Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$; $\quad d\tau = dx\,dy\,dz$

Gradient: $\nabla t = \frac{\partial t}{\partial x}\,\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\,\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\,\hat{\mathbf{z}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Curl: $\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$

Laplacian: $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$; $\quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient: $\nabla t = \frac{\partial t}{\partial r}\,\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\,\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\,\hat{\boldsymbol{\phi}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$

Curl:
$$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}}$$
$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian: $\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$; $\quad d\tau = s\,ds\,d\phi\,dz$

Gradient: $\nabla t = \frac{\partial t}{\partial s}\,\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\,\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\,\hat{\mathbf{z}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$

Curl: $\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$

Laplacian: $\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$

FUNDAMENTAL CONSTANTS

$\epsilon_0 = 8.85 \times 10^{-12}\, \mathrm{C^2/Nm^2}$	(permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7}\, \mathrm{N/A^2}$	(permeability of free space)
$c = 3.00 \times 10^{8}\, \mathrm{m/s}$	(speed of light)
$e = 1.60 \times 10^{-19}\, \mathrm{C}$	(charge of the electron)
$m = 9.11 \times 10^{-31}\, \mathrm{kg}$	(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{r}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{r}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \tan^{-1}\left(\sqrt{x^2+y^2}/z\right) \\ \phi = \tan^{-1}(y/x) \end{cases} \qquad \begin{cases} \hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \end{cases}$$

Cylindrical

$$\begin{cases} x = s\cos\phi \\ y = s\sin\phi \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{s}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{s}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\begin{cases} s = \sqrt{x^2+y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{s}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$\mathbf{F} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{r^2}\hat{r}$ Coulomb's Law $\quad \mathbf{F} = Q\mathbf{E} + Q\mathbf{V}\times\mathbf{B}$ Lorentz force

$\mathbf{E} = \dfrac{1}{4\pi\varepsilon_0}\iiint \dfrac{\rho(\mathbf{r}')}{r^2}\hat{r}\, d\tau'$ electric field from continuous charge distribution

$\oiint \mathbf{E} \bullet d\mathbf{a} = \frac{1}{\varepsilon_0} Q_{\text{encl}}$ Gauss' Law (integral) $\oint \mathbf{E} \bullet d\mathbf{l} = 0$ (statics)

$V(\mathbf{r}) = -\int_{\wp}^{\mathbf{r}} \mathbf{E} \bullet d\mathbf{l}$ Electrostatic potential, and $\mathbf{E} = -\nabla V$ (statics)

$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$ Poisson's equation, and $\nabla^2 V = 0$ Laplace's equation (regions of no charge)

$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$ and $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho(\mathbf{r}')}{r} d\tau'$ (setting reference point at infinity)

$\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{n}}$ and $\frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} = \frac{1}{\varepsilon_0}\sigma$ boundary conditions

$D^{\perp}_{\text{above}} - D^{\perp}_{\text{below}} = \sigma_f$

$V(\mathbf{b}) - V(\mathbf{a}) = \frac{W}{Q}$ $\quad W = \frac{1}{2}\sum_{i=1}^{n} q_i V(\mathbf{r}_i)$ and $W = \frac{1}{2}\iiint \rho V \, d\tau$

$Q = CV$ capacitor

$\nabla^2 V = -\frac{1}{\varepsilon_0}\rho$ Poisson's equation $\quad \nabla^2 V = 0$ Laplace's equation

$V(r) = \frac{1}{4\pi\varepsilon_0} \sum_{0}^{\infty} \frac{1}{r^{n+1}} \iiint (r')^n P_n(\cos\alpha)\rho(\mathbf{r}')d\tau'$ multipole expansion

$\vec{p} = \iiint \vec{r}'\rho(\vec{r}')d\tau'$ $\quad \vec{p} = \sum_{1}^{n} q_i \vec{r}_i'$ $\quad V_{\text{dip}}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p} \bullet \hat{r}}{r^2}$ dipole moment

