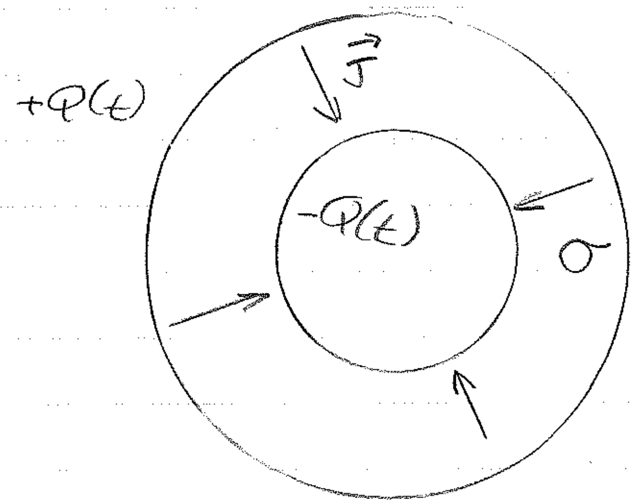
Physics 513, Electrodynamics I
Department of Physics, University of Washington
Autumn quarter 2020
December 3, 2020, 11am
On-line lecture

Administrative:
1. The draft of this lecture is posted at
`faculty.washington.edu/ljrberg/AUT20_PHYS513`
2. Office hours are today after class at 12:30.

Lecture: Magnetostatics, Faraday's Law. Quasi-Static Fields. (Jackson chapter 5).
Section 5.8-12 The fields M and H. The magnetized sphere via scalar and vector potentials. Linear magnetic media: Magnetic susceptibility and permeability. Macroscopic boundary conditions on B and H (and A).

COMMENT ON HOMEWORK PROBLEM.
"LEAKY" SPHERICAL CAPACITOR



σ SMALL BUT NON-ZERO.

WE NOW HAVE MANY WAYS TO SHOW $B=0$ EVERYWHERE. E.g.,

a. SYMMETRY & $\vec{\nabla}\cdot\vec{B}=0$.
b. $\vec{A}\sim\hat{r}$ AND $\vec{\nabla}\times\vec{A}=0$
c. DIRECT COMPUTATION OF $\vec{B}$ FROM BIOT - SAVART LAW. WITH $\vec{\nabla}\times\vec{J}=0$.

d. (HOMEWORK) LORENTZ CONDITION

$$\vec{\nabla}\cdot\vec{A}+\frac{1}{c^2}\frac{\partial}{\partial t}\Phi=0$$

VIA $\Phi(t)$.

MAGNETIC MATERIALS II (AND BOUNDARY-VALUE PROBLEMS).

WE PURSUE A PATH SIMILAR TO THAT OF DIELECTRIC MATERIALS IN AN ELECTRIC FIELD.

THE GOVERNING EQUATION IS AMPÉRE'S LAW

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$, WHERE $\vec{J}$

INCLUDES TRUE CURRENTS, $\vec{J}_{TRUE}$, MAGNETIZATION CURRENTS $(\vec{\nabla} \times \vec{M})$, AND DISPLACEMENT $(\partial/\partial t \, \vec{D})$, etc.

Q! CAN YOU THINK OF OTHER CURRENTS?

RECALL THE PATH OF DIELECTRIC MATERIALS IN ELECTROSTATICS! WE SEPARATED FROM THE TOTAL FIELD $\vec{E}$ (WHICH HAS SOURCES ALL CHARGES, INCLUDING POLARIZATION CHARGES), A FIELD $\vec{D}$ WHOSE SOURCES ARE "FREE" CHARGES.

IN MAGNETOSTATICS, WE SEPARATE FROM THE TOTAL FIELD $\vec{B}$ (WHOSE SOURCES ARE ALL CURRENTS, INCLUDING MAGNETIZATION CURRENTS), A PART WHOSE SOURCES OMIT MAGNETIZATION CURRENTS:

$$\vec{\nabla}\times\underbrace{(\vec{B}-\mu_0\vec{M})}_{\mu_0\vec{H}} = \mu_0\left(\vec{J}_{TRUE}+\frac{d}{dt}\vec{D}\right).$$

$$\vec{H} = \vec{B}/\mu_0 - \vec{M} \qquad \text{WHERE}$$

$$\vec{\nabla}\times\vec{H} = \vec{J}_{TRUE} + \frac{d}{dt}\vec{D}.$$

$\vec{H}$ IS THE "MAGNETIC FIELD INTENSITY" OR (CONFUSINGLY) THE "MAGNETIC FIELD".

