Physics 513, Electrodynamics I
Department of Physics, University of Washington
Autumn quarter 2020
December 1, 2020, 11am PST
On-line lecture

Administrative:
1. Homework 9 posted to the course web site.

Lecture: (a) Magnetostatics, Faraday's Law, Quasi-Static Fields. (Jackson chapter 5) (b) Maxwell's Equations, Macroscopic E&M, Conservation Laws. (Jackson chapter 6)

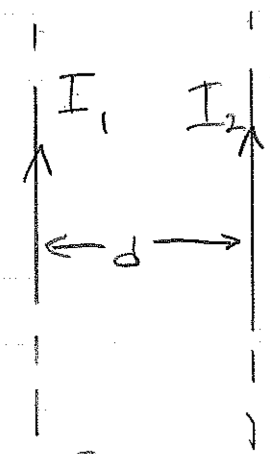
B and the magnetic term in the Lorentz force.
Biot Savart Law.
Ampère's Law (static form).
Scalar magnetic potential (for certain cases).
Vector magnetic potential (most general).
Ohm's law and currents.
Electromotive force.
Gauge freedom and the Lorentz condition.
Magnetization and effective magnetization currents.
The displacement current and continuity.

MAGNETOSTATICS (JACKSON §5.1-6).

ITEMS FROM LAST LECTURE:

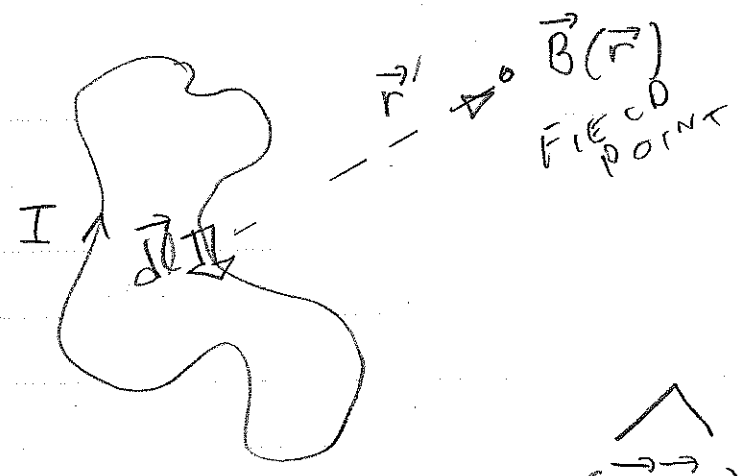
$\vec{N} = \vec{\mu} \times \vec{B}$ (TORQUE);

$\vec{\nabla} \cdot \vec{J} + \frac{\partial}{\partial t}\rho = 0$ (LOCAL CONTINUITY);



$$\frac{dF}{dl} = \frac{\mu_0}{2\pi} I_1 I_2 \frac{1}{d}$$

$$\vec{F} = I\vec{l} \times \vec{B} \quad (\vec{F} = q\vec{v} \times \vec{B})$$



$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \oint I\, d\vec{l} \times \frac{\widehat{(\vec{r}-\vec{r}')}}{|\vec{r}-\vec{r}'|^2}$$

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ (STATIC AMPÉRE'S LAW);

$\vec{\nabla} \cdot \vec{B} = 0$ (ALWAYS—WITH OUR CONVENTION; MAGNETIC "GAUSS'S LAW");

$\vec{J} = \sigma \vec{E}$ (OHM'S LAW: ENGINEERING);

$\vec{E} \cdot \vec{J}$ (RATE OF ENERGY THE CURRENTS EXPEND);

$\mathcal{E} = \oint \vec{E} \cdot d\vec{l}$ ("ELECTROMOTIVE FORCE"; REQUIRES NON-CONSERVATIVE SOURCES FOR $\mathcal{E} \neq 0$);

MAGNETIC POTENTIAL.

