

[since all lengths cancel on the right side of Eq. (2)], which in mks units for which 1 abampere = 10 coulomb/second, becomes

$$10^{-5} \text{ newton} \equiv 10^{-5} \frac{\text{kilogram-meter}}{\text{second}^2} = \frac{\mu_0}{4\pi} \frac{10^2 \text{ (coulomb)}^2}{\text{(second)}^2}$$

or

$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{kilogram-meter}}{\text{(coulomb)}^2} = 4\pi \times 10^{-7} \frac{\text{henry}}{\text{meter}}.$$

The constant $\epsilon_0$ is now obtained from the relation $\epsilon_0\mu_0 = 1/c^2$,

$$\epsilon_0 = \left(\frac{10^7}{4\pi c^2} \simeq \frac{1}{36\pi \times 10^9}\right)\left(\frac{\text{(coulomb)}^2 \text{ (second)}^2}{\text{kilogram (meter)}^3} = \frac{\text{farad}}{\text{meter}}\right).$$

What has been done here is to define $\mu_0$ in terms of the arbitrarily chosen size and dimension of an electrical unit, whereupon $\epsilon_0$ is automatically fixed if the system is to be consistent with the mechanical units and the *experimental* value of the fundamental constant $c$.

Electrical units which are referred to mechanical standards via defining relations containing fixed numerical constants are called *absolute* electrical units. Actually, the accuracy with which the emu and esu could be realized in terms of their defining equations was until recently insufficient for practical purposes: during the period when the accuracy of verification of absolute units was inferior to the reproducibility of standards, the practical units were based on such standards* as the *international* ampere and the *international* ohm. Improvements in techniques have resulted in greatly improved absolute electrical measurements, so that the former international standards have been relegated to the role of secondary standards. Hence the value $\mu_0 = 4\pi \times 10^{-7}$ as an exact numerical constant refers to the practical units as absolute rather than international units. Note that $\epsilon_0$ in the mks system depends on the experimental relation between the velocity of electromagnetic radiation to the length and time standards (although $\mu_0$ does not); this corresponds to the explicit presence of $c$ in the Gaussian and Heaviside-Lorentz systems. In the "natural" system the velocity of light itself constitutes a standard. It is clear that an experimental measurement of the velocity of light can only provide a measure of the ratio of the velocity of propagation of electromagnetic radiation to the ratio of length and time standards. Hence the resultant number can never have any fundamental significance in an absolute sense, but is of great practical utility in providing independent accurate means of relating the length and time standards.

Clearly, the physical content of the fundamental relations is the same in all systems of units. It is easy to translate the laws of electrodynamics from one system to another: for vacuum conditions, the relations in this book are written so that the transformation

$$cB_{\text{mks}} \rightarrow B_{\text{Gaussian}},$$
$$\epsilon_0 \rightarrow (4\pi)^{-1},$$
$$\mu_0\epsilon_0 \rightarrow 1/c^2$$

will effect a reduction to their Gaussian equivalents. Table I–2 contains a brief summary of fundamental electromagnetic relations valid *in vacuo* as they appear in the various systems of units. For convenience in the numerical conversion of units, a list of the most important conversion factors is given in Table I–1. Table I–4 contains other numerical constants and functional relations useful in applications involving atomic particles.

* For example, the international ampere was defined as "the value of the unvarying current which on passing through a solution of silver nitrate in water in accordance with standard specifications deposits silver at the rate of 0.001118 gm per second."

The situation regarding the equations in material media is somewhat more complex than that for vacuum conditions and is frequently misunderstood. To gain some insight into the problem, let us consider the fundamental process by which Maxwell's equations in media are generated from the vacuum relations.

The vacuum source equations have the general form

$$D(F) = S, \tag{4}$$

where $D$ is a linear differential operator acting on the field $F$ and $S$ is the source. In media $S$ is broken up in terms of an "accessible" (macroscopic) source $S$ and an "inaccessible" source $S_p$, i.e.,

$$D(F) = S + S_p. \tag{5}$$

$S_p$ is then derived from an auxiliary quantity $F_p$ by the same differential operator $D$ such that

$$S_p = D(F_p) \tag{6}$$

and hence

$$D(F - F_p) = S. \tag{7}$$

A partial field $F_H = F - F_p$ can thus be defined such that

$$D(F_H) = S, \tag{8}$$

i.e., such that this field is derived from the "accessible" (often called "true") sources only. In Table I-3 this general statement is illustrated in terms of the actual electrodynamic quantities.