$\vec{E}_{\text{dip}} = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})\vec{p}$

$\mathbf{F}_{\text{mag}} = \int I \, d\mathbf{l} \times \mathbf{B}$

$\mathbf{K} = d\mathbf{I} / d\ell_{\perp}$ $\quad \mathbf{J} = d\mathbf{I} / da_{\perp}$ surface and volume currents

$\nabla \bullet \mathbf{J} + \frac{d\rho}{dt} = 0$ conserved current (continuity equation for current)ß

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}' \times \hat{r}}{|\mathbf{r}-\mathbf{r}'|^2} \quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K} \times \hat{r}}{|\mathbf{r}-\mathbf{r}'|^2} da' \quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I \int \frac{\mathbf{J} \times \hat{r}}{|\mathbf{r}-\mathbf{r}'|^2} d\tau'$$

$\oint \mathbf{B} \bullet d\mathbf{l} = \mu_0 I_{\text{enc}}$ static

$\mathbf{B} = \nabla \times \mathbf{A}$ magnetic vector potential. $\nabla \bullet \mathbf{A} = 0$ “Coulomb gauge” convention

$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$ (for Coulomb gauge)

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\tau' \quad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} da' \quad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{l}'$$

$$\oint \mathbf{A} \bullet d\mathbf{l} = \Phi_m$$

$B^{\perp}_{\text{above}} - B^{\perp}_{\text{below}} = 0 \quad B^{\parallel}_{\text{above}} - B^{\parallel}_{\text{below}} = \mu_0 K \quad \mathbf{B}_{\text{above}} - \mathbf{B}_{\text{below}} = \mu_0 \mathbf{K} \times \hat{n}$

$\mathbf{H}^{\parallel}_{\text{above}} - \mathbf{H}^{\parallel}_{\text{below}} = \mu_0 \mathbf{K}_f \times \hat{n}$

$\mathbf{A}_{\text{above}} - \mathbf{A}_{\text{below}} = 0 \qquad \frac{\partial \mathbf{A}_{\text{above}}}{\partial n} - \frac{\partial \mathbf{A}_{\text{below}}}{\partial n} = -\mu_0 \mathbf{K}$

$\mathbf{m} = I \iint \hat{n}\, da = I\mathbf{a} \qquad \mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{r}}{r^2}$

$\mathbf{N} = \mathbf{m} \times \mathbf{B} \quad \mathbf{F} = \nabla(\mathbf{m} \bullet \mathbf{B}) \quad U = -\mathbf{m} \bullet \mathbf{B}$

$\mathbf{J}_b = \nabla \times \mathbf{M} \quad \mathbf{K}_b = \mathbf{M} \times \hat{n}$

$\oint \mathbf{H} \bullet d\mathbf{l} = I_{\text{f,encl}}$

$H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} = -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}}) \quad \mathbf{H}^{\parallel}_{\text{above}} - \mathbf{H}^{\parallel}_{\text{below}} = \mathbf{K}_f \times \hat{n}$

$\mu = \mu_0 (1 + \chi_m)$

$\mathbf{J} = \sigma \mathbf{E} \quad V = IR \quad P = VI = I^2 R \quad \mathbf{J}_P = \frac{\partial \mathbf{P}}{\partial t}$ polarization current

$\varepsilon = \oint \mathbf{E} \bullet d\ell \quad \varepsilon = -\frac{d\Phi_M}{dt}$

$$\Phi_2 = M\, I_1 \quad \Phi = LI \quad \mathcal{E}_2 = -M\frac{dI_1}{dt} \quad \mathcal{E} = -L\frac{dI}{dt} \quad W = \frac{1}{2}LI^2$$

$$\nabla \bullet \left(\frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}\right) + \frac{\partial}{\partial t}\left(\frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2\right) = -\mathbf{E}\bullet\mathbf{J}$$ Poynting's theorem

$$\frac{dW}{dt} = -\frac{d}{dt}\iiint\left(\frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2\right)d\tau - \oiint\left(\frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}\right)\bullet\hat{n}\,da$$ Poynting's theorem

