

## VECTOR DERIVATIVES

**Cartesian.**  $d\mathbf{l} = dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}}; \quad d\tau = dx dy dz$

*Gradient:*  $\nabla t = \frac{\partial t}{\partial x} \hat{\mathbf{x}} + \frac{\partial t}{\partial y} \hat{\mathbf{y}} + \frac{\partial t}{\partial z} \hat{\mathbf{z}}$

*Divergence:*  $\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

*Curl:*  $\nabla \times \mathbf{v} = \left( \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{\mathbf{x}} + \left( \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{\mathbf{y}} + \left( \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{\mathbf{z}}$

*Laplacian:*  $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

**Spherical.**  $d\mathbf{l} = dr \hat{\mathbf{r}} + r d\theta \hat{\boldsymbol{\theta}} + r \sin\theta d\phi \hat{\boldsymbol{\phi}}; \quad d\tau = r^2 \sin\theta dr d\theta d\phi$

*Gradient:*  $\nabla t = \frac{\partial t}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial t}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{r \sin\theta} \frac{\partial t}{\partial \phi} \hat{\boldsymbol{\phi}}$

*Divergence:*  $\nabla \cdot \mathbf{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta v_\theta) + \frac{1}{r \sin\theta} \frac{\partial v_\phi}{\partial \phi}$

*Curl:*  $\nabla \times \mathbf{v} = \frac{1}{r \sin\theta} \left[ \frac{\partial}{\partial \theta} (\sin\theta v_\phi) - \frac{\partial v_\theta}{\partial \phi} \right] \hat{\mathbf{r}}$   
 $+ \frac{1}{r} \left[ \frac{1}{\sin\theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\phi) \right] \hat{\boldsymbol{\theta}} + \frac{1}{r} \left[ \frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\boldsymbol{\phi}}$

*Laplacian:*  $\nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left( \sin\theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 t}{\partial \phi^2}$

**Cylindrical.**  $d\mathbf{l} = ds \hat{\mathbf{s}} + s d\phi \hat{\boldsymbol{\phi}} + dz \hat{\mathbf{z}}; \quad d\tau = s ds d\phi dz$

*Gradient:*  $\nabla t = \frac{\partial t}{\partial s} \hat{\mathbf{s}} + \frac{1}{s} \frac{\partial t}{\partial \phi} \hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z} \hat{\mathbf{z}}$

*Divergence:*  $\nabla \cdot \mathbf{v} = \frac{1}{s} \frac{\partial}{\partial s} (s v_s) + \frac{1}{s} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$

*Curl:*  $\nabla \times \mathbf{v} = \left[ \frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right] \hat{\mathbf{s}} + \left[ \frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s} \right] \hat{\boldsymbol{\phi}} + \frac{1}{s} \left[ \frac{\partial}{\partial s} (s v_\phi) - \frac{\partial v_s}{\partial \phi} \right] \hat{\mathbf{z}}$

*Laplacian:*  $\nabla^2 t = \frac{1}{s} \frac{\partial}{\partial s} \left( s \frac{\partial t}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$

## FUNDAMENTAL CONSTANTS

$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$	(permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$	(permeability of free space)
$c = 3.00 \times 10^8 \text{ m/s}$	(speed of light)
$e = 1.60 \times 10^{-19} \text{ C}$	(charge of the electron)
$m = 9.11 \times 10^{-31} \text{ kg}$	(mass of the electron)