WE CAN AT THIS POINT DEFINE WHAT WE MEAN BY "QUASI-STATIC":

$$\frac{dD}{dt} \ll J_{TRUE} \quad \text{IS QUASI-STATIC}$$

AND FOR QUASI-STATIC SYSTEMS

$$\vec{\nabla}\times\vec{H} = \vec{J}_{TRUE} \quad \text{AND}$$

$$\oint \vec{H}\cdot d\vec{l} = I_{TRUE}.$$

WE SAW FOR MAGNETIC MATERIALS (WHERE $\vec{J}_{TRUE} = 0$) IT WAS SENSIBLE TO INTRODUCE A MAGNETIC SCALAR POTENTIAL. THIS HAS COULOMB-LIKE "CHARGES" SOURCING THE POTENTIAL. WHAT'S THE NATURE OF THIS MAGNETIC "CHARGE".

DON'T BE UNDER THE IMPRESSION STANDARD ELECTRODYNAMICS HAS FREE MAGNETIC CHARGE, THIS "MAGNETIC CHARGE" IS FICTICIOUS.

FOR A PERMANENT MAGNET, THERE'S NO TRUE CURRENTS:

$$\vec{\nabla} \times \vec{H} = 0, \quad \text{AND} \quad \vec{\nabla} \times \vec{B} = \vec{\nabla} \times \vec{M} \neq 0.$$

HOWEVER,

$$\vec{\nabla} \cdot \vec{H} = -\vec{\nabla} \cdot \vec{M}.$$

$\vec{H}$ FOR A PERMANENT MAGNET HAS THE ROLE OF $\vec{E}$ IN ELECTROSTATICS $\left(\vec{\nabla} \times \vec{E} = 0, \ \vec{\nabla} \cdot \vec{E} = \rho/\varepsilon_0\right)$.

WE CAN SIMPLY READ OFF THE MAGNETIC CHARGE DENSITY

$$\rho_M = -\vec{\nabla} \cdot \vec{M}.$$

AND $\sigma_M = \vec{M}\cdot\hat{n}$ Q: WHY?

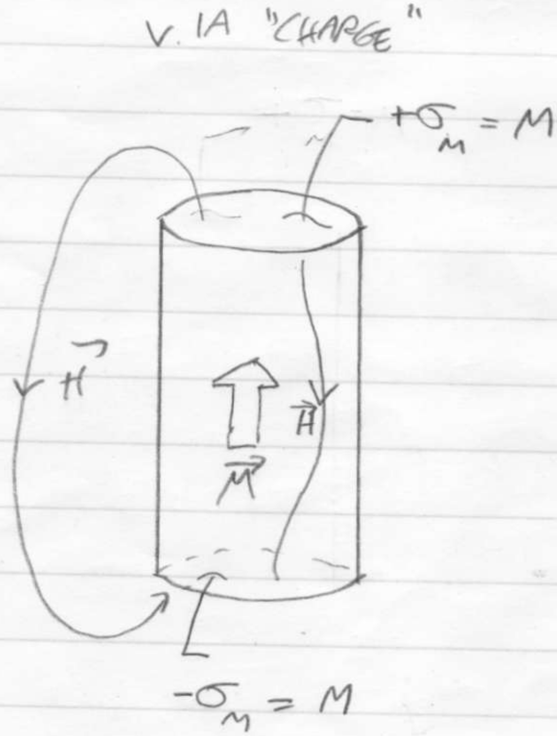
AND CONTINUING THE PARALLEL WITH ELECTROSTATICS:

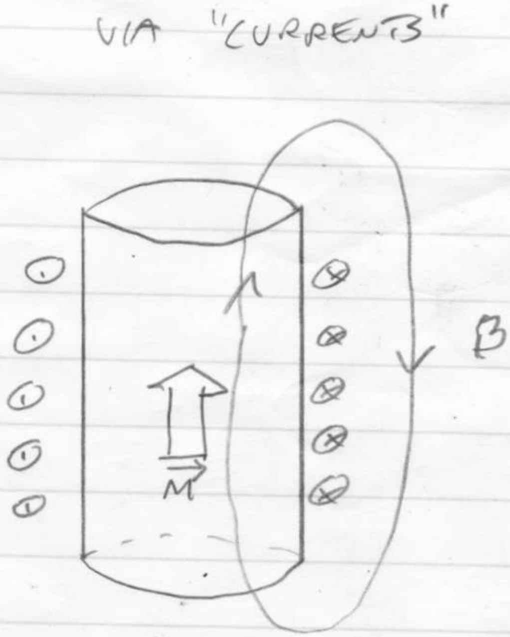
FOR $\vec{H} = -\vec{\nabla}\Phi_M$,

$$\Phi_M(\vec{r}) = \frac{1}{4\pi}\iiint \frac{(-\vec{\nabla}\cdot\vec{M})}{|\vec{r}-\vec{r}'|}\,dv'$$