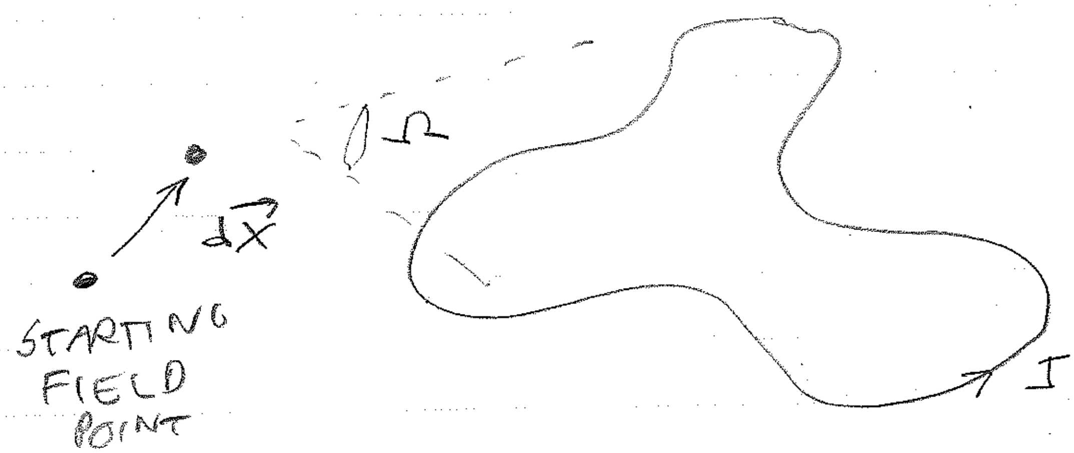
START WITH THE SOMEWHAT OBSCURE MAGNETIC SCALAR POTENTIAL Φ_M.

DEFINED BY $\vec{B} = -\mu_0 \vec{\nabla} \Phi_M$.

A MAGNETIC FIELD WITH SUCH A FORM IS CONSERVATIVE; FOR ANY CLOSED PATH

$$\oint \vec{B} \cdot d\vec{l} = 0 \qquad \text{(Q: WHY?)}.$$

WE CONSIDERED A CURRENT LOOP (SOURCE OF $\vec{B}$) AND EVALUATED HOW Φ_M CHANGES ON MOVING THE FIELD POINT BY $d\vec{x}$;



WE SAW THIS MATH WAS SIMILAR TO THE CHANGE IN THE ELECTROSTATIC POTENTIAL ON MOVING THE FIELD POINT NEAR A DIPOLE-CHARGE LAYER. ANYWAY,

$$d\Phi_M = \frac{I}{4\pi} d\Omega$$

WHERE $d\Omega$ IS THE CHANGE IN SOLID ANGLE OF THE LOOP AS VIEWED BY THE FIELD POINT.

WHEREAS THE DIPOLE-CHARGE LAYER HAD A DEFINITE POSITION, AND A "MICROSCOPIC" ANALYSIS OF THE LAYER SHOWS IT TO BE "+" AND "−" SURFACE CHARGE LAYERS SEPARATED BY A SMALL GAP; THE DISCONTINUITY ARISES AT A DEFINITE SURFACE,

THE "SURFACE" FOR THE MAGNETIC POTENTIAL IS (ALMOST) ARBITRARY; THE POSITION OF THE SURFACE HAS NO PHYSICAL MEANING,

AND IT DOESN'T MAKE PHYSICAL SENSE TO SAY THE DISCONTUNITY IN Φ_M OCCURS AT A SPECIFIC PLACE. ALL YOU KNOW IS

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I,$$

SO LONG AS THE INTEGRATION PATH THREADS THE LOOP EXACTLY ONCE.

FIRSTLY: THIS IS THE GEOMETRIC DERIVATION OF AMPÈRE'S LAW.

SECONDLY, IF THE SYSTEM INCLUDES CURRENTS, THEN

$$\oint \vec{B} \cdot d\vec{l} \neq 0$$

MEANS IT'S NOT THE CASE IN GENERAL THAT $\vec{B}$ HAS FORM

$$\vec{B} = -\mu_0 \vec{\nabla} \Phi_M.$$

Q: WHY?