$$dF_i = T_{ij}\,da_j \qquad \frac{dF_i}{d\tau_i} = \frac{\partial T_{ij}}{\partial x_j} \qquad T_{ij} = \varepsilon_0\left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right)$$

$$\frac{d}{dt}\left(P_i(\text{mech}) + P_i(\text{fields})\right) = \iint T_{ij}\,a_j \quad \frac{d}{d\tau}\mathbf{P}(\text{fields}) = \mu_0\varepsilon_0\mathbf{S} \quad \frac{d}{d\tau}\mathbf{L}(\text{fields}) = \mathbf{r}\times\mu_0\varepsilon_0\mathbf{S}$$

$$\frac{\partial^2 f}{\partial z^2} - \frac{1}{V^2}\frac{\partial^2 f}{\partial t^2} = 0 \quad \lambda = 2\pi/k \quad T = 2\pi/kV \quad \nu = 1/T \quad \omega = 2\pi\nu = kV$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

vacuum $\nabla^2\mathbf{E} - \mu_0\varepsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2} = 0 \quad \nabla^2\mathbf{B} - \mu_0\varepsilon_0\frac{\partial^2\mathbf{B}}{\partial t^2} = 0 \quad c = 1/\sqrt{\mu_0\varepsilon_0}$

$$\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i(kz-\omega t)} \quad \tilde{\mathbf{B}}(z,t) = \tilde{\mathbf{B}}_0 e^{i(kz-\omega t)} \quad E_0 = cB_0$$

$$\langle u\rangle = \frac{1}{2}\varepsilon_0E_0^2 \quad \langle\mathbf{S}\rangle = \frac{1}{2}c\varepsilon_0E_0^2\hat{k} \quad I = \langle S\rangle \quad \langle\mathbf{p}\rangle = \frac{1}{2c}\varepsilon_0E_0^2\hat{k}$$

lossless $\nabla^2\mathbf{E} - \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2} = 0 \quad \nabla^2\mathbf{B} - \mu\varepsilon\frac{\partial^2\mathbf{B}}{\partial t^2} = 0 \quad V = 1/\sqrt{\mu\varepsilon} = c/n \quad n = \sqrt{\mu\varepsilon/\mu_0\varepsilon_0}$

$$u = \frac{1}{2}\left(\varepsilon E^2 + \frac{1}{\mu}B^2\right) \quad \mathbf{S} = \frac{1}{\mu}\mathbf{E}\times\mathbf{B} \quad I = \frac{1}{2}\varepsilon VE_0^2$$

$$\tan\theta_B \cong n_2/n_1 \quad \sin\theta_c = n_2/n_1$$

lossy $\nabla^2\mathbf{E} - \mu\sigma\frac{\partial\mathbf{E}}{\partial t} - \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2} = 0 \qquad \nabla^2\mathbf{B} - \mu\sigma\frac{\partial\mathbf{B}}{\partial t} - \mu\varepsilon\frac{\partial^2\mathbf{B}}{\partial t^2} = 0$

$$\tilde{k}^2 = \mu\varepsilon\omega^2 + i\mu\sigma\omega \quad d = 1/\operatorname{Im}\tilde{k}$$

$$\mathrm{Re}\,\tilde{k} = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}+1\right]^{1/2} \quad \mathrm{Im}\,\tilde{k} = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}-1\right]^{1/2}$$

poor conductor $\mathrm{Re}\,\tilde{k} = \sqrt{\mu\varepsilon}\,\omega \quad \mathrm{Im}\,\tilde{k} = \sqrt{\frac{\mu}{\varepsilon}}\frac{\sigma}{2}$

good conductor $\mathrm{Re}\,\tilde{k} = \mathrm{Im}\,\tilde{k} = \sqrt{\frac{\omega\sigma\mu}{2}}$

$$\ddot{y} + \gamma\dot{y} + \omega_0^2 y = \frac{e}{m}E(t) \qquad E(t) = E_0 e^{i\omega t} \qquad y(t) = y_0 e^{i\omega t} \qquad y_0 = \frac{e/m}{\left(\omega_0^2 - \omega^2\right) - i\gamma\omega}E_0$$

$$\varepsilon = \varepsilon_0 + \frac{Nfe^2}{m}\frac{1}{\left(\omega_0^2 - \omega^2\right)} - i\gamma\omega$$

dilute $n = \frac{c}{\omega}\mathrm{Re}\,\tilde{k} = 1 + \frac{1}{2\varepsilon_0}\frac{Nfe^2}{m}\frac{\left(\omega_0^2 - \omega^2\right)}{\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2}$

$$\alpha = 2\,\mathrm{Im}\,\tilde{k} = 1 + \frac{\omega}{c\varepsilon_0}\frac{Nfe^2}{m}\frac{\gamma\omega}{\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2}$$

plasma $\sigma = \frac{Nfe^2}{m}\frac{1}{\gamma - i\omega}$ dilute plasma $\sigma = \frac{Nfe^2}{m}\frac{1}{-i\omega} \qquad \omega_p = \sqrt{\frac{Nfe^2}{\varepsilon_0 m}}$

dilute plasma $\tilde{k}^2 = \frac{1}{c^2}\left\{\omega^2 - \omega_p^2\right\}$

waveguide transverse fields

$$E_x = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial E_z}{\partial x} - \omega\frac{\partial B_z}{\partial y}\right)$$

$$E_y = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right)$$

$$B_x = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)$$

$$B_y = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial B_z}{\partial y} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right)$$

waveguide longitudinal fields $$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\left(\omega/c^2\right)-k^2\right]E_z=0$$
$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\left(\omega/c^2\right)-k^2\right]B_z=0$$

"waveguide equation" $\frac{1}{\lambda_g^2}+\frac{1}{\lambda_c^2}=\frac{1}{\lambda_0^2}$

TE & TM guided wavenumber $k^2=\left(\frac{\omega}{c}\right)^2-\pi^2\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]$

TE & TM cutoff angular frequency $\omega_{mn}=c\pi\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$

phase velocity $v=\omega/k$ group velocity $\frac{1}{dk/d\omega}$

rectangular resonant cavity angular frequency $\omega_{nml}=c\sqrt{\left(\frac{n\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2+\left(\frac{l\pi}{c}\right)^2}$

transmission line, vacuum dielectric, lossless

$Z_0=$ characteristic impedance of line

$Z_L=$ load impedance

$$V(z)=V_0^+e^{-i\frac{\omega}{c}z}+V_0^-e^{+i\frac{\omega}{c}z}$$
$$\Gamma=\frac{V_0^-}{V_0^+}=\frac{Z_L-Z_0}{Z_L+Z_0}$$
$$I(z)=V_0^+/Z_0e^{-i\frac{\omega}{c}z}+V_0^-/Z_0e^{+i\frac{\omega}{c}z}$$
$$V(z)=V_0^+\left[e^{-i\frac{\omega}{c}z}+\Gamma e^{+i\frac{\omega}{c}z}\right]$$
$$I(z)=V_0^+/Z_0\left[e^{-i\frac{\omega}{c}z}+\Gamma e^{+i\frac{\omega}{c}z}\right]$$

dynamical potentials $\mathbf{E}=-\nabla V-\partial\mathbf{A}/\partial t$, $\mathbf{B}=\nabla\times\mathbf{A}$

potential equations $\nabla^2V+\frac{\partial}{\partial t}\nabla\bullet\mathbf{A}=-\rho/\varepsilon_0$ $\left(\nabla^2\mathbf{A}-\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}\right)-\nabla\left(\nabla\bullet\mathbf{A}+\frac{1}{c^2}\frac{\partial V}{\partial t}\right)=-\mu_0\mathbf{J}$

gauge transformations $A' = A + \nabla\lambda \quad V' = V - \partial\lambda / \partial t$

Coulomb gauge $\nabla \bullet \mathbf{A} = 0$, Lorentz gauge $\nabla \bullet \mathbf{A} + \frac{1}{c^2}\partial V / \partial t = 0$

retarded time $t_r = t - r / c$

retarded potentials $V(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(\mathbf{r}',t_r)}{r}d\tau' \quad \mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\iiint \frac{\mathbf{J}(\mathbf{r}',t_r)}{r}d\tau'$

Jefimenko's equations

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\iiint\left[\frac{\rho(\mathbf{r}',t_r)}{r^2}\hat{r} + \frac{\dot{\rho}(\mathbf{r}',t_r)}{cr}\hat{r} - \frac{\dot{\mathbf{J}}(\mathbf{r}',t_r)}{c^2 r}\right]d\tau'$$

$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\iiint\left[\frac{\mathbf{J}(\mathbf{r}',t_r)}{r^2} + \frac{\dot{\mathbf{J}}(\mathbf{r}',t_r)}{cr}\hat{r}\right]\times\hat{r}\, d\tau'$$

Liénard-Wiechert potentials

$$V(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\frac{qc}{(rc - \mathbf{r}\bullet\mathbf{v})}$$

$$\mathbf{A}(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\frac{qc\mathbf{v}}{(rc - \mathbf{r}\bullet\mathbf{v})} = \frac{\mathbf{v}}{c^2}V(\mathbf{r},t)$$

Point charges $\mathbf{E}(\mathbf{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{r}{(\mathbf{r}\bullet u)^3}\left[(c^2 - v^2)\mathbf{u} + \mathbf{r}\times\mathbf{u}\times\mathbf{a}\right], \quad \mathbf{u} = c\hat{\mathbf{r}} - \mathbf{v}$

radiation fields $\mathbf{E}_{\text{rad}}(\mathbf{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{r}{(\mathbf{r}\bullet u)^3}[\mathbf{r}\times\mathbf{u}\times\mathbf{a}], \quad \mathbf{S}_{\text{rad}}(\mathbf{r},t) = \frac{1}{\mu_0 c}E_{\text{rad}}^2\hat{\mathbf{r}}$

charge at rest at time t_r $\mathbf{E}_{\text{rad}}(\mathbf{r},t) = \frac{\mu_0 q}{4\pi r}\left[(\hat{\mathbf{r}}\bullet\mathbf{a})\hat{\mathbf{r}} - \mathbf{a}\right] \quad \mathbf{S}_{\text{rad}}(\mathbf{r},t) = \frac{\mu_0 q^2 a^2}{16\pi^2 c}\frac{\sin^2\theta}{r^2}\hat{\mathbf{r}}$

Larmor formula $p = \frac{\mu_0 q^2 a^2}{6\pi c}$

generalization of Larmor formula $P = \frac{\mu_0 q^2\gamma^6}{6\pi c}\left(a^2 - \left|\frac{\mathbf{v}\times\mathbf{a}}{c}\right|^2\right), \quad \gamma^2 = \frac{1}{1-(v/c)^2}$

Maxwell's Equations

In general:

$$\begin{cases} \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{cases}$$

In matter:

$$\begin{cases} \nabla \cdot \mathbf{D} = \rho_f \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \end{cases}$$

Auxiliary Fields

Definitions:

$$\begin{cases} \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \end{cases}$$

Linear media:

$$\begin{cases} \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}, \quad \mathbf{D} = \epsilon \mathbf{E} \\ \mathbf{M} = \chi_m \mathbf{H}, \quad \mathbf{H} = \frac{1}{\mu}\mathbf{B} \end{cases}$$

Potentials

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

Lorentz force law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Energy, Momentum, and Power

Energy: $U = \frac{1}{2}\int \left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) d\tau$

Momentum: $\mathbf{P} = \epsilon_0 \int (\mathbf{E} \times \mathbf{B})\, d\tau$

Poynting vector: $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$