



**Physics 513, Electrodynamics I**  
**Department of Physics, University of Washington**  
**Autumn quarter 2020**  
**October 1, 2020, 11am**  
**On-line lecture**

***Administrative:***

- 1. All lectures are on Zoom; see the course web-site [faculty.washington.edu/ljrberg/AUT20\\_PHYS513](http://faculty.washington.edu/ljrberg/AUT20_PHYS513)**
- 2. The exams are take-home: a midterm & “final” (technically midterm 2).**
- 3. Homework and exam submission is via email; submission instructions are on the homework/exam.**
- 4. We’ll go over the syllabus.**
- 5. We’ll go over the schedule.**

***Lecture: Loose ends and preliminary remarks.***

**Units and the speed of light  $c$ .**

**Electric & magnetic media: the fields/sources  $P$  and  $M$ .**

**Where to start? Charge of the electron and the  $1/r^2$  law.**

**The electric field  $E$ ; on the issue of the “reality” of the field.**

**On the question of quantization of charge.**

**On the question of magnetic monopoles.**

**Where does Classical Electrodynamics fail?**

**Starting place: Maxwell’s equations and the Lorentz force law.**

# Physics 513, Autumn Quarter 2020

## Prof. Leslie J Rosenberg, Department of Physics, University of Washington

### General information:

Physics 513, the first course in graduate electrodynamics

Textbook: J.D. Jackson, "Classical Electrodynamics," third edition

Because the campus is largely closed due to COVID-19, this entire course is on-line throughout the quarter. Zoom lectures for the course are Tuesdays & Thursdays 11:00-12:20 Pacific time. The lectures aren't recorded, but lecture notes will typically be posted on this web site. The first lecture is Thursday, October 1.

Join the on-line Zoom lectures at URL <https://washington.zoom.us/j/99341867892> (you'll need your UWnetID credentials, then enter the Zoom "SSO" of "washington" or "washington.zoom.edu" depending on your system).

### Course Instructor:

Prof. Leslie J Rosenberg

Email: [ljrosenberg@phys.washington.edu](mailto:ljrosenberg@phys.washington.edu)

Office: Physics & Astronomy Building, room C503

Office Hours Thursdays 12:30 via Zoom URL <https://washington.zoom.us/j/99341867892> (you'll need your UWnetID credentials, then enter the Zoom "SSO" of "washington" or "washington.zoom.edu" depending on your system) or by appointment.

Telephone: (206) 221-5856

### TAs/graders:

Kyle Aitken [kaitken@uw.edu](mailto:kaitken@uw.edu)

Isaac Shelby [ishelby@uw.edu](mailto:ishelby@uw.edu)

### Useful Information:

- [Readings, Lectures and Exams](#)
- On-Line Lecture Notes

- Special Lectures
- Homework
- Midterm-exam information.  
The midterm exam will be posted on this web site on Friday, November 6, at 4pm Pacific time. It's due via email on Monday, November 9, at 4pm Pacific time. The exam contains more detailed submission instructions. I'll post more exam information as the exam date approaches.
- Final-exam information.  
The final exam (more properly, the second midterm exam) will be posted here on Thursday, December 10, at 4pm Pacific time. It's due via email on Friday, December 11, at 4pm Pacific time. The exam contains more detailed submission instructions. I'll post more exam information as the exam date approaches.

### Recent course news:

- [28Sep2020 9:00] The first class day is Thursday, October 1. Zoom lectures are Tuesdays and Thursdays, see above.

### Lecture Instructor's Comments

Welcome to Physics 513, the first of a three-quarter sequence of graduate classical electrodynamics. This is a wonderful topic, it's challenging and stimulating. I think it's the most enjoyable and edifying course in the Physics graduate curriculum. Electrodynamics is crucial for understanding the underpinnings of the physical and biological sciences. It's also essential for modern technology. Going forward in your career, a familiarity with Jackson chapters 1-16 will allow you to converse sensibly with your colleagues.

There are many ways to approach electrodynamics. Roughly speaking, there's the formal approach (starting with the Lagrangian of electrodynamics) of Landau and Lifshitz and the historical approach (starting with electric charges and electrostatics) of Jackson. We'll use Jackson's text "Classical Electrodynamics". You might want more details or other topics than found in Jackson, or perhaps you'd like an alternative approach, and I assure you there are many approaches. In which case you might want to look at, e.g., Panofsky and Phillips "Classical Electricity and Magnetism". Two very good, very readable, books for some of the more formal aspects of the classical field theory but with fewer applications are Landau and Lifshitz "The Classical Theory of Fields" and "Electrodynamics of Continuous Media". Another nice thing about Landau and Lifshitz "Fields" is halfway through the text, General Relativity enters rather seamlessly. There's a movement in teaching graduate electrodynamics to present the topic in the language of differential forms, but we won't do so. However, if you're interested in this approach, a good introduction is Misner, Thorne and Wheeler "Gravitation" chapter 2. A slightly more elementary but very good graduate electrodynamics text is Slater and Frank "Electromagnetism". Most homework problems in this course, and indeed the majority of homework problems in most texts, are adapted from Smythe, "Static and Dynamic Electricity", a challenging text with an archaic and hard-to-read notation; Smythe adapted the problems in that text from a compilation of Oxford University graduate exam problems. A more modern electrodynamics text is Zangwell,