THIRDLY! WHEN WE GET TO MAGNETIC MATERIALS, WE'LL SEE THERE'S LOTS OF SITUATIONS (e.g., HARD FERROMAGNETS) WHERE THERE ARE NO CURRENTS, OR (e.g, IRON-CORE ELECTRO-MAGNETS) WHERE ALMOST ALL THE MAGNETIC FIELD COMES FROM THE MATERIAL NOT THE BIOT-SAVART CURRENTS.

THIS CAN BE EXPLOITED, AS WE SHALL SEE. BUT IT'S NOT THE TRADITIONAL PATH.

VECTOR POTENTIAL (THE USUAL "MAGNETIC POTENTIAL"); $\vec{A}$.

WE WANT $\vec{B} = \vec{\nabla} \times \vec{A}$ (AS A SIMPLE FORM OF THE VECTOR POTENTIAL).

FROM THE (BULK) BIOT-SAVART LAW

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint \vec{J}(\vec{r}\,') \times \frac{\widehat{(\vec{r} - \vec{r}\,')}}{|\vec{r} - \vec{r}\,'|^2} \, dv'$$

AND $\vec{\nabla} \dfrac{1}{|\vec{r} - \vec{r}\,'|} = \dfrac{\widehat{\vec{r} - \vec{r}\,'}}{|\vec{r} - \vec{r}\,'|^2}$,

$$\vec{B}(\vec{r}) = \vec{\nabla} \times \frac{\mu_0}{4\pi} \iiint \frac{\vec{J}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \, dv'$$

AND IDENTIFIED

$$\vec{A} = \frac{\mu_0}{4\pi} \iiint \frac{J(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \, dv'$$

TO WHICH ANYTHING WITH VANISHING CURL CAN BE ADDED.

$\vec{A}$ IS MUCH EASIER TO CALCULATE THAN $\vec{B}$ FROM THE BIOT-SAVART LAW.

ONE MORE COMMENT. FROM AMPÈRE'S LAW: "GAUGES" I:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J},$$

$$\nabla \times \{\nabla \times A\} = \mu_0 \vec{J}$$

$$-\nabla^2 \vec{A} + \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) = \mu_0 \vec{J}$$

WE'RE FREE TO ADD ANYTHING TO $\vec{A}$ WHOSE CURL VANISHES. A GRADIENT OF A SCALAR (λ) HAS NO CURL. LET'S ADD THIS GRADIENT TO $\vec{A}$, AND SET $\vec{\nabla} \cdot \vec{A} = 0$; THAT IS:

$$\vec{A}' = \vec{A} + \vec{\nabla}'\lambda,$$

REQUIRE $\vec{\nabla}' \cdot \vec{A}' = 0$;

$$\nabla'^2 \lambda = -\vec{\nabla} \cdot \vec{A};$$

WHICH IS A POISSON EQUATION WITH

$$\lambda(\vec{r}) = \frac{1}{4\pi} \iiint \frac{\vec{\nabla}' \cdot A(\vec{r}')}{|\vec{r} - \vec{r}'|} \, dV'$$

$\Rightarrow$ THERE'S A WAY, USING "GAUGE FREEDOM" TO HAVE $\vec{\nabla} \cdot \vec{A} = \mu_0 \vec{J}$; THIS IS THREE POISSON'S EQUATIONS FOR $\vec{A}$. WE'LL SEE MUCH MORE OF "GAUGES".

SOME EXAMPLES OF $\vec{A}$:

I. LONG STRAIGHT CURRENT-CARRYING WIRE.

- SINCE CURRENTS GO OFF TO ∞, WE'LL HAVE THE SAME ISSUES AS FOR THE ELECTROSTATIC POTENTIAL OF THE LONG CHARGED WIRE; CAN'T SET $\Phi(r\to\infty)=0$.