Ambiguity arises because in this general formulation quantities of the type $F_p$ (i.e., **P** and **M**) appear in a dual role. On the one hand, in the relations

$$\rho_P = -\nabla \cdot \mathbf{P}, \tag{9}$$

$$\mathbf{j}_M = \nabla \times \mathbf{M}, \tag{10}$$

**P** and **M** represent purely *source* quantities—they describe certain charge and current distributions. In both relations, on the other hand, **P** and **M** can be viewed as *fields*, namely, those electric or magnetic fields whose sources are $\rho_P$ or $\mathbf{j}_M$ respectively. In relations of the type

$$\epsilon_0 \mathbf{E} = \mathbf{D} - \mathbf{P}, \tag{11}$$

$$\mathbf{B}/\mu_0 = \mathbf{H} + \mathbf{M}, \tag{12}$$

as written in the now conventional mks system, **P** and **E** or **B** and **M** are given different units: **P** is measured in coulombs/meter$^2$, while **E** is in volts/meter; **M** is in amperes/meter, while **B** is in webers/meter$^2$. This convention emphasizes the roles of **M** and **P** as current and charge descriptions. The "partial field" aspect is, however, equally valid, and is somewhat obscured by the constants in Eqs. (11) and (12). In cgs units, Eqs. (11) and (12) become

$$\mathbf{E} = \mathbf{D} - 4\pi\mathbf{P}, \qquad \mathbf{B} = \mathbf{H} + 4\pi\mathbf{M} \text{ (Gaussian)},$$

$$\mathbf{E} = \mathbf{D} - \mathbf{P}, \qquad \mathbf{B} = \mathbf{H} + \mathbf{M} \text{ (Heaviside-Lorentz)}.$$

Here the units of all quantities are the same, and hence the "partial field" aspect is emphasized.

To solve field problems, it is in general necessary to specify the "constitutive equation" giving **P**(**E**) and **M**(**B**). If these are of linear form, such as

$$\mathbf{P} = \epsilon_0(\kappa - 1)\mathbf{E}, \tag{13}$$

it is often said that **E** represents an "intensive variable" and **P** an "extensive variable," i.e., **E** is cause and **P** effect. This point of view is emphasized by the fact that $\mathbf{E} \cdot \delta\mathbf{P}$ represents the differential of work done in this case. From this aspect the use of different units for **E** and **P** appears justified. Actually the cause-effect situation is very much less clear when permanent polarization (electrets) or permanent magnets are considered. It should be noted that the basic relations (11) and (12) are *additive*; on the other hand, relations of type (13) are not at all general.

We may summarize by saying that the question of whether **E**, **P**, and **D** (or **B**, **M**, and **H**) should have the same units is fairly irrelevant; in fact, an understanding of the *dual* physical function of **P** and **M** is the principal requirement for clarity in the classical theory of electric and magnetic media.

TABLE I-1

CONVERSION FACTORS

| Multiply the number of mks units below | by | to obtain the number of Gaussian (cgs) units of |
|---|---|---|
| ampere | $10^{-1}$ | current in abamperes |
| ampere/meter$^2$ | $10^{-5}$ | current density in abampere/cm$^2$ |
| coulomb | $3 \times 10^{9}$* | charge in esu |
| coulomb/meter$^3$ | $3 \times 10^{3}$* | charge density in esu/cm$^3$ |
| farad = coulomb/volt | $9 \times 10^{-11}$* | capacitance in cm |
| henry = volt sec/ampere | $10^9$ | inductance in emu |
| joule | $10^7$ | energy in ergs |
| newton | $10^5$ | force in dynes |
| ohm = volt/ampere | $\frac{1}{30}$* | resistance in esu of potential per abampere |
| | $\frac{1}{9} \times 10^{11}$* | resistance in esu |
| volt | $\frac{1}{300}$* | potential in esu |
| volt/meter | $\frac{1}{3} \times 10^{-4}$* | electric field intensity **E** in esu |
| coulomb/meter$^2$ | $12\pi \times 10^{5}$* | electric displacement **D** in esu |
| weber = volt second | $10^8$ | magnetic flux in maxwells |
| weber/meter$^2$ | $10^4$ | flux density **B** in gauss |
| ampere-turns/meter | $4\pi/10^3$ | field intensity **H** in oersteds |
| mho/meter | $\frac{3}{10}$* | conductivity in abamperes/cm$^2$/esu of field intensity |
| ampere turns | $4\pi/10$ | magnetomotive force in gilberts |