"Modern Electrodynamics", it has good reviews, but I haven't yet gone through it in detail. It also has some new problems. There's no perfect text, and every text has gems scattered throughout.

Mathematical methods are interspersed throughout the course as needed, Jackson is good about introducing the necessary mathematics background. For a math refresher, there are many suitable texts, e.g., Dennery and Krzywicki "Mathematics for Physicists". I think the gold standard of mathematical physics texts are the two volumes of Morse and Feshbach, "Methods of Mathematical Physics".

That said, for the first quarter we'll follow Jackson's text somewhat closely.

**Syllabus** The syllabus for 513 starts with chapter 1 in Jackson, the beginnings of electrostatics. With luck, this quarter we'll be able to get through chapter 5, magnetostatics. We'll follow the text in more or less the text ordering, though we'll sometimes supplement Jackson's presentation with added material. See above for a link to the readings and lectures. Try to read the relevant text and added material before class; this will take time but there's a big payoff in understanding. Alas, I've found in teaching this course the syllabus readings become out of sync with the lectures, so you'll have to pay attention.

**Grading** 40% of your grade is assigned to the midterm exam, 40% to the "final" (second midterm) exam, 20% to the homework.

- **Midterm and "final" exam:** There will be a midterm exam and a "final" exam (more properly a second midterm exam). Both are take-home exams. Exams are to be your own work; you aren't permitted to collaborate with any other person.
- **Note that there are no make-up exams or make-up homework.** Students with outside professional, service, or career commitments (i.e. military service, professional conference presentation, etc.) conflicting exactly with the exam dates must contact the instructor in the first two weeks of the quarter to establish alternate procedures. I'm quite flexible in this regard but you'll need to contact me beforehand. Students who miss an exam or homework due to illness should contact the instructor as soon as you're reasonably able to discuss alternate procedures. Except for debilitating illness or other crisis, students who miss an exam or homework without making prior arrangements with the instructor will get a zero for that score. Except for illness and circumstances noted above, a final grade of 0.0 may be assigned to any student who misses a midterm or final exam.

- **Homework:**

Lecture homework will generally be assigned and collected weekly. Typically, homework is assigned Monday afternoon (Pacific time) and is due via email the next Monday afternoon (Pacific time). The assignments contain more submission details. The graders will consider neatness and logic of presentation, points will be deducted for lack of either. Words help in explaining your solution. Briefly, if your homework is a messy, incoherent scrawl, the graders won't evaluate your homework. I strongly encourage you to work collaboratively, but your submitted work must be your own. A typical assignment has four problems. One problem is straightforward (at the Griffiths level), two are at the Jackson level, and one is more challenging. Typically, not all problems are graded. I typically post solutions on the due-day, I welcome comments and corrections to the solutions and clever alternate approaches.

- **Communication:**

For administrative issues, it's best to contact me via email. But, for physics questions, I discourage email (unless the question answer is of the "yes/no" variety). Physics is best discussed at Zoom office hours. Also, don't hesitate to make a Zoom appointment to talk with me at other times.

- **Misconduct:**

Academic misconduct is a serious offense. See <https://www.washington.edu/cssc/facultystaff/academic-misconduct/> for policy and reporting procedures. The above grading description applies only to students who uphold academic integrity; grades will be lowered, possibly to zero, for students found to have engaged in academic misconduct. There could also be other sanctions. Respectful conduct is expected of all, see <https://www.washington.edu/studentconduct/>. Students are encouraged to bring any instances of problematic behavior to the attention of the instructor and to contact SafeCampus (206-685-7233 anytime) with concerns for yourself or others.

- **Religious Accommodations:**

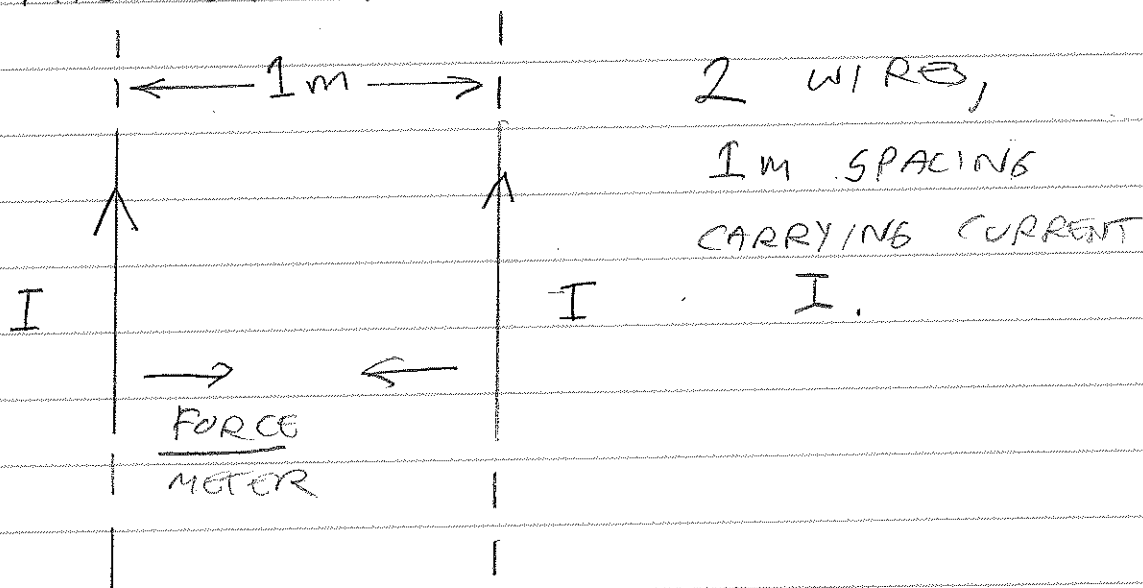
Washington state law requires that UW develop a policy for accommodation of student absences or significant hardship due to reasons of faith or conscience, or for organized religious activities. The UW's policy, including more information about how to request an accommodation, is available at [Religious Accommodations Policy](https://registrar.washington.edu/staffandfaculty/religious-accommodations-policy/) (<https://registrar.washington.edu/staffandfaculty/religious-accommodations-policy/>). Accommodations must be requested within the first two weeks of this course using the Religious Accommodations Request form (<https://registrar.washington.edu/students/religious-accommodations-request/>).



## LOOSE ENDS &amp; PRELIMINARY REMARKS

- How you think about ELECTRODYNAMICS TO A LARGE EXTENT DEPENDS ON YOUR CHOICE OF A SYSTEM OF UNITS; THIS IS A DIFFERENT SITUATION THAN FOR CLASSICAL MECHANICS.
- For 513 & 514, THE TEXT USES "MKSA" UNITS; FOR 515 THE TEXT USES "ESU" ("GAUSSIAN") UNITS.
- WHAT'S THE STARTING POINT OF MKSA UNITS?

IT STARTS WITH MAGNETOSTATICS AND THE DEFINITION OF THE AMPÈRE. THE CONCEPTUAL EXPERIMENT IS:



(2)

AS YOU WELL KNOW, THE WIRES ATTRACT.  
THERE'S A FORCE PER LENGTH.  
THE AMPÈRE OF CURRENT IS  
THAT CURRENT RESULTING IN A  
FORCE OF

$$2 \times 10^{-7} \text{ N / METER EXACTLY.}$$

THIS LEADS TO AN EXACT DEFINITION

$$\mu_0 (\text{MKSA}) = 4\pi \times 10^{-7} \text{ H/m}$$

EXACTLY.

THIS APPEARS, IN, EG,

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}.$$

AS YOU WELL KNOW, ELECTRODYNAMICS  
CONTAINS A FUNDAMENTAL SCALE:  $c$ .  
THIS, AS MENTIONED, DIFFERS  
FROM CLASSICAL MECHANICS WHICH  
CONTAINS NO CONSTANTS EMBEDDED  
IN THE THEORY.



AS YOU WELL KNOW, DYNAMICS IN ELECTROMAGNETISM FORCES THE RELATION

$$c^2 = \frac{1}{\epsilon_0 \mu_0}$$

WITH  $c$  MEASURED, AND  $\mu_0$  DEFINED,  $\epsilon_0$  IS A DERIVED QUANTITY IN MKSA UNITS AND IN EXPRESSIONS LIKE

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0.$$

HISTORICALLY, THE "EMU" DEFINED BY

$$\frac{\mu_0}{4\pi} = 1 \quad (\text{DEFINES THE ABAMPERE}).$$

HISTORICALLY, THE "ESU" DEFINED BY

$$\frac{1}{4\pi\epsilon_0} = 1$$

(IN BOTH CASES, PLUS CGS UNITS FOR MECHANICAL QUANTITIES).

(4)

o RATIONALIZED VS. UNRATIONALIZED  
e.g.,  $4\pi$ 's IN

$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0, \quad \vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$$

THIS IS CALLED THE "RATIONALIZED FORM";  $4\pi$  APPEARS IN THE FORCE LAW BUT NOT IN MAXWELL'S EQUATIONS.

HERE'S "UNRATIONALIZED" FORM (ESU)

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho, \quad \vec{F} = \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$$

AND, THERE'S INDEED A SYSTEM OF "RATIONALIZED" ESU:

"HEAVYSIDE-LORENTZ" UNITS,

THIS HAS A NICE FEATURE OF  
 $\vec{E} = \vec{D} - 4\pi\vec{P}$  BECOMES  $\vec{E} = \vec{D} - \vec{P}$ .

WITHOUT MEDIA, IT'S EASY TO CONVERT BETWEEN MKSA AND ESU!

$$c B_{\text{MKSA}} \rightarrow B_{\text{ESU}}$$

$$\epsilon_0 \rightarrow \frac{1}{4\pi}$$

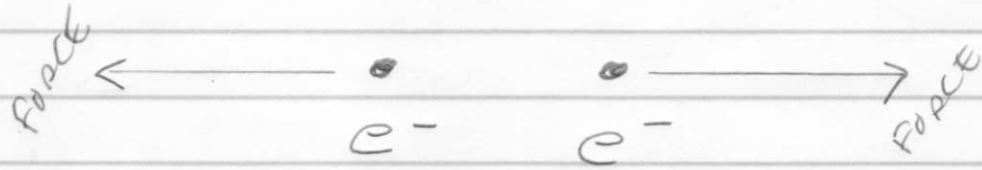
$$\mu_0 \epsilon_0 \rightarrow \frac{1}{c^2}$$

FOR NOW, LET'S JUST SAY THE SITUATION INVOLVING ELECTRIC & MAGNETIC MEDIA IS MORE COMPLICATED, AMBIGUITIES ARISE BECAUSE POLARIZATIONS  $\vec{P}$  &  $\vec{M}$  HAVE TWO ROLES.

• IN  $\rho_p = -\nabla \cdot \vec{P}$  AND  $\vec{J}_M = \nabla \times \vec{M}$ ,  
WHERE  $\vec{P}$  &  $\vec{M}$  ARE SOURCES.

• IN  $\epsilon_0 \vec{E} = \vec{D} - \vec{P}$  AND  $\vec{B}/\mu_0 = \vec{H} + \vec{M}$ ,  
WHERE  $\vec{P}$  AND  $\vec{M}$  ARE FIELDS,  
NAMELY THOSE FIELDS LEADING TO SOURCES  $\rho_p$  AND  $\vec{J}_M$ .

THIS COURSE STARTS WITH ELECTROSTATICS,  
AND THIS STARTS WITH ELECTRONS.



COULD THIS FORCE BE GRAVITATIONAL?

NO, IT'S MUCH TOO STRONG.

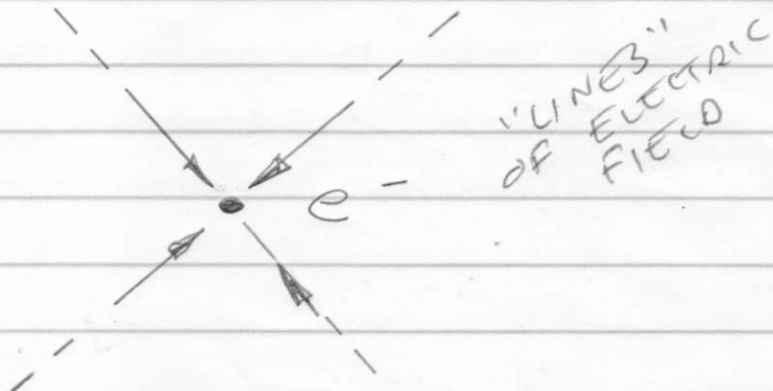
FURTHER, THE FORCE IS REPULSIVE  
(THIS LAST IS VERY DEEP; SEE  
FEYNMAN'S NOTES ON GRAVITATION).

CAVENDISH EMPIRICALLY CHAR-  
ACTERIZED THIS FORCE AS

$$|\vec{F}| \sim \frac{1}{r^2} \sim q_{e^-}^2$$

THE ELECTROSTATIC FORCE.

EARLY ON, "ACTION AT A DISTANCE"  
WAS SURPLANTED BY AN ELECTRIC  
FIELD



### CONVENTIONS!

1. THE FIELD AROUND AN ISOLATED  $e^-$  IS PURELY ELECTRIC, (IT NEEDN'T BE SO, THIS IS A CONVENTION.)

2. FOR THE  $e^-$ , LINES OF  $e^-$  POINT INWARD. (THIS, TOO IS A CONVENTION.)

YOU COULD HAVE CHOSEN OTHER CONVENTIONS! ONE SUCH IS

$$\vec{E} \leftrightarrow \vec{B} \quad \text{A "DUALITY"}$$

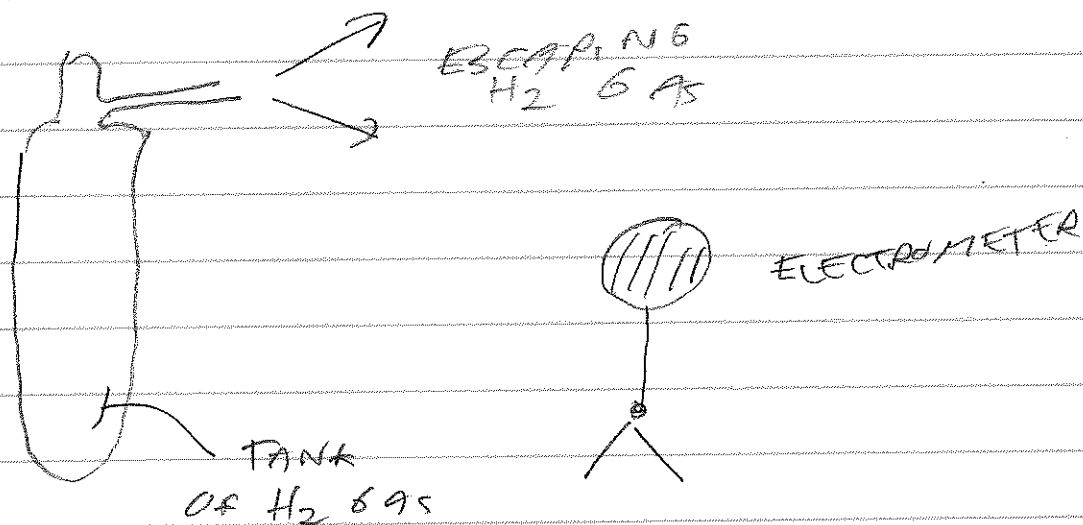
$$e^- \rightarrow e^+ \quad \text{"C" PARITY.}$$

WE'LL BE MORE PRECISE ABOUT THIS LATER SINCE "DUALITY" AND "C" PARITY ARE SUCH USEFUL PROPERTIES OF ELECTRODYNAMICS (SEE JACKSON P. 273).

A COROLLARY IS WE ARE LOGICALLY FREE TO DEFINE THE ELECTRON AS FULLY ELECTRIC, FULLY MAGNETIC, OR A COMBINATION.

BUT, OF COURSE, OUR CONVENTION IS THE ELECTRON IS FULLY ELECTRIC.

Q! WITH OUR CONVENTION, ARE OTHER CHARGED PARTICLES FULLY ELECTRIC? J. G. KING'S EXPERIMENT.



START WITH A FULL TANK OF  $H_2$ , OBSERVE THAT IT'S ELECTRICALLY NEUTRAL (NO  $\vec{E}$  FIELD). IF  $|z_{e^-}| \neq |z_{H_2}|$  THERE'S A DEFICIT OR EXCESS OF OTHER CHARGE IN OR ON THE TANK.

LET  $H_2$  GAS ESCAPE, WHEN THE TANK IS EMPTY, THE DEFICIT OR EXCESS CHARGE REMAINS! THERE WOULD BE A REMNANT  $E$ -FIELD (QUICKLY NEUTRALIZED) ATTACHED TO THIS CHARGE.

KING FOUND

$$\frac{|g_{e^-}| - |g_p|}{|g_{e^-}|} \lesssim 10^{-24}$$

I DON'T KNOW OF A FUNDAMENTAL PRINCIPLE THAT REQUIRES THIS TO BE SO.

Q: DOES THE PROTON HAVE ANY MAGNETIC CHARGE? (THIS, WITH OUR CONVENTION FOR  $e^-$  CHARGE.)

IF SO, THE VAST NUMBERS OF  $p$  IN THE EARTH WOULD SOURCE A MAGNETIC FIELD WITH  $|F| \sim 1/r^2$ ; SUCH A FIELD IS UNSEEN AND  $g_{M,p}$  IS VANISHINGLY SMALL.

BUT, YOU SAY, SUPPOSE FREE MAGNETIC CHARGES NEUTRALIZES THE EARTH'S MAGNETIC CHARGE,

THIS IS ADDRESSED IN THE "PARKER BOUND", FROM FREE MAGNETIC CHARGES "SHORTING" OF ASTRO-PHYSICAL FIELDS. WE'LL COME BACK TO THIS, BUT!

MAGNETIC CHARGES, IF THEY EXIST, WOULD BE EXTREMELY RARE.

WE'LL DEFER FOR NOW THE QUESTION OF HOW "BIG" THE MAGNETIC CHARGE QUANT OF THE ELECTRON CHARGE WOULD BE.

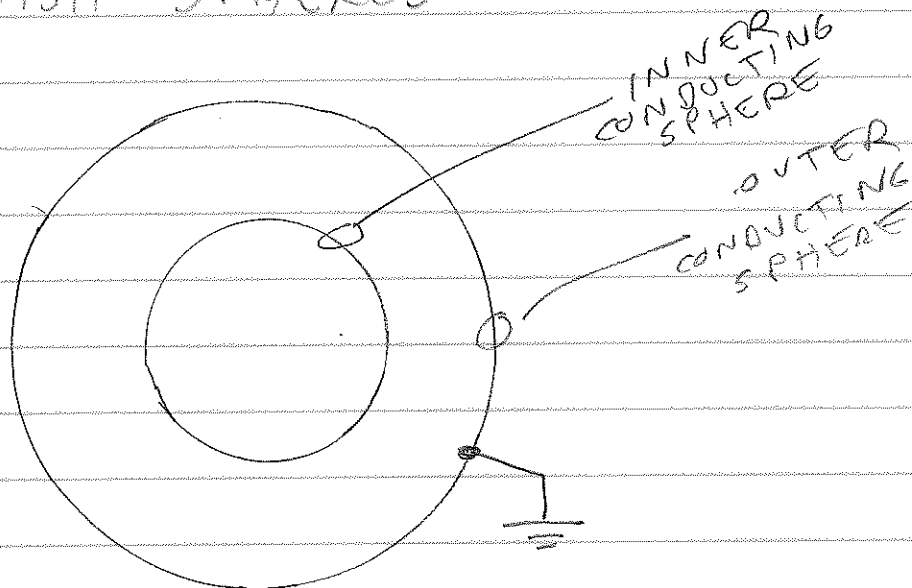


NEVERTHELESS, MAGNETIC CHARGES ARE VERY APPEALING!

1. THEY PROVIDE A CLASSICAL EXPLANATION FOR QUANTIZATION OF ELECTRIC CHARGE (SEE DIRAC'S ARGUMENT, JACKSON P. 275)
2. "GRAND UNIFIED THEORIES" CONTAIN HEAVY ("GUT"-SCALE) MAGNETIC MONOPOLES.
3. MAGNETIC MONOPOLES ARE SO ATTRACTIVE AND THEIR ABSENCE SO PUZZLING, THAT THEORISTS ARE KEPT BUSY AND EMPLOYED, WHICH I SUPPOSE IS A GOOD THING.

Q: How "GOOD" IS THE  $1/r^2$  LAW?

"CAVENDISH SPHERES"



PLACE CHARGE  $Q_{INNER}$  ON INNER SPHERE, THEN MEASURE INDUCED CHARGE  $Q_{OUTER}$  ON OUTER SPHERE.

FOR  $1/r^2$ :  $Q_{INNER} = -Q_{OUTER}$ ,

CAVENDISH FOUND

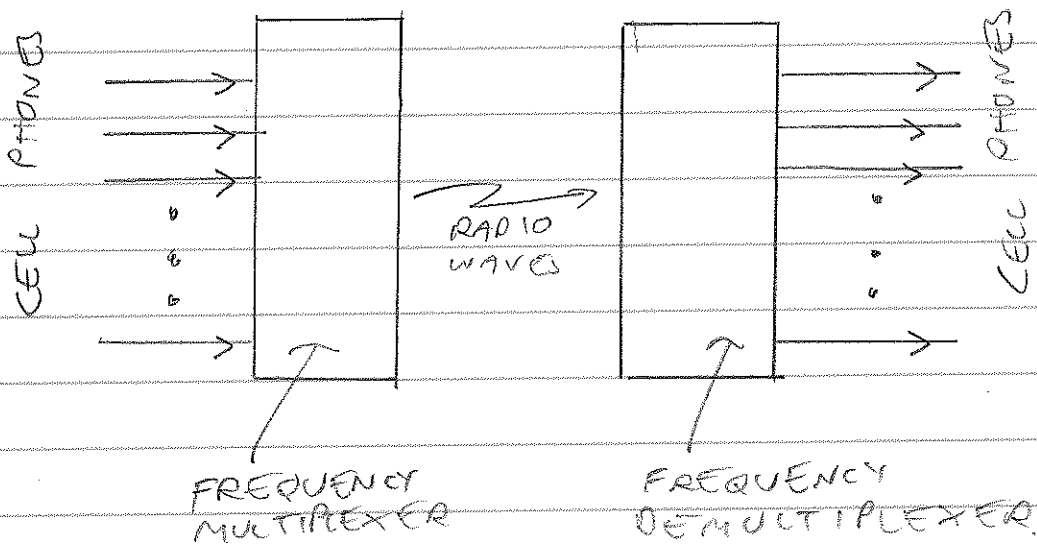
FOR  $\frac{1}{r^2 + \epsilon}$ ,  $|\epsilon| \lesssim 2\%$ .

THESE DAYS  $|\epsilon| < 10^{-15}$  OR SO,

# LINEAR SUPERPOSITION

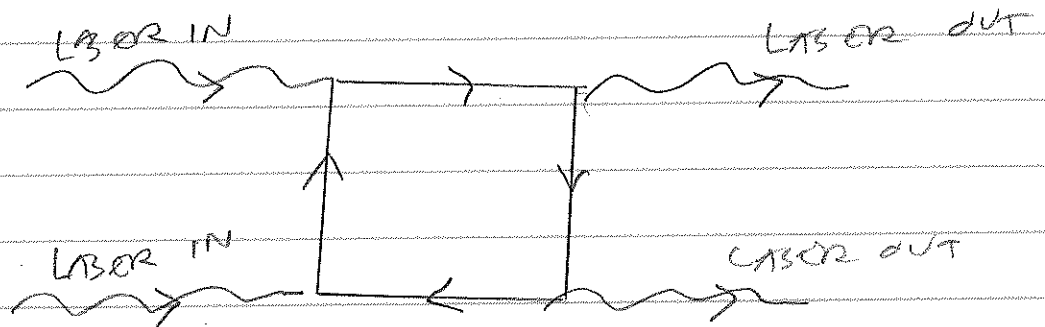
MAXWELL'S EQUATIONS ARE LINEAR  
IN  $\vec{E}$  AND  $\vec{B}$ .

## EXAMPLE



NO EVIDENCE (IN VACUUM) FOR  
A RADIO SIGNAL AT ONE FREQUENCY  
ENDING UP AT ANOTHER FREQUENCY.

SUBTLETY! THIS QED PROCESS  
HAS BEEN OBSERVED



# CONSTANCY OF THE SPEED OF LIGHT (LORENTZ INVARIANCE) OF ELECTRO-DYNAMICS REQUIRES (IN VACUUM)

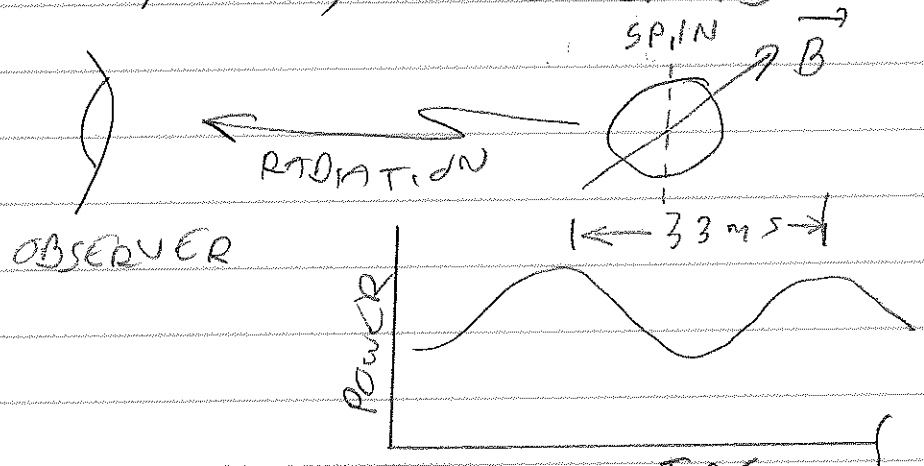
$$\frac{1}{c^2} = \epsilon_0 / \mu_0$$

WITH  $\epsilon_0$  AND  $\mu_0$  MEASURED IN STATIC OR DYNAMIC PROCESSES, AS YOU PREFER, THIS IS ASTOUNDING: STATIC QUANTITIES  $\epsilon_0$  AND  $\mu_0$  FIX THE "MOST" DYNAMICAL CONSTANT  $c$ .

Q: How constant is  $c$ ?  
 $c$  IS CONSTANT (UP TO WHAT WE CAN MEASURE) FROM

$$f < 10^9 \text{ Hz TO } f > 10^{26} \text{ Hz}$$

VIA, E.G., THE CRAB PULSAR



FROM RADIO TO HARD X-RAY  
 $\Delta V/V \lesssim 10^{-35}$

### INTRODUCTION OF FIELDS.

HISTORICALLY,  $\vec{E}$  AND  $\vec{B}$  FIELDS APPEAR IN THE LORENTZ FORCE LAW<sup>6</sup>:  
 $\vec{E} \sim \text{FORCE/UNIT CHARGE}$ ;  
 $\vec{B} \sim \text{FORCE/UNIT CURRENT}$ .

IT THEREFORE SEEMS THE UTILITY OF  $\vec{E}$  AND  $\vec{B}$  ARE CONVENIENT REPLACEMENTS FOR THE FORCES PRODUCED BY CHARGES AND CURRENTS.

BUT, THERE ARE OTHER REASONS TO INTRODUCE  $\vec{E}$  AND  $\vec{B}$ !

1. CONCEPTUALLY DECOUPLE SOURCES FROM OBJECTS EXPERIENCING FORCES.
  - o IF  $\vec{E}$  AND  $\vec{B}$  FROM DIFFERENT SOURCE DISTRIBUTIONS ARE THE SAME, THEN THE FORCES ACTING ARE THE SAME
2.  $\vec{E}$  AND  $\vec{B}$  CAN EXIST WHERE THERE ARE NO SOURCES. THE FIELDS CAN BE VIEWED AS CARRYING ENERGY, MOMENTUM, ETC.

3. THE FIELD CONCEPT IS ONE OF THE MOST POWERFUL IDEAS IN PHYSICS, ARISING IN, E.G., GRAVITATION, QUANTUM MECHANICS AND CONDENSED MATTER.

CAN YOU DEVELOP CLASSICAL ELECTRODYNAMICS WITHOUT FIELDS? YES, WITH SOME EXCEPTIONS.

ARE THERE PROBLEMS WITH FIELDS? YES.

1. THE SELF-ENERGY (ELECTROSTATIC BINDING ENERGY) OF THE ELECTRON IS HUGE. THIS LEADS TO THE IDEA OF "RENORMALIZATION".

2. "RADIATION REACTION": THE FORCE OF AN ACCELERATED CHARGED PARTICLE FEELS ON ITSELF. THIS "CHALLENGES" CAUSALITY AND/OR ENERGY CONSERVATION.

YOU COULD ELIMINATE FIELDS AND REVERT TO "ACTION-AT-A-DISTANCE", BUT THIS REQUIRES CONTORTIONS. (E.G., WHEELER-FEYNMAN ELECTRODYNAMICS: REV. MOD. PHYS. VOL 21 (1949) P 245.)

BACK TO MAXWELL'S EQUATIONS:  
THE SOURCES  $\rho(\vec{r}, t)$  AND  $\vec{J}(\vec{r}, t)$  ARE MOSTLY ASSUMED TO BE CONTINUOUS FUNCTIONS OF  $\vec{r}$ . BUT, WE'LL OFTEN CONSIDER LOCALIZED DISTRIBUTIONS APPROXIMATED AS POINTS.

AS YOU WELL KNOW, AT A MICROSCOPIC LEVEL, CHARGE IS QUANTIZED!

MILLIKAN OIL DROP:  $|q| = n |e|$

QUARKS  $|q_q| = \frac{1}{3} |e|$  OR  $\frac{2}{3} |e|$

THIS PUTS US AT THE STARTING PLACE OF THIS COURSE.

• MAXWELL'S EQUATIONS (IN VACUUM): THESE CONNECT SOURCES TO FIELDS

$$\begin{aligned}
\vec{\nabla} \cdot \vec{E} &= \rho / \epsilon_0 && \text{GAUSS'S LAW} \\
\vec{\nabla} \cdot \vec{B} &= 0 && \text{GAUSS'S LAW} \\
\vec{\nabla} \times \vec{E} &+ \frac{d\vec{B}}{dt} = 0 && \text{FARADAY'S LAW} \\
\vec{\nabla} \times \vec{B} &- \epsilon_0 \mu_0 \frac{d\vec{E}}{dt} = \mu_0 \vec{J} && \text{AMPERE'S LAW}
\end{aligned}$$

THESE EQUATIONS CONTAIN LOCAL CURRENT CONSERVATION

$$\vec{\nabla} \cdot \vec{J} + \frac{d\rho}{dt} = 0.$$

• WHEN CONNECTING FIELDS TO FORCES:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

LORENTZ FORCE LAW.