$$+\frac{1}{4\pi}\oiint \frac{\vec{M}\cdot\hat{n}}{|\vec{r}-\vec{r}'|}\,da'$$

QUALITATIVELY, FOR A PERMANENT BAR MAGNET:





IN EARLY ELECTRODYNAMICS, THIS RAISED A PHILOSOPHICAL QUESTION; THE FIELDS OF A PERMANENT MAGNET CAN BE DESCRIBED IN TERMS OF $\vec{B}$ OR $\vec{H}$ (THAT IS, IN TERMS OF EQUIVALENT "CURRENTS" OR EQUIVALENT "CHARGES").

IS $\vec{B}$ OR $\vec{H}$ MORE FUNDAMENTAL?

THIS QUESTION WAS REDUCED TO: CONSIDER A CHARGE q MOVING AT VELOCITY $\vec{V}$ IN A MAGNET. SUPPOSE THE FORCE ACTING ON THE CHARGE HAS FORM

$$\vec{F} = q\,\vec{V} \times \vec{Z}.$$

SHOULD $\vec{Z}$ BE $\vec{B}$ OR $\vec{H}$, OR PERHAPS A SUPERPOSITION OF THE TWO?

THERE WAS, HISTORICALLY, GREAT CONFUSION ON THIS FOR $V \ll C$.

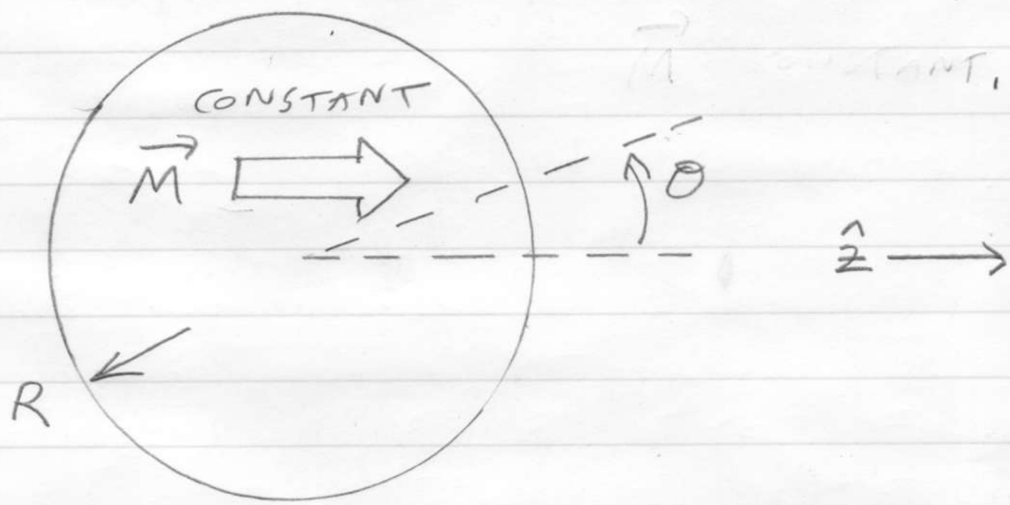
IF YOU'RE INTERESTED:

G.H. WANNIER, PHYS. REV. (1947) <u>72</u> 304;

D. J. HUGHES, "PILE NEUTRON RESEARCH", ADDISON-WESLEY (1953), § 11.4 AND § 10.6.

EXAMPLE: MAGNETIZED SPHERE (SIMILAR TO DIELECTRIC SPHERE IN AN $\vec{E}$ FIELD).

LET'S DO IT WITH "CHARGES".



HERE, THE USE OF THE SCALAR MAGNETIC POTENTIAL IS VALID. Q: WHY?

$$\Phi_M = \frac{1}{4\pi}\iiint \frac{(-\vec{\nabla}\cdot\vec{M})}{|\vec{r}-\vec{r}\,'|}\,d\tau' + \frac{1}{4\pi}\oint\!\!\oint \frac{\vec{M}\cdot\hat{n}}{|\vec{r}-\vec{r}\,'|}\,da'$$

$\vec{M}\cdot\hat{n} = M\cos\theta$ (THAT "$\cos\theta$" AGAIN!) IS THE SURFACE "CHARGE".