## SPHERICAL AND CYLINDRICAL COORDINATES

### Spherical

$$\begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{cases} \quad \begin{cases} \hat{\mathbf{x}} = \sin \theta \cos \phi \hat{\mathbf{r}} + \cos \theta \cos \phi \hat{\boldsymbol{\theta}} - \sin \phi \hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin \theta \sin \phi \hat{\mathbf{r}} + \cos \theta \sin \phi \hat{\boldsymbol{\theta}} + \cos \phi \hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \cos \theta \hat{\mathbf{r}} - \sin \theta \hat{\boldsymbol{\theta}} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \tan^{-1}(\sqrt{x^2 + y^2}/z) \\ \phi = \tan^{-1}(y/x) \end{cases} \quad \begin{cases} \hat{\mathbf{r}} = \sin \theta \cos \phi \hat{\mathbf{x}} + \sin \theta \sin \phi \hat{\mathbf{y}} + \cos \theta \hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} = \cos \theta \cos \phi \hat{\mathbf{x}} + \cos \theta \sin \phi \hat{\mathbf{y}} - \sin \theta \hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} = -\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{y}} \end{cases}$$

### Cylindrical

$$\begin{cases} x = s \cos \phi \\ y = s \sin \phi \\ z = z \end{cases} \quad \begin{cases} \hat{\mathbf{x}} = \cos \phi \hat{\mathbf{s}} - \sin \phi \hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin \phi \hat{\mathbf{s}} + \cos \phi \hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\begin{cases} s = \sqrt{x^2 + y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases} \quad \begin{cases} \hat{\mathbf{s}} = \cos \phi \hat{\mathbf{x}} + \sin \phi \hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} = -\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{y}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \hat{\mathbf{r}} \quad \text{Coulomb's Law} \quad \mathbf{F} = Q\mathbf{E} + Q\mathbf{V} \times \mathbf{B} \quad \text{Lorentz force}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \iiint \frac{\rho(\mathbf{r}')}{r^2} \hat{\mathbf{r}} \, d\tau' \quad \text{electric field from continuous charge distribution}$$

$$\oiint \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{encl}} \text{ Gauss' Law (integral)} \quad \oint \mathbf{E} \cdot d\mathbf{l} = 0 \text{ (statics)}$$

$$V(\mathbf{r}) = -\int_{\phi}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l} \text{ Electrostatic potential, and } \mathbf{E} = -\nabla V \text{ (statics)}$$

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \text{ Poisson's equation, and } \nabla^2 V = 0 \text{ Laplace's equation (regions of no charge)}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \text{ and } V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \iiint \frac{\rho(\mathbf{r}')}{r} d\tau' \text{ (setting reference point at infinity)}$$

$$\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0} \hat{\mathbf{n}} \text{ and } \frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} = \frac{1}{\epsilon_0} \sigma \text{ boundary conditions}$$

$$D_{\text{above}}^{\perp} - D_{\text{below}}^{\perp} = \sigma_f$$

$$V(\mathbf{b}) - V(\mathbf{a}) = \frac{W}{Q} \quad W = \frac{1}{2} \sum_{i=1}^n q_i V(\mathbf{r}_i) \text{ and } W = \frac{1}{2} \iiint \rho V d\tau$$

$Q = CV$  capacitor

$$\nabla^2 V = -\frac{1}{\epsilon_0} \rho \text{ Poisson's equation} \quad \nabla^2 V = 0 \text{ Laplace's equation}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \sum_0^{\infty} \frac{1}{r^{n+1}} \iiint (r')^n P_n(\cos \alpha) \rho(\mathbf{r}') d\tau' \text{ multipole expansion}$$

$$\vec{p} = \iiint \vec{r}' \rho(\vec{r}') d\tau' \quad \vec{p} = \sum_1^n q_i \vec{r}'_i \quad V_{\text{dip}}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{\mathbf{r}}}{r^2} \text{ dipole moment}$$

$$\vec{E}_{\text{dip}} = \frac{P}{4\pi\epsilon_0 r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\theta}) \vec{p}$$

$$\mathbf{F}_{\text{mag}} = \int I d\mathbf{l} \times \mathbf{B}$$

$$\mathbf{K} = d\mathbf{I} / d\ell_{\perp} \quad \mathbf{J} = d\mathbf{I} / da_{\perp} \text{ surface and volume currents}$$

$$\nabla \cdot \mathbf{J} + \frac{d\rho}{dt} = 0 \text{ conserved current (continuity equation for current)}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}' \times \hat{\mathbf{r}}}{|\mathbf{r} - \mathbf{r}'|^2} \quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K} \times \hat{\mathbf{r}}}{|\mathbf{r} - \mathbf{r}'|^2} da' \quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I \int \frac{\mathbf{J} \times \hat{\mathbf{r}}}{|\mathbf{r} - \mathbf{r}'|^2} d\tau'$$