* In all conversion factors marked by an asterisk 3 is used for $c/10^{10}$, where in this definition $c$ is measured in cgs units. If higher accuracy is desired, a more precise value of $c$ must be substituted. DuMond and Cohen give $c = 299792.5$ km/sec. (J. W. M. DuMond and E. R. Cohen, 1961 adjustment of natural constants, to be published in *Annals of Physics* and *Nuovo Cimento*.)

## Table I–2

FUNDAMENTAL ELECTROMAGNETIC RELATIONS VALID "IN VACUO" AS THEY APPEAR IN THE VARIOUS SYSTEMS OF UNITS

| mks (rationalized) | Gaussian* (cgs) | Heaviside-Lorentz (cgs) | "Natural" units $c = \hbar = 1$ (rationalized) |
|---|---|---|---|
| $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$ | $\nabla \cdot \mathbf{E} = 4\pi\rho$ | $\nabla \cdot \mathbf{E} = \rho$ | $\nabla \cdot \mathbf{E} = \rho$ |
| $\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\rho \mathbf{r}}{r^3}\, dv$ | $\mathbf{E} = \int \frac{\rho \mathbf{r}}{r^3}\, dv$ | $\mathbf{E} = \frac{1}{4\pi} \int \frac{\rho \mathbf{r}}{r^3}\, dv$ | $\mathbf{E} = \frac{1}{4\pi} \int \frac{\rho \mathbf{r}}{r^3}\, dv$ |
| $\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)$ <br> $\mathbf{B} = \frac{\mu_0}{4\pi} \int \frac{\left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right) \times \mathbf{r}}{r^3}\, dv$ | $\nabla \times \mathbf{B} = 4\pi \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}$ <br> $\mathbf{B} = \int \frac{\left(\mathbf{j} + \frac{1}{4\pi c} \frac{\partial \mathbf{E}}{\partial t}\right) \times \mathbf{r}}{r^3}\, dv$ | $\nabla \times \mathbf{B} = \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}$ <br> $\mathbf{B} = \frac{1}{4\pi} \int \frac{\left(\mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}\right) \times \mathbf{r}}{r^3}\, dv$ | $\nabla \times \mathbf{B} = \mathbf{j} + \frac{\partial \mathbf{E}}{\partial t}$ <br> $\mathbf{B} = \frac{1}{4\pi} \int \frac{\left(\mathbf{j} + \frac{\partial \mathbf{E}}{\partial t}\right) \times \mathbf{r}}{r^3}\, dv$ |
| $\nabla \cdot \mathbf{B} = 0$ <br> $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ | $\nabla \cdot \mathbf{B} = 0$ <br> $c\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ | $\nabla \cdot \mathbf{B} = 0$ <br> $c\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ | $\nabla \cdot \mathbf{B} = 0$ <br> $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ |
| $\mathbf{F} = e(\mathbf{E} + \mathbf{u} \times \mathbf{B})$ | $\mathbf{F} = e\left(\mathbf{E} + \frac{\mathbf{u}}{c} \times \mathbf{B}\right)$ | $\mathbf{F} = e\left(\mathbf{E} + \frac{\mathbf{u}}{c} \times \mathbf{B}\right)$ | $\mathbf{F} = e(\mathbf{E} + \mathbf{u} \times \mathbf{B})$ |

| | | | |
|---|---|---|---|
| $\mathbf{B} = \nabla \times \mathbf{A}$<br>$\mathbf{E} = -\nabla\phi - \dfrac{\partial \mathbf{A}}{\partial t}$ | $\mathbf{B} = \nabla \times \mathbf{A}$<br>$\mathbf{E} = -\nabla\phi - \dfrac{1}{c}\dfrac{\partial \mathbf{A}}{\partial t}$ | $\mathbf{B} = \nabla \times \mathbf{A}$<br>$\mathbf{E} = -\nabla\phi - \dfrac{1}{c}\dfrac{\partial \mathbf{A}}{\partial t}$ | $\mathbf{B} = \nabla \times \mathbf{A}$<br>$\mathbf{E} = -\nabla\phi - \dfrac{\partial \mathbf{A}}{\partial t}$ |
| $\nabla \cdot \mathbf{j} + \dfrac{\partial \rho}{\partial t} = 0$<br>$\nabla \cdot \mathbf{A} + \dfrac{1}{c^2}\dfrac{\partial \phi}{\partial t} = 0$ | $\nabla \cdot \mathbf{j} + \dfrac{1}{c}\dfrac{\partial \rho}{\partial t} = 0$<br>$\nabla \cdot \mathbf{A} + \dfrac{1}{c}\dfrac{\partial \phi}{\partial t} = 0$ | $\nabla \cdot \mathbf{j} + \dfrac{1}{c}\dfrac{\partial \rho}{\partial t} = 0$<br>$\nabla \cdot \mathbf{A} + \dfrac{1}{c}\dfrac{\partial \phi}{\partial t} = 0$ | $\nabla \cdot \mathbf{j} + \dfrac{\partial \rho}{\partial t} = 0$<br>$\nabla \cdot \mathbf{A} + \dfrac{\partial \phi}{\partial t} = 0$ |
| $F^{ij} = \begin{pmatrix} 0 & -cB_z & cB_y & +E_x \\ cB_z & 0 & -cB_x & +E_y \\ -cB_y & cB_x & 0 & +E_z \\ -E_x & -E_y & -E_z & 0 \end{pmatrix}$<br>$\dfrac{\partial F^{ij}}{\partial x^i} = \dfrac{j^j}{\epsilon_0}$ | $F^{ij} = \begin{pmatrix} 0 & -B_z & B_y & +E_x \\ B_z & 0 & -B_x & +E_y \\ -B_y & B_x & 0 & +E_z \\ -E_x & -E_y & -E_z & 0 \end{pmatrix}$<br>$\dfrac{\partial F^{ij}}{\partial x^i} = 4\pi j^j$ | $F^{ij}$ has same form as in Gaussian units<br>$\dfrac{\partial F^{ij}}{\partial x^i} = j^j$ | $F^{ij}$ has same form as in Gaussian units<br>$\dfrac{\partial F^{ij}}{\partial x^i} = j^j$ |

* In some textbooks employing Gaussian units **j** is measured in esu. In that case the equations are those given here except that **j** appears with a factor $1/c$.

TABLE I–3

DEFINITION OF FIELDS FROM SOURCES (MKS SYSTEM)

| | Electric | Magnetic | Equivalent covariant description |
|---|---|---|---|
| Vacuum (all sources "accessible") | $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$ | $\nabla \times \mathbf{B} = \mu_0 \left( \mathbf{j} + \epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} \right)$ | $\dfrac{\partial F^{ij}}{\partial x^i} = \dfrac{j^j}{\epsilon_0}$ |
| Material media, sources separated | $\nabla \cdot \mathbf{E} = \dfrac{\rho + \rho_P}{\epsilon_0}$ | $\nabla \times \mathbf{B} = \mu_0 \left( \mathbf{j} + \mathbf{j}_M + \mathbf{j}_P + \epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} \right)$ | $\dfrac{\partial F^{ij}}{\partial x^i} = \dfrac{1}{\epsilon_0} (j^j + j^j_M)$ |
| Inaccessible sources defined from auxiliary function | $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\epsilon_0} + \dfrac{(-\nabla \cdot \mathbf{P})}{\epsilon_0}$ | $\nabla \times \mathbf{B} = \mu_0 \left( \mathbf{j} + \nabla \times \mathbf{M} + \dfrac{\partial \mathbf{P}}{\partial t} + \epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} \right)$ | $\dfrac{\partial F^{ij}}{\partial x^i} = \dfrac{1}{\epsilon_0} \left( j^j + \dfrac{\partial M^{ij}}{\partial x^i} \right)$ |
| Definition of partial field | $\mathbf{D} = \epsilon_0 \mathbf{E} - (-\mathbf{P})$ | $\mathbf{H} = \dfrac{\mathbf{B}}{\mu_0} - \mathbf{M}$ | $H^{ij} = \epsilon_0 F^{ij} - M^{ij}$ |
| Field equations in media | $\nabla \cdot \mathbf{D} = \rho$ | $\nabla \times \mathbf{H} = \mathbf{j} + \dfrac{\partial \mathbf{D}}{\partial t}$ | $\dfrac{\partial H^{ij}}{\partial x^i} = j^j$ |

## TABLE I–4. USEFUL NUMERICAL RELATIONS

A. *Some atomic constants.**

| cgs Gaussian | cgs Heaviside-Lorentz | mks | Name |
|---|---|---|---|
| $\frac{e^2}{\hbar c} = \frac{1}{137.039}$ | $\frac{e^2}{4\pi\hbar c} = \frac{1}{137.039}$ | $\frac{e^2}{4\pi\hbar c\epsilon_0} = \frac{1}{137.039}$ | Fine structure constant |
| $\frac{e^2}{mc^2} = 2.81776 \times 10^{-13}$ cm | $\frac{e^2}{4\pi mc^2} = 2.81776 \times 10^{-13}$ cm | $\frac{e^2}{4\pi\epsilon_0 mc^2} = 2.81776 \times 10^{-15}$ m | Classical electron radius |
| $\frac{\hbar^2}{me^2} = 5.29166 \times 10^{-9}$ cm | $\frac{4\pi\hbar^2}{me^2} = 5.29166 \times 10^{-9}$ cm | $\frac{4\pi\epsilon_0\hbar^2}{me^2} = 5.29166 \times 10^{-11}$ m | Bohr radius |

B. *Relations useful if energy of a particle is measured in electron volts.**

$$E_0 = m_0c^2 = 0.51097 \text{ Mev} \qquad \text{for electron}$$
$$E_0 = m_0c^2 = 938.21 \text{ Mev} \qquad \text{for proton}$$

1. "Magnetic rigidity" $B\rho$ of particle of kinetic energy $T$ carrying charge $e$.

*cgs Gaussian*

$$B\rho = \frac{10^8}{c}\sqrt{T^2 + 2TE_0} \sim \frac{\sqrt{T^2 + 2TE_0}}{300}$$

*mks*

$$B\rho = \frac{\sqrt{T^2 + 2TE_0}}{c} \sim \frac{\sqrt{T^2 + 2TE_0}}{3 \times 10^8}$$

2. Tension $\tau$ of wire carrying current $I$ having same orbit in magnetic field as particle of kinetic energy $T$ carrying electronic charge $e$.

*cgs Gaussian*

$$\tau = I\sqrt{T^2 + 2TE_0} \times \frac{10^8}{c} \sim \frac{I\sqrt{T^2 + 2TE_0}}{300}$$

*mks*

$$\tau c = I\sqrt{T^2 + 2TE_0}$$

3. Velocity $u$, momentum $p$, kinetic energy $T$, total energy $E$.

$$\frac{E}{E_0} = \cosh\theta = \frac{T}{E_0} + 1 = \sqrt{\left(\frac{cp}{E_0}\right)^2 + 1} = \frac{1}{\sqrt{1 - (u^2/c^2)}}$$

$$\frac{cp}{E_0} = \sinh\theta = \sqrt{\left(\frac{T}{E_0}\right)^2 + 2\frac{T}{E_0}} = \sqrt{\left(\frac{E}{E_0}\right)^2 - 1} = \frac{(u/c)}{\sqrt{1 - (u^2/c^2)}}$$

$$\frac{u}{c} = \tanh\theta = \frac{\sqrt{2TE_0 + T^2}}{T + E_0} = \frac{\sqrt{E^2 - E_0^2}}{E} = \frac{cp}{\sqrt{(cp)^2 + E_0^2}}$$

$$\frac{T}{E_0} = \cosh\theta - 1 = \frac{1}{\sqrt{1 - (u^2/c^2)}} - 1 = \frac{E}{E_0} - 1 = \sqrt{\left(\frac{cp}{E_0}\right)^2 + 1} - 1$$

* The numerical values here are consistent with those given by J. W. M. DuMond and E. R. Cohen, *loc. cit.*