SHOW THAT GREEN'S FUNCTION $G(\vec{r},\vec{r}')$ OBEYS RECIPROCITY. THAT IS, SHOW $G(\vec{r},\vec{r}') = G(\vec{r}',\vec{r})$.

(I'M SURPRISED JACKSON DOESN'T SHOW THIS.)

A GENERIC COMMENT: YOU MIGHT HAVE ANTICAPED THIS SINCE THE DELTA-FUNCTION SOURCE IN POISSON'S EQUATION IS SYMMETRICAL IN $\vec{r}$ AND $\vec{r}'$. ANYWAY...

LET $G(\vec{r},\vec{r}_1)$ BE THE GREEN'S FUNCTION FOR A SOURCE AT $\vec{r}_1$. LET $G(\vec{r},\vec{r}_2)$ BE THE GREEN'S FUNCTION FOR A SOURCE AT $\vec{r}_2$. APPLY GREEN'S THEOREM (JACKSON EQN. 1.35) WITH $G(\vec{r},\vec{r}_1)$ AND $G(\vec{r},\vec{r}_2)$ THE TWO FIELDS:

$$\iiint \left\{ G(\vec{r},\vec{r}_1)\nabla^2 G(\vec{r},\vec{r}_2) - G(\vec{r},\vec{r}_2)\nabla^2 G(\vec{r},\vec{r}_1) \right\} \cdot dV$$

$$+ \oiint \left\{ G(\vec{r},\vec{r}_2)\frac{\partial G(\vec{r},\vec{r}_1)}{\partial n} - G(\vec{r},\vec{r}_1)\frac{\partial G(\vec{r},\vec{r}_2)}{\partial n} \right\} dA$$

WITH G VANISHING OVER THE SURFACE AND OBEYING

$$\nabla^2 G(\vec{r}, \vec{r}_1) = \frac{-1}{4\pi}\delta(\vec{r} - \vec{r}_1)$$

$$\nabla^2 G(\vec{r}, \vec{r}_2) = \frac{-1}{4\pi}\delta(\vec{r} - \vec{r}_2),$$

THE VOLUME INTEGRAL IS

$$G(\vec{r}_2, \vec{r}_1) = G(\vec{r}_1, \vec{r}_2).$$

SINCE $\vec{r}_1$ AND $\vec{r}_2$ ARE ARBITRARY, THIS RELATION IS GENERAL.