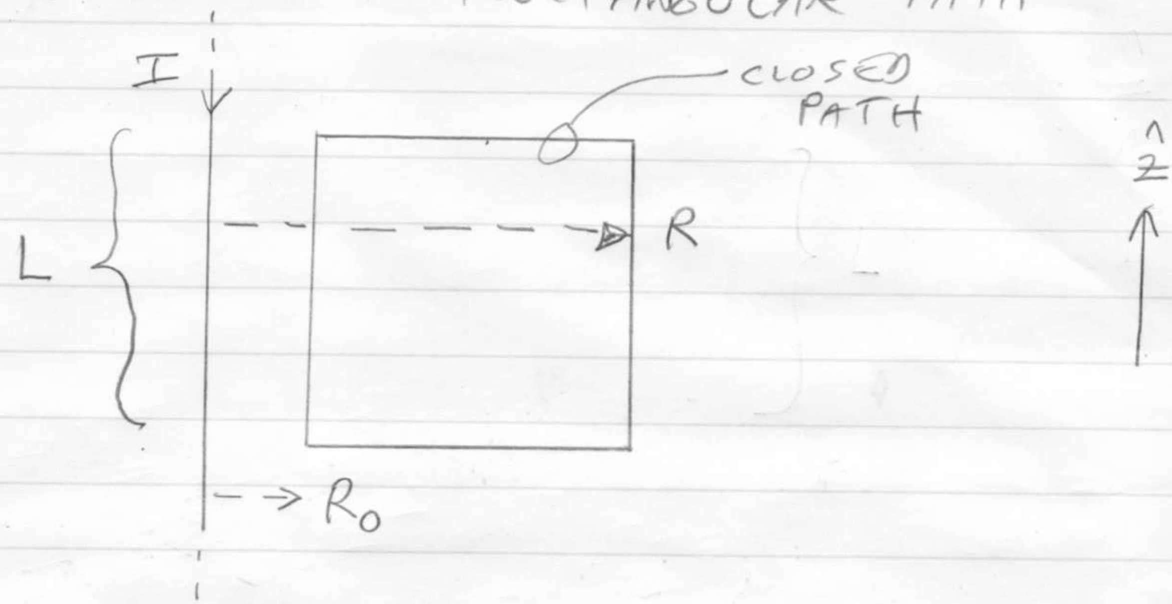
- $\vec{A}$ PICKS UP ITS VECTOR NATURE FROM THE CURRENTS; IN THIS CASE $\vec{A} \sim \vec{I}$.

/ SUPPOSE YOU COULD FIND $\vec{A}$ FROM BRUTE-FORCE INTEGRATION OF

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{\substack{\text{ALL} \\ \text{CURRENTS}}} \frac{\vec{I}}{|\vec{r}-\vec{r}'|}\, dr'.$$

BUT THERE'S SO MUCH SYMMETRY IN THIS PROBLEM, ITS PROBABLY EASIER TO EVALUATE $\oint \vec{A} \cdot d\vec{l}$

AROUND A RECTANGULAR PATH



$$\oint \vec{A} \cdot d\vec{l} = A(R)L - A(R_0)L \quad \text{Q: WHY?}$$

$$= \iint \vec{\nabla} \times \vec{A} \, da = \iint \vec{B} \cdot \hat{n} \, da = \iint B \, da.$$

RECALL $B(r) = \frac{\mu_0}{2\pi} \frac{I}{r}$, SO

$$A(R) - A(R_0) = \frac{\mu_0}{2\pi} \ln(R/R_0) \quad (\text{ALONG } \hat{z})$$

$\vec{A}(R_0)$ IS A REFERENCE POTENTIAL.

NOTICE: $\vec{\nabla} \cdot \vec{A} = \frac{\partial A_z}{\partial R} = 0$

IN THIS CASE.

ALSO:

$$\vec{B} = \vec{\nabla} \times \vec{A} = -\frac{\partial A_z}{\partial R} \hat{\phi} = \frac{\mu_0}{2\pi} \frac{I}{R} \hat{\phi}.$$

IF YOU'RE INTERESTED IN THE "BRUTE FORCE" CALCULATION OF $\vec{A}$, IT'S DONE IN MANY PLACES.