$\vec{\nabla}\cdot\vec{M} = 0$ (NO BULK "CHARGE").

NOW WE PROCEED AS FOR ELECTROSTATICS OF THE POTENTIAL OF A SPHERICAL SURFACE WITH $\sigma \cos\theta$ PASTED ON:

- SEPARATE VARIABLES

$$\Phi_M(r,\theta) = \sum_\ell \left(a_\ell r^\ell + b_\ell \frac{1}{r^{\ell+1}}\right) P_\ell(\cos\theta)$$

- "OUTSIDE" $\Phi_{OUT}(r,\theta) = \sum_\ell \frac{b_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$

"INSIDE" $\Phi_{IN}(r,\theta) = \sum_\ell a_\ell r^\ell P_\ell(\cos\theta)$.

- THE POTENTIAL IS CONTINUOUS AT $r=R$:

$$\sum_\ell a_\ell R^\ell P_\ell(\cos\theta) = \sum_\ell \frac{b_\ell}{R^{\ell+1}} P_\ell(\cos\theta)$$

TERM-BY-TERM. Q: WHY?

$$b_\ell = a_\ell R^{2\ell+1}.$$

- BECASE OF THE SURFACE "CHARGE", $d\Phi/dr$ HAS A DISCONTINUITY

$$\frac{d}{dr}\Phi_{OUT}\Big|_R - \frac{d}{dr}\Phi_{IN}\Big|_R = -\sigma\cos\theta.$$

GIVES $\sum_\ell (2\ell+1) a_\ell r^{\ell-1} P_\ell(\cos\theta) = \sigma \cos\theta$.

EACH P_ℓ IS INDEPENDENT (OR, APPLY ORTHOGONALITY)

- ONLY $\ell = 1$ SURVIVES.

$$a_1 = \frac{\sigma}{3}.$$

THIS PROCEDURE SHOULD BE FAMILIAR. HENCE

$$\Phi_{IN}(r,\theta) = \frac{\sigma}{3} r \cos\theta;$$

$$\Phi_{OUT}(r,\theta) = \frac{\sigma}{3} \frac{R^3}{r^2} \cos\theta.$$

THE FIELDS ARE

$$\vec{H}_{IN} = -\vec{\nabla}\Phi_{IN} = -\frac{1}{3}\vec{M} \qquad (M = \sigma).$$

$$\vec{B}_{IN} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

$$= \mu_0 \frac{2}{3} \vec{M}.$$

INSIDE, $\vec{B}$ AND $\vec{H}$ ARE IN OPPOSITE DIRECTIONS.
AND $\vec{B}$ AND $\vec{H}$ ARE UNIFORM.
OUTSIDE, OF COURSE, $\vec{B} \sim \vec{H}$.

RECALL $\Phi_{OUT}(r,\theta) = \frac{\sigma}{3} \frac{R^3}{r^2} \cos\theta$. $(\sigma = M)$

ALSO RECALL THE FIELDS OF AN ELECTROSTATIC DIPOLE

$$\Phi^{(2)}_{ELEC} = \frac{1}{4\pi\varepsilon_0} \frac{|\vec{p}| \cos\theta}{r^2}.$$

HENCE WE IDENTIFY FOR

$$\Phi^{(2)}_{MAG} = \frac{1}{4\pi} \frac{|\vec{m}| \cos\theta}{r^2}$$

WITH $\frac{1}{3} M R^3 = \frac{1}{4\pi} |\vec{m}|$.

THE EXTERIOR FIELD IS A PURE MAGNETIC DIPOLE IN $\vec{H}$, AND $\vec{B} = \mu_0 \vec{H}$ OUTSIDE.

WE SEE AGAIN THE "$\cos\theta$" BEHAVIOR FROM ELECTROSTATICS.

EXAMPLE: SAME MAGNETIZED SPHERE WITH MAGNETIZATION CURRENTS.

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\iiint \frac{\vec{\nabla}' \times \vec{M}}{|\vec{r}-\vec{r}'|}\,dv'$$

$$\frac{\mu_0}{4\pi}\oiint \frac{\vec{M}\times\hat{n}'}{|\vec{r}-\vec{r}'|}\,da'.$$

THE $\vec{\nabla}' \times \vec{M}$ BULK CURRENT VANISHES AND WE'RE LEFT WITH CIRCULATING SURFACE CURRENTS; FOR $\vec{M} \sim \hat{z}$;

$$\vec{K} = \vec{M}\times\hat{n} = M\sin\theta\,\hat{\phi}$$
$$= M\sin\theta\{-\sin\phi\,\hat{x} + \cos\phi\,\hat{y}\}.$$

(SEE JACKSON P. 199).

THE PROBLEM IS SYMMETRIC IN ϕ, SO CHOOSE $\phi = 0$ FOR THE FIELD POINT. THE X-COMPONENT OF $\vec{A}$ IS ODD IN ϕ, SO THAT COMPONENT VANISHES ON INTEGRATING $\phi \in [-\pi, +\pi]$. HENCE WE KEEP $\vec{K} = M\sin\theta\cos\phi\,\hat{y}$. FOR A FIELD POINT AT $\phi = 0$.

KEEP IN MIND $\vec{A}$ HAS SYMMETRY IN ϕ, SO THIS PARTICULAR $\vec{A}(\phi=0)$ HAS THE SAME MAGNITUDE FOR ALL ϕ. THAT IS

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} M R^2 \oint \frac{\sin\theta' \cos\phi'}{|\vec{r}-\vec{r}'|} d\Omega' \hat{\phi}$$

(JACKSON EQN 5.109).

RECALL $Y_1^1(\theta,\phi) = -\sqrt{\frac{3}{8\pi}} \sin\theta \, e^{i\phi}$

ALSO RECALL THE $1/r$ EXPANSION

$$\frac{1}{|\vec{r}-\vec{r}'|} = 4\pi \sum_{\ell,m} \frac{1}{2\ell+1} \frac{r_<^\ell}{r_>^{\ell+1}} Y_\ell^{m*}(\theta,\phi) Y_\ell^m(\theta,\phi).$$

ONCE WE APPLY ORTHOGONALITY TO $\vec{A}$ AT THE SURFACE, ONLY TERMS WITH $\sin\theta \cos\phi$ SURVIVE, THIS IS $\mathrm{Re}\, Y_1^1$:

$$\frac{1}{|\vec{r}-\vec{r}'|} \rightarrow 4\pi \frac{1}{3} \frac{r_<}{r_>^2} \sin\theta \cos\phi$$

$$\vec{A}_{IN}(\vec{r}) = \frac{\mu_0}{3} M r \sin\theta \, \hat{\phi}$$

$$A_{OUT}(\vec{r}) \; \frac{\mu_0}{3} M R^2 \frac{R}{r^2} \sin\theta \, \hat{\phi}$$

Q! SHOW THIS?

THE $\vec{B}$ FIELD IS OBTAINED BY $\vec{\nabla} \times A$: SEE JACKSON EQN. 5.38.

THIS HAS, IN GENERAL, $\hat{\theta}$ AND $\hat{r}$ COMPONENTS. FOR $\vec{A} \sim \hat{\phi}$,

INSIDE

$$B_\theta = -\frac{1}{r}\frac{d}{dr}(rA_\phi) \sim \sin\theta,$$

$$B_r = \frac{1}{r\sin\theta}\frac{d}{d\theta}(\sin\theta \, A_\phi)$$

$$\sim \cos\theta$$

THIS DESCRIBES A CONSTANT $\vec{B} \sim \hat{z}$.

SIMILARLY, OUTSIDE

$$B_r \sim \cos\theta / r^3; \quad B_\theta \sim \sin\theta / r^3$$

WITH CARE EVALUATING THE CONSTANTS, THIS DESCRIBES A PURE DIPOLE FIELD WITH MAGNETIC DIPOLE MOMENT

$$\vec{m} = \vec{M} \tfrac{4}{3}\pi r^3.$$

(MY EXPERIENCE HAS BEEN IT'S EASIER TO USE THE SCALAR POTENTIAL.)

RECALL IN ELECTROSTATICS, WE INTRODUCED THE POLARIZATION DENSITY $\vec{P}$, THEN THE ELECTRIC SUSCEPTIBILITY χ_E FOR LINEAR MEDIA VIA

$$\vec{P} = \varepsilon_0 \chi_E \vec{E},$$

THE $\vec{D}$ FIELD VIA

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P},$$

AND THE DIELECTRIC CONSTANT VIA

$$\vec{D} = \varepsilon \vec{E} \qquad (\varepsilon = \varepsilon_0\{1 + \chi_E\}).$$

WE'LL FOLLOW A SIMILAR PATH FOR MAGNETIC MATERIALS. THE BIG DIFFERENCE IS A PERMANENT ELECTRIC POLARIZATION IS RARE, WHILE A PERMANENT MAGNETIC POLARIZATION IS COMMON. THIS COMPLICATES MAGNETIC EFFECTS IN MATERIALS.

"PERMEABLE" MEDIA:
MAGNETIC SUSCEPTIBILITY, PERMEABILITY, AND BOUNDARY CONDITIONS.

WE'LL START WITH AN IDEAL LINEAR MEDIUM ($\vec{M} \sim \vec{H}$), (NOTICE FOR DIELECTRICS $\vec{P} \sim \vec{E}$; BUT HERE $\vec{M} \sim \vec{H}$.)

(WITH NO TRUE CURRENTS ANYWHERE, $\vec{M} = 0$ FOR LINEAR MEDIA.)

RECALL $\vec{\nabla} \cdot \vec{B} = 0$ AND $\vec{\nabla} \times \vec{H} = \vec{J}_{TRUE}$.

WITH $\vec{M}$ OF FORM $\vec{M} = \chi_M \vec{H}$
(χ_M THE "MAGNETIC SUSCEPTIBILITY)

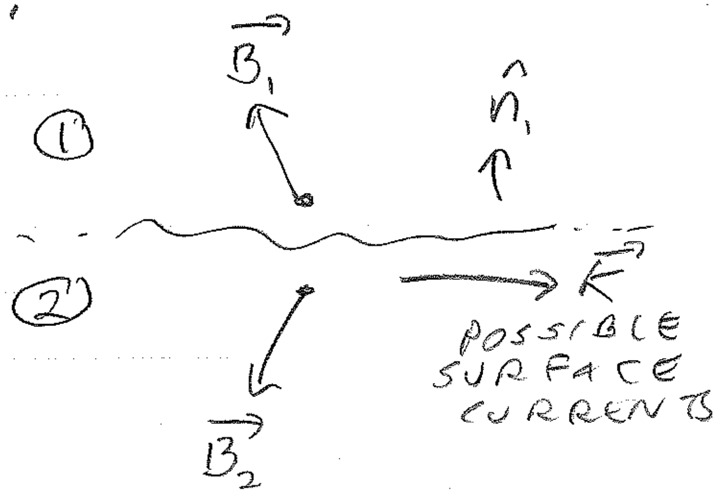
$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (1 + \chi_M) H = \mu \vec{H}$$

(μ THE PERMEABILITY).

LET'S EXPLORE HOW $\vec{B}$ AND $\vec{H}$ CHANGE ON CROSSING AN ABRUPT CHANGE IN PERMEABILITY: BOUNDARY CONDITIONS.

BOUNDARY CONDITIONS ON $\vec{B}, \vec{H}$ (AND $\vec{A}$):

$\vec{\nabla} \cdot \vec{B} = 0$



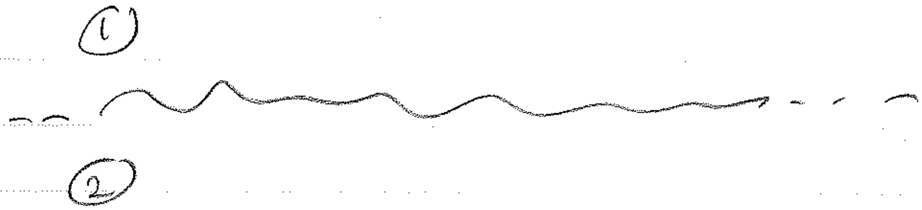
$$\hat{n}_1 \cdot (\vec{B}_2 - \vec{B}_1) = \hat{n}_1 \cdot (\mu_2 \vec{H}_2 - \mu_1 \vec{H}_1) = 0.$$

$\vec{\nabla} \times \vec{H} = \vec{J}_{TRUE}$

WE AGAIN TAKE A "SQUAT" LOOP STRADDLING THE BOUNDARY.

$$\hat{n}_1 \times (\vec{H}_2 - \vec{H}_1) = \hat{n}_1 \times \left(\frac{\vec{B}_2}{\mu_2} - \frac{\vec{B}_1}{\mu_1} \right) = \vec{K}.$$

BOUNDARY CONDITIONS ON $\vec{A}$.
WE'LL FIND THESE FROM BOUNDARY CONDITIONS ON $\vec{B}$.



- THE NORMAL COMPONENT OF $\vec{B}$ IS CONSERVED! HENCE THE MAGNETIC FLUX ACROSS THE BOUNDARY IS CONSERVED.

  RECALL $\oint \vec{A} \cdot d\vec{l}$ IS THE MAGNETIC FLUX THREADING THE LOOP. FOR A LOOP ON THE BOUNDARY, $\oint \vec{A} \cdot d\vec{l}$ IS CONSERVED CROSSING THE BOUNDARY.

  FOR A "THIN" LOOP:

$$A_{\parallel_1} = A_{\parallel_2} \, ;$$

  THE TANGENTIAL (PARALLEL) COMPONENTS OF $\vec{A}$ ARE CONSERVED ACROSS THE BOUNDARY.

- WITH NO TRUE SURFACE CURRENTS $\vec{K}$, THE TANGENTIAL COMPONENTS OF $\vec{H}$ ARE CONSERVED ON CROSSING THE BOUNDARY.

$$\frac{1}{\mu_1}\left[\vec{\nabla}\times\vec{A}\right]_{\parallel_1} = \frac{1}{\mu_2}\left[\vec{\nabla}\times\vec{A}\right]_{\parallel_2}$$

WE SHOULD PONDER THIS; THE SET OF BOUNDARY CONDITIONS ON $\vec{A}$ CONSIST OF 4 CONDITION; THERE WERE 2 CONDITIONS FOR THE ELECTROSTATIC CASE.

IN PRACTICE, BOUNDARY-CONDITION PROBLEMS INVOLVING $\vec{A}$ ARE USUALLY DIFFICULT.

NOW, WE USE $\vec{A}$ TO FIND SOLUTIONS TO MAGNETOSTATIC PROBLEMS.

UNIQUENESS: THE TANGENTIAL COMPONENTS OF $\vec{A}$ (OR $\vec{B}$) ON A SURFACE (ALMOST) UNIQUELY DETERMINE $\vec{A}$ WITHIN THE BOUNDED VOLUME.

LET'S RETURN TO THE POISSON-LIKE EQUATION FOR $\vec{A}$:

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = -\nabla^2\vec{A} + \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) = \mu_0\vec{J}$$

(CARE SHOULD BE TAKEN IN EVALUATING $\nabla^2\vec{A}$. IN CARTESIAN COORDINATES EACH COMPONENT IS $\nabla^2 A_x$, e.g. IN NON-CARTESIAN COORDINATES, THE, e.g., x-COMPONENT OF $\nabla^2\vec{A}$ IS $[-\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) + \vec{\nabla}\cdot(\vec{\nabla}\cdot\vec{A})]_x$; NOTICE YOU DON'T FIND $\nabla^2\vec{A}$ IN THE BACK OF JACKSON.)

RECALL "COULOMB GAUGE".
FOR STATIONARY CURRENTS $\vec{\nabla}\cdot\vec{A}=0$,

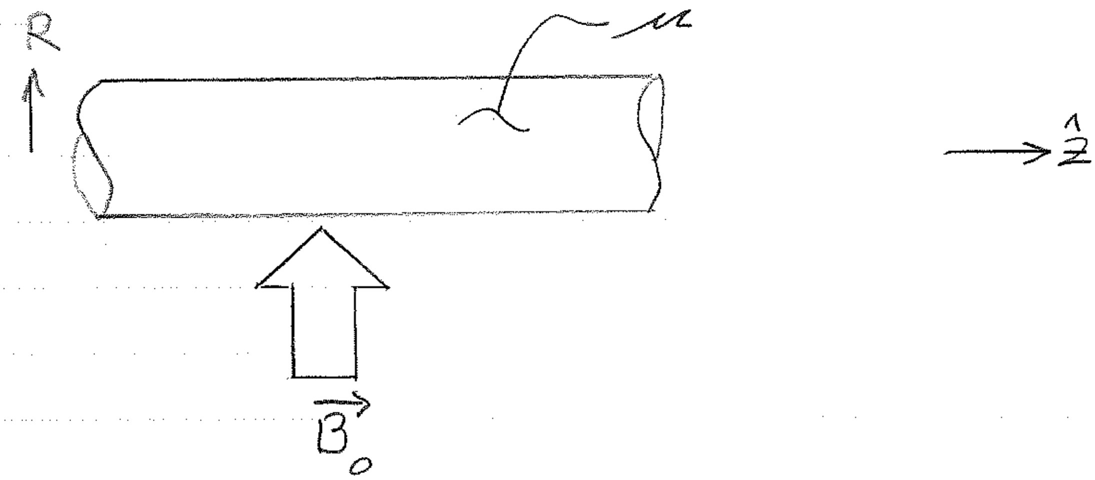
HENCE $\nabla^2\vec{A} = -\mu_0 J$, THE VECTOR FORM OF POISSON'S EQUATION, WITH SOLUTION

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\iiint \frac{\vec{J}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,dr',$$

(YOU RECALL FOR A CURRENT FLOWING DOWN A LONG STRAIGHT WIRE, WE FOUND $\vec{\nabla}\cdot\vec{A}=0$.)

NOW, WE'D LIKE THIS SOLUTION TO $\nabla^2\vec{A}$ SUBJECT TO BOUNDARY CONDITIONS. THIS IS CONSIDERABLY MORE COMPLICATED THAN THAT FOR $\nabla^2\Phi$ SINCE $\vec{\nabla}\cdot\vec{A}=0$ ADDS RELATIONS BETWEEN COMPONENTS OF $\vec{A}$.

EXAMPLE: LONG PERMEABLE ROD IN TRANSVERSE $\vec{B}$ FIELD. FIND $\vec{A}$.



THE SYSTEM IS TRANSLATIONALLY INVARIANT; ALL $\partial/\partial z$ VANISH AND $\vec{B} = \vec{\nabla} \times \vec{A}$ IS

$$B_x = \frac{\partial}{\partial y} A_z \quad \text{AND} \quad B_y = -\frac{\partial}{\partial x} A_z.$$

Q: WHAT HAPPENED TO B_z?

LAPLACE'S AND POISSON'S EQUATIONS WITH TRANSLATIONAL INVARIANCE ARE:

$$\nabla^2 A_z = 0 \quad \text{AND} \quad \nabla^2 A_z = -\mu \vec{J}.$$

SIMILAR TO THE PROBLEM OF THE DIELECTRIC ROD IN AN ELECTRIC FIELD, THERE ARE SEPARATE SOLUTIONS FOR $r < R$ AND $r > R$.

INSIDE: $A_z = \sum_\ell (a_n \cos \ell\theta + b_n \sin \ell\theta) r^\ell$

(RECALL LECTURE OCT. 22, 2020.)

Q: WHY ARE $r^{-\ell}$ TERMS ABSENT?

OUTSIDE: AT LARGE DISTANCES THE POTENTIAL SHOULD BE THAT FOR CONSTANT $\vec{B}_0$ (SAY $\vec{B}_0 = B_0 \hat{x}$)

Q: WHY IS SUCH A DISTANT FIELD $A_z = B_0 y = B_0 r \sin\theta$?

THE OUTSIDE SOLUTION HAS FORM

$$A_z = B_0 r \sin\theta + \sum_\ell (C_\ell \cos \ell\theta + d_\ell \sin\theta) r^{-\ell}.$$

$+ B_0 r \sin\theta$

Q: WHY ARE $r^{+\ell}$ TERMS ABSENT?

APPLY BOUNDARY CONDITIONS:

1. $\vec{\nabla} \cdot \vec{B} = 0$; $\vec{B} \cdot \hat{n}$ IS CONTINUOUS ACROSS $r = R$, SO A_z IS LIKEWISE CONTINUOUS.

2. $\vec{\nabla} \times \vec{H} = 0$; THE TRANSVERSE COMPONENT OF $\vec{H}$ ACROSS $r = R$ IS CONTINUOUS, SO:

$$\frac{1}{\mu} \frac{d}{dr} A_z(r < R) \Big|_{r=R} = \frac{1}{\mu_0} \frac{d}{dr} A_z(r > R) \Big|_{r=R}.$$

Q: WHY THIS FORM?

AT THIS POINT THE STRUCTURE OF THE EQUATIONS IS THE SAME AS THAT OF THE DIELECTRIC ROD IN A TRANSVERSE $\vec{E}$ FIELD.

$$A_z(r < R) = \frac{2\mu}{\mu + \mu_0} B_0 \, r \sin\theta$$

$$A_z(r > R) = \left(1 + \frac{\mu - \mu_0}{\mu + \mu_0} \left\{\frac{R}{r}\right\}^2\right) \cdot B_0 \, r \sin\theta$$