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \quad \text{static}$$

$\mathbf{B} = \nabla \times \mathbf{A}$  magnetic vector potential.  $\nabla \cdot \mathbf{A} = 0$  “Coulomb gauge” convention

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J} \quad (\text{for Coulomb gauge})$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\tau' \quad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} da' \quad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{l}'$$

$$\oint \mathbf{A} \cdot d\mathbf{l} = \Phi_m$$

$$B_{\text{above}}^\perp - B_{\text{below}}^\perp = 0 \quad B_{\text{above}}^\parallel - B_{\text{below}}^\parallel = \mu_0 K \quad \mathbf{B}_{\text{above}} - \mathbf{B}_{\text{below}} = \mu_0 \mathbf{K} \times \hat{\mathbf{n}}$$

$$\mathbf{H}_{\text{above}}^\parallel - \mathbf{H}_{\text{below}}^\parallel = \mu_0 \mathbf{K}_f \times \hat{\mathbf{n}}$$

$$\mathbf{A}_{\text{above}} - \mathbf{A}_{\text{below}} = 0 \quad \frac{\partial \mathbf{A}_{\text{above}}}{\partial n} - \frac{\partial \mathbf{A}_{\text{below}}}{\partial n} = -\mu_0 \mathbf{K}$$

$$\mathbf{m} = I \iint \hat{\mathbf{n}} da = I \mathbf{a} \quad \mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^2}$$

$$\mathbf{N} = \mathbf{m} \times \mathbf{B} \quad \mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B}) \quad U = -\mathbf{m} \cdot \mathbf{B}$$

$$\mathbf{J}_b = \nabla \times \mathbf{M} \quad \mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}} \quad \text{surface and volume currents}$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{f,encl}}$$

$$H_{\text{above}}^\perp - H_{\text{below}}^\perp = -(M_{\text{above}}^\perp - M_{\text{below}}^\perp) \quad \mathbf{H}_{\text{above}}^\parallel - \mathbf{H}_{\text{below}}^\parallel = \mathbf{K}_f \times \hat{\mathbf{n}}$$

$$\mu = \mu_0 (1 + \chi_m)$$

$$\mathbf{J} = \sigma \mathbf{E} \quad V = IR \quad P = VI = I^2 R \quad \mathbf{J}_p = \frac{\partial \mathbf{P}}{\partial t} \quad \text{polarization current}$$

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} \quad \mathcal{E} = -\frac{d\Phi_M}{dt}$$

$$\Phi_2 = M I_1 \quad \Phi = LI \quad \mathcal{E}_2 = -M \frac{dI_1}{dt} \quad \mathcal{E} = -L \frac{dI}{dt} \quad W = \frac{1}{2} LI^2$$

$$\nabla \cdot \left( \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \right) + \frac{\partial}{\partial t} \left( \frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2 \right) = -\mathbf{E} \cdot \mathbf{J} \quad \text{Poynting's theorem}$$

$$\frac{dW}{dt} = -\frac{d}{dt} \iiint \left( \frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2 \right) d\tau - \iint \left( \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \right) \cdot \hat{n} da \quad \text{Poynting's theorem}$$

$$dF_i = T_{ij} da_j \quad \frac{dF_i}{d\tau} = \frac{\partial T_{ij}}{\partial x_j} \quad T_{ij} = \epsilon_0 \left( E_i E_j - \frac{1}{2} \delta_{ij} E^2 \right) + \frac{1}{\mu_0} \left( B_i B_j - \frac{1}{2} \delta_{ij} B^2 \right)$$

$$\frac{d}{dt} (P_i(\text{mech}) + P_i(\text{fields})) = \iint T_{ij} a_j \quad \frac{d}{d\tau} \mathbf{P}(\text{fields}) = \mu_0 \epsilon_0 \mathbf{S} \quad \frac{d}{d\tau} \mathbf{L}(\text{fields}) = \mathbf{r} \times \mu_0 \epsilon_0 \mathbf{S}$$

$$\frac{\partial^2 f}{\partial z^2} - \frac{1}{V^2} \frac{\partial^2 f}{\partial t^2} = 0 \quad \lambda = 2\pi/k \quad T = 2\pi/kV \quad v = 1/T \quad \omega = 2\pi\nu = kV$$

$$e^{i\theta} = \cos\theta + i \sin\theta$$

$$\text{vacuum} \quad \nabla^2 \mathbf{E} - \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad \nabla^2 \mathbf{B} - \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0 \quad c = 1/\sqrt{\mu_0 \epsilon_0}$$

$$\tilde{\mathbf{E}}(z, t) = \tilde{\mathbf{E}}_0 e^{i(kz - \omega t)} \quad \tilde{\mathbf{B}}(z, t) = \tilde{\mathbf{B}}_0 e^{i(kz - \omega t)} \quad E_0 = cB_0$$

$$\langle u \rangle = \frac{1}{2} \epsilon_0 E_0^2 \quad \langle \mathbf{S} \rangle = \frac{1}{2} c \epsilon_0 E_0^2 \hat{k} \quad I = \langle S \rangle \quad \langle \mathbf{p} \rangle = \frac{1}{2c} \epsilon_0 E_0^2 \hat{k}$$

$$\text{lossless} \quad \nabla^2 \mathbf{E} - \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad \nabla^2 \mathbf{B} - \mu \epsilon \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0 \quad V = 1/\sqrt{\mu \epsilon} = c/n \quad n = \sqrt{\mu \epsilon / \mu_0 \epsilon_0}$$

$$u = \frac{1}{2} \left( \epsilon E^2 + \frac{1}{\mu} B^2 \right) \quad \mathbf{S} = \frac{1}{\mu} \mathbf{E} \times \mathbf{B} \quad I = \frac{1}{2} \epsilon V E_0^2$$

$$\tan \theta_B \cong n_2 / n_1 \quad \sin \theta_c = n_2 / n_1$$

$$\text{lossy} \quad \nabla^2 \mathbf{E} - \mu \sigma \frac{\partial \mathbf{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad \nabla^2 \mathbf{B} - \mu \sigma \frac{\partial \mathbf{B}}{\partial t} - \mu \epsilon \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$$

$$\tilde{k}^2 = \mu \epsilon \omega^2 + i \mu \sigma \omega \quad d = 1 / \text{Im} \tilde{k}$$

$$\operatorname{Re} \tilde{k} = \omega \sqrt{\frac{\varepsilon \mu}{2} \left[ \sqrt{1 + \left( \frac{\sigma}{\varepsilon \omega} \right)^2} + 1 \right]^{1/2}} \quad \operatorname{Im} \tilde{k} = \omega \sqrt{\frac{\varepsilon \mu}{2} \left[ \sqrt{1 + \left( \frac{\sigma}{\varepsilon \omega} \right)^2} - 1 \right]^{1/2}}$$

poor conductor  $\operatorname{Re} \tilde{k} = \sqrt{\mu \varepsilon} \omega \quad \operatorname{Im} \tilde{k} = \sqrt{\frac{\mu}{\varepsilon}} \frac{\sigma}{2}$

good conductor  $\operatorname{Re} \tilde{k} = \operatorname{Im} \tilde{k} = \sqrt{\frac{\omega \sigma \mu}{2}}$

$$\ddot{y} + \gamma \dot{y} + \omega_0^2 y = \frac{e}{m} E(t) \quad E(t) = E_0 e^{i\omega t} \quad y(t) = y_0 e^{i\omega t} \quad y_0 = \frac{e/m}{(\omega_0^2 - \omega^2) - i\gamma\omega} E_0$$

$$\varepsilon = \varepsilon_0 + \frac{Nfe^2}{m} \frac{1}{(\omega_0^2 - \omega^2)} - i\gamma\omega$$

dilute  $n = \frac{c}{\omega} \operatorname{Re} \tilde{k} = 1 + \frac{1}{2\varepsilon_0} \frac{Nfe^2}{m} \frac{(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$

$$\alpha = 2 \operatorname{Im} \tilde{k} = 1 + \frac{\omega}{c\varepsilon_0} \frac{Nfe^2}{m} \frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$$

plasma  $\sigma = \frac{Nfe^2}{m} \frac{1}{\gamma - i\omega}$  dilute plasma  $\sigma = \frac{Nfe^2}{m} \frac{1}{-i\omega} \quad \omega_p = \sqrt{\frac{Nfe^2}{\varepsilon_0 m}}$

dilute plasma  $\tilde{k}^2 = \frac{1}{c^2} \{ \omega^2 - \omega_p^2 \}$

waveguide transverse fields

$$E_x = \frac{i}{(\omega/c)^2 - k^2} \left( k \frac{\partial E_z}{\partial x} - \omega \frac{\partial B_z}{\partial y} \right)$$

$$E_y = \frac{i}{(\omega/c)^2 - k^2} \left( k \frac{\partial E_z}{\partial y} - \omega \frac{\partial B_z}{\partial x} \right)$$

$$B_x = \frac{i}{(\omega/c)^2 - k^2} \left( k \frac{\partial B_z}{\partial x} - \frac{\omega}{c^2} \frac{\partial E_z}{\partial y} \right)$$

$$B_y = \frac{i}{(\omega/c)^2 - k^2} \left( k \frac{\partial B_z}{\partial y} - \frac{\omega}{c^2} \frac{\partial E_z}{\partial x} \right)$$

waveguide longitudinal fields 
$$\begin{cases} \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + (\omega / c^2) - k^2 \right] E_z = 0 \\ \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + (\omega / c^2) - k^2 \right] B_z = 0 \end{cases}$$

“waveguide equation” 
$$\frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2} = \frac{1}{\lambda_0^2}$$

TE & TM guided wavenumber 
$$k^2 = \left( \frac{\omega}{c} \right)^2 - \pi^2 \left[ \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 \right]$$

TE & TM cutoff angular frequency 
$$\omega_{mn} = c\pi \sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2}$$

phase velocity  $v = \omega / k$       group velocity  $\frac{1}{dk / d\omega}$

rectangular resonant cavity angular frequency 
$$\omega_{nml} = c \sqrt{\left( \frac{n\pi}{a} \right)^2 + \left( \frac{m\pi}{b} \right)^2 + \left( \frac{l\pi}{c} \right)^2}$$

$Z_0$  = characteristic impedance of line

$Z_L$  = load impedance

$$V(z) = V_0^+ e^{-i\frac{\omega}{c}z} + V_0^- e^{+i\frac{\omega}{c}z}$$

transmission line, vacuum dielectric, lossless 
$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$I(z) = V_0^+ / Z_0 e^{-i\frac{\omega}{c}z} + V_0^- / Z_0 e^{+i\frac{\omega}{c}z}$$

$$V(z) = V_0^+ \left[ e^{-i\frac{\omega}{c}z} + \Gamma e^{+i\frac{\omega}{c}z} \right]$$

$$I(z) = V_0^+ / Z_0 \left[ e^{-i\frac{\omega}{c}z} + \Gamma e^{+i\frac{\omega}{c}z} \right]$$

dynamical potentials  $\mathbf{E} = -\nabla V - \partial \mathbf{A} / \partial t$ ,  $\mathbf{B} = \nabla \times \mathbf{A}$

potential equations 
$$\nabla^2 V + \frac{\partial}{\partial t} \nabla \cdot \mathbf{A} = -\rho / \epsilon_0 \quad \left( \nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} \right) - \nabla \left( \nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial V}{\partial t} \right) = -\mu_0 \mathbf{J}$$

gauge transformations  $A' = A + \nabla \lambda$      $V' = V - \partial \lambda / \partial t$

Coulomb gauge  $\nabla \cdot \mathbf{A} = 0$ , Lorentz gauge  $\nabla \cdot \mathbf{A} + \frac{1}{c^2} \partial V / \partial t = 0$

retarded time  $t_r = t - r / c$

retarded potentials  $V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \iiint \frac{\rho(\mathbf{r}', t_r)}{r} d\tau'$ ,  $\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \iiint \frac{\mathbf{J}(\mathbf{r}', t_r)}{r} d\tau'$

Jefimenko's equations  $\mathbf{E}(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \iiint \left[ \frac{\rho(\mathbf{r}', t_r)}{r^2} \hat{\mathbf{r}} + \frac{\dot{\rho}(\mathbf{r}', t_r)}{cr} \hat{\mathbf{r}} - \frac{\dot{\mathbf{J}}(\mathbf{r}', t_r)}{c^2 r} \right] d\tau'$

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \iiint \left[ \frac{\mathbf{J}(\mathbf{r}', t_r)}{r^2} + \frac{\dot{\mathbf{J}}(\mathbf{r}', t_r)}{cr} \right] \times \hat{\mathbf{r}} d\tau'$$

Liénard-Wiechert potentials

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{qc}{(rc - \mathbf{r} \cdot \mathbf{v})}$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{qc\mathbf{v}}{(rc - \mathbf{r} \cdot \mathbf{v})} = \frac{\mathbf{v}}{c^2} V(\mathbf{r}, t)$$

Point charges  $\mathbf{E}(\mathbf{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{r}{(\mathbf{r} \cdot \mathbf{u})^3} \left[ (c^2 - v^2) \mathbf{u} + \mathbf{r} \times \mathbf{u} \times \mathbf{a} \right]$ ,  $\mathbf{u} = c\hat{\mathbf{r}} - \mathbf{v}$

radiation fields  $\mathbf{E}_{\text{rad}}(\mathbf{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{r}{(\mathbf{r} \cdot \mathbf{u})^3} [\mathbf{r} \times \mathbf{u} \times \mathbf{a}]$ ,  $\mathbf{S}_{\text{rad}}(\mathbf{r}, t) = \frac{1}{\mu_0 c} E_{\text{rad}}^2 \hat{\mathbf{r}}$

charge at rest at time  $t_r$   $\mathbf{E}_{\text{rad}}(\mathbf{r}, t) = \frac{\mu_0 q}{4\pi r} [(\hat{\mathbf{r}} \cdot \mathbf{a}) \hat{\mathbf{r}} - \mathbf{a}]$   $\mathbf{S}_{\text{rad}}(\mathbf{r}, t) = \frac{\mu_0 q^2 a^2 \sin^2 \theta}{16\pi^2 c} \frac{1}{r^2} \hat{\mathbf{r}}$

magnetic dipole radiation  $\langle P \rangle = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3}$  electric dipole radiation  $\langle P \rangle = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$

Larmor formula  $p = \frac{\mu_0 q^2 a^2}{6\pi c}$

generalization of Larmor formula  $P = \frac{\mu_0 q^2 \gamma^6}{6\pi c} \left( a^2 - \left| \frac{\mathbf{v} \times \mathbf{a}}{c} \right|^2 \right)$ ,  $\gamma^2 = \frac{1}{1 - (v/c)^2}$

Abraham-Lorentz radiation-reaction force  $\mathbf{F} = \frac{\mu_0 q^2}{6\pi c} \dot{\mathbf{a}}$

relativistic velocity addition  $V_{AC} = \frac{V_{AB} + V_{BC}}{1 + V_{AB} V_{BC} / c^2}$



velocity factor  $\beta = v/c$       boost factor  $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

Lorentz transformation

$$\begin{aligned} x' &= \gamma(x - vt) & x^{0'} &= \gamma(x^0 - \beta x^1) \\ y' &= y & x^{1'} &= \gamma(x^1 - \beta x^0) \\ z' &= z & x^{2'} &= x^2 \\ t' &= \gamma\left(t - \frac{v}{c^2}x\right) & x^{3'} &= x^3 \end{aligned}$$

or

Lorentz transformation in matrix form

$$\begin{pmatrix} x^{0'} \\ x^{2'} \\ x^{3'} \\ x^{4'} \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^2 \\ x^3 \\ x^4 \end{pmatrix} \quad \text{or } x^{\mu'} = \Lambda_{\nu}^{\mu} x^{\nu}$$

scalar product  $a^{\mu}a_{\mu} = a^{\mu}g_{\mu\nu}a^{\nu}$  with  $g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$  (note summation notation)

spacelike  $a^{\mu}a_{\mu} > 0$       timelike  $a^{\mu}a_{\mu} < 0$       lightlike  $a^{\mu}a_{\mu} = 0$

invariant interval  $I = (\Delta x)^{\mu}(\Delta x)_{\mu} = -c^2t^2 + d^2$

proper time  $d\tau = dt / \gamma$       4-velocity  $\eta = \frac{dx^{\mu}}{d\tau}$

4-momentum  $p^{\mu} = m\eta^{\mu}$      $\vec{p} = m\vec{\eta}$      $p^0c = E$      $p^{\mu}p_{\mu} = -m^2c^2$      $E^2 - p^2c^2 = m^2c^4$

Compton wavelength  $\lambda_c = h/mc$

relativistic force  $\mathbf{F} = \frac{d\mathbf{P}}{dt}$  with  $\mathbf{P}$  the relativistic momentum

transformation of force for particle at rest in unprimed frame  $\mathbf{F}_{\perp}' = \mathbf{F}_{\perp} / \gamma$      $F_{\parallel}' = F_{\parallel}$

Minkowski force  $K^{\mu} = \frac{dp^{\mu}}{d\tau}$     center of energy  $\mathbf{R}_E = \frac{1}{M} \sum_i m_i \mathbf{r}_i$      $\mathbf{P}_{\text{tot}} = \frac{E}{c^2} \frac{d\mathbf{R}_E}{d\tau}$

$$\begin{aligned}
 & E'_x = E_x \quad E'_y = \gamma(E_y - vB_z) \quad E'_z = \gamma(E_z + vB_y) \\
 \text{Transformation of fields} \quad & B'_x = B_x \quad B'_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right) \quad B'_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right)
 \end{aligned}$$

$$\text{Field strength tensor } F^{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix}$$

### Maxwell's Equations

*In general:*

$$\begin{cases} \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{cases}$$

*In matter:*

$$\begin{cases} \nabla \cdot \mathbf{D} = \rho_f \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \end{cases}$$

### Auxiliary Fields

*Definitions:*

$$\begin{cases} \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M} \end{cases}$$

*Linear media:*

$$\begin{cases} \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}, & \mathbf{D} = \epsilon \mathbf{E} \\ \mathbf{M} = \chi_m \mathbf{H}, & \mathbf{H} = \frac{1}{\mu} \mathbf{B} \end{cases}$$

### Potentials

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A}$$

### Lorentz force law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

### Energy, Momentum, and Power

*Energy:* 
$$U = \frac{1}{2} \int (\epsilon_0 E^2 + \frac{1}{\mu_0} B^2) d\tau$$

*Momentum:* 
$$\mathbf{P} = \epsilon_0 \int (\mathbf{E} \times \mathbf{B}) d\tau$$

*Poynting vector:* 
$$\mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B})$$