(YOU'LL NEED

$$\int_{-L}^{+L} \frac{d\ell}{\sqrt{R^2 + \ell^2}} = \ln\left[1 + \frac{4L^2}{R^2}\right],$$

).

MORE EXAMPLES OF $\vec{A}$.

2. CURRENT LOOP.

NEAR THE LOOP, THE FIELDS ARE QUITE COMPLICATED. SEE JACKSON §5.5.

FAR FROM THE LOOP, THE FIELD IS A (NEARLY PURE) DIPOLE WITH DIPOLE MOMENT

$$\vec{m} = I \tfrac{1}{2} \oint \vec{r}' \times d\ell$$

(JACKSON EQN P. 186).

$$\left\{ \begin{array}{l} = I \times (\text{AREA OF LOOP}), \\ \text{ASSUMING PLANAR LOOP;} \\ \text{JACKSON EQN. 5.57.} \end{array} \right\}$$

RECALL THE ELECTROSTATIC POTENTIAL OF AN ELECTRIC DIPOLE $\vec{p}$:

$$\Phi = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}.$$

BY ANALOGY:

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2} \quad \text{(PURE DIPOLE).}$$

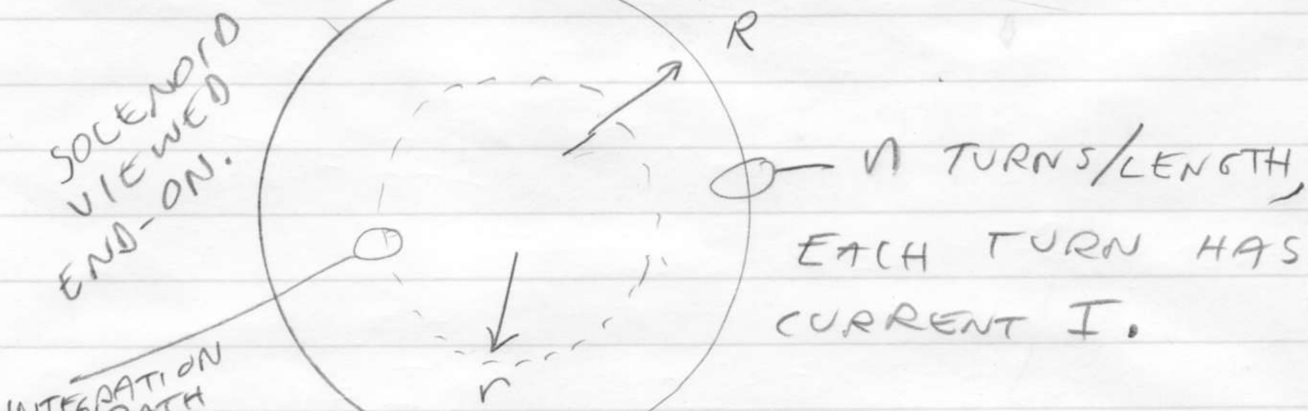
(JACKSON EQN. 5.55).

3. LONG SOLENOID.

WE'LL AGAIN USE

$$\oint \vec{A} \cdot d\vec{l} = \iint \vec{B} \cdot \hat{n} \, da.$$

WE CALL $\iint \vec{B} \cdot \hat{n} \, da = \Phi_M$ THE "MAGNETIC FLUX", HENCE THE * "FUSSY" NAME FOR $\vec{B}$ "FLUX DENSITY". (DON'T CONFUSE Φ_M WITH SCALAR POTENTIAL.)



INSIDE: $|\vec{B}| = \mu_0 n I$ $(\sim \hat{z}$: WHY?).

TAKE THE PATH A CIRCLE OF RADIUS r:

$$A(\vec{r}) = \tfrac{1}{2} r \mu_0 n I. \quad (\sim \hat{\phi}).$$

OUTSIDE:

$$A(r) = \frac{1}{2} \frac{R^2}{r} \mu_0 n I \neq 0. \quad (\sim \hat{\phi})$$

* SOME ALSO CALL $\vec{B}$ THE "MAGNETIC INDUCTION".

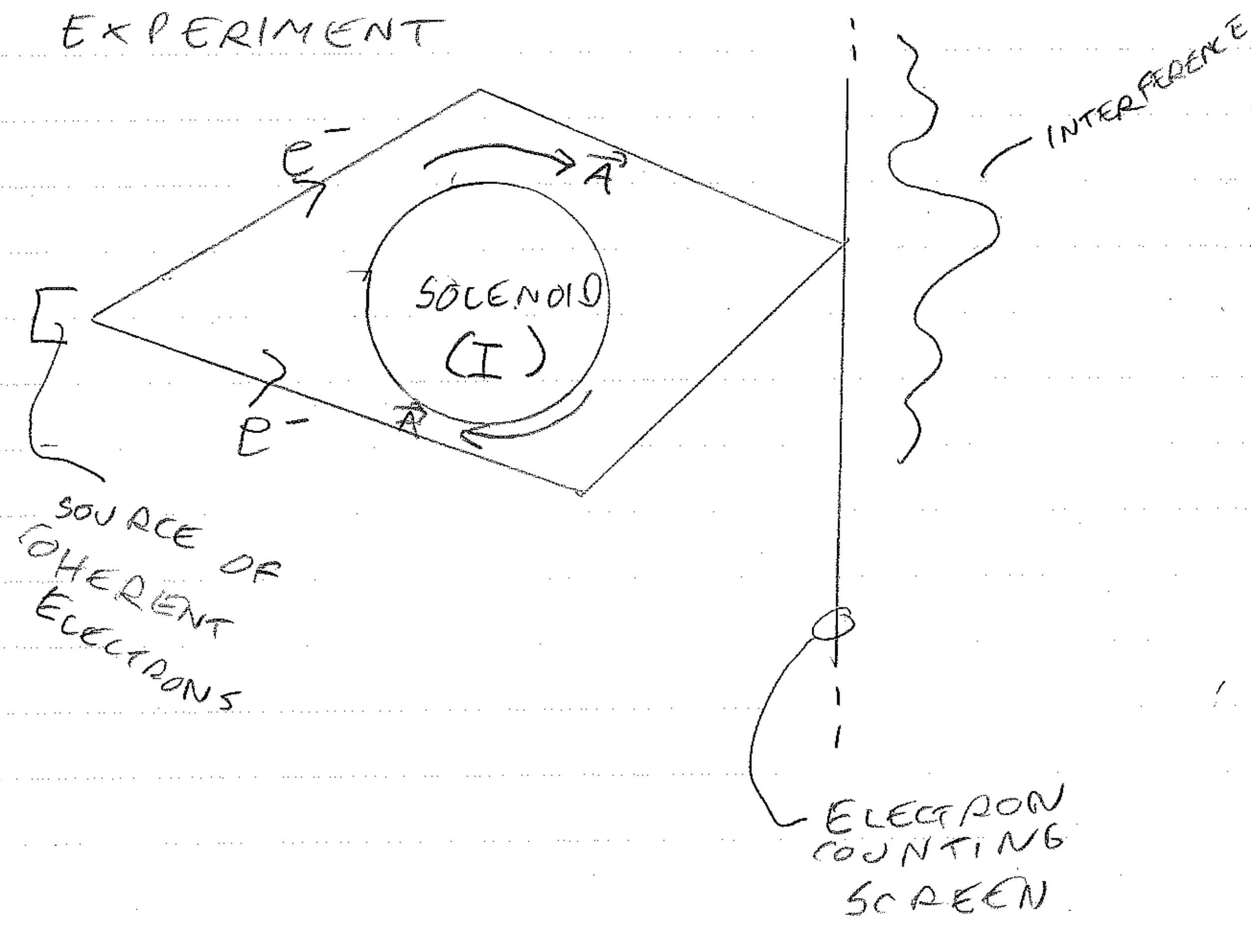
INDEED, THERE'S A POTENTIAL WHERE THERE'S NO FIELD. THIS IS PROFOUND.

RECALL FROM QUANTUM MECHANICS THE CHANGE IN PHASE OF THE ELECTRON WAVE FUNCTION ALONG SOME PATH IN A MAGNETIC FIELD IS

$$\psi = \psi_0 e^{ie/\hbar c \int_{PATH} \vec{A} \cdot d\vec{l}}.$$

NOTICE $\vec{A}$ APPEARS IN THE PHASE.

CONSIDER THIS "AHARNOV-BOHM" EXPERIMENT



NOTICE THE ELECTRONS PICK UP OPPOSITE PHASE DIFFERENCES! THERE'S CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE AT THE SCREEN.

THE INTERFERENCE PATTERN CHANGES AS YOU CHANGE I OF THE SOLENOID. THAT IS: THERE ARE PHYSICAL EFFECTS DUE TO THE SOLENOID MAGNETIC FIELD EVEN THOUGH THE ELECTRONS TRAVERSED A FIELD-FREE REGION.

WE WON'T TALK MORE ABOUT THIS HERE, BUT THIS EFFECT IS PROFOUND. IT SUGGESTS $\vec{A}$ IS MORE THAN A MATHEMATICAL "TRICK" TO INFER $\vec{B}$.

MAGNETIZATION AND "EFFECTIVE" MAGNETIZATION CURRENTS.

AS A MODEL, WE BORROW FROM ELECTROSTATICS, WHERE A BULK POLARIZATION DENSITY $\vec{P}$ LEADS TO ELECTROSTATIC POTENTIAL

$$\Phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \oiint \frac{\vec{P}(\vec{r}\,') \cdot \hat{n}}{|\vec{r}-\vec{r}\,'|}\, da'$$

$$+ \frac{1}{4\pi\varepsilon_0} \iiint \frac{\left(-\vec{\nabla}' \cdot \vec{P}(\vec{r}\,')\right)}{|\vec{r}-\vec{r}\,'|}\, dv'.$$

WE COULD THEN READ OFF THE POLARIZATION CHARGES

$$\sigma_p = \vec{P} \cdot \hat{n}; \quad \rho_p = -\vec{\nabla} \cdot \vec{P}.$$

THIS CAME FROM OUR DISCUSSION OF THE $1/r$ EXPANSION.

NOW, SUPPOSE THERE'S A DISTRIBUTED MAGNETIC DIPOLE MOMENT $\vec{M}$. THE $1/r$ EXPANSION IN $\vec{A}$ LEADS TO (JACKSON EQN 5.77)

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\iiint \vec{M} \times \vec{\nabla}\frac{1}{|\vec{r}-\vec{r}'|}\, dv'$$

Q: WHAT HAPPENED TO THE MONOPOLE TERM $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\iiint \frac{\vec{J}(\vec{r}')}{|\vec{r}'|}\, dv'$

A: HOMEWORK.

$\vec{M}(\vec{r}') \times \vec{\nabla}\frac{1}{|\vec{r}-\vec{r}'|}$ IS PART OF $\vec{\nabla} \times \frac{\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|}$:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\iiint \nabla' \times \frac{\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|}\, dv'$$

"DIVERGENCE THEOREM" FOR THE CURL.

$$+ \frac{\mu_0}{4\pi}\iiint \frac{1}{r}\vec{\nabla}' \times \vec{M}(\vec{r}')\, dv'$$

$$- \oiint \frac{\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|} \times \hat{n}\, da$$

WE CAN THEREFORE READ OFF EQUIVALENT BULK (J_M) AND SURFACE CURRENTS (K_M):

$$\vec{J}_M = \vec{\nabla} \times \vec{M}; \qquad \vec{K}_M = \vec{M} \times \hat{n}$$

THIS IS WHERE JACKSON STARTS TO BRING IN SOME TIME DEPENDENCE. THE "VACUUM DISPLACEMENT CURRENT" IN STATIONARY MEDIA.

SO FAR, WE HAVE CURRENTS

$$\vec{J}_{TOTAL} = \vec{J}_{TRUE} + \frac{\partial \vec{P}}{\partial t} + \vec{\nabla} \times \vec{M},$$

WITH CONTINUITY EQUATION

$$\vec{\nabla} \cdot \vec{J}_{TOTAL} + \frac{\partial}{\partial t} \rho_{TOTAL} = 0.$$

THESE COMBINE TO

$$\vec{\nabla} \cdot \vec{J}_{TRUE} + \vec{\nabla} \cdot \frac{\partial \vec{P}}{\partial t} + \vec{\nabla} \cdot \vec{\nabla} \times \vec{M} + \vec{\nabla} \cdot \frac{\partial}{\partial t} \rho_{TOTAL} = 0.$$

WE ALSO HAVE $\vec{\nabla} \cdot \vec{E} = \rho_{TOTAL} / \varepsilon_0$ AND $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$. HENCE

$$\vec{\nabla} \cdot \left(\vec{J}_{TRUE} + \frac{\partial \vec{D}}{\partial t} + \vec{\nabla} \times \vec{M} \right) = 0$$

HENCE $\vec{\nabla} \cdot \left(J_{TOTAL} + \frac{\partial}{\partial t} \vec{D} \right) = 0.$

Q: WHAT HAPPENED TO $\vec{\nabla} \times \vec{M}$?

RECALL, FOR MAGNETOSTATICS,

$$\vec{\nabla} \cdot \vec{J} = 0.$$

IF WE ADD IN A "CURRENT"

$$\frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{IN VACUUM,}$$

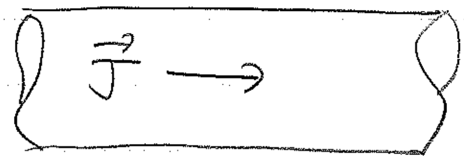
THEN WE HAVE THE APPEARANCE OF STATICS FOR A DYNAMIC SYSTEM.

- $\frac{\partial \vec{D}}{\partial t}$ IS THE "DISPLACEMENT CURRENT".

- ADDING IN $\frac{\partial \vec{D}}{\partial t}$ ALLOWS STATIONARY CURRENTS PLUS CHARGE ACCUMULATION TO BE TREATED AS STATIONARY.

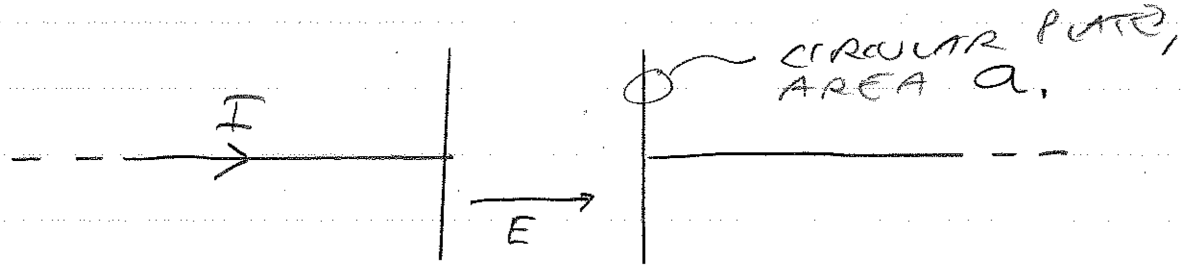
- THIS DISPLACEMENT CURRENT IS A ALSO A SOURCE OF MAGNETIC FIELDS! THIS IS CRUCIAL FOR RADIATION OF ALL TYPES.

EXAMPLE: LONG STRAIGHT CURRENT-CARRYING WIRE:



NO CHARGE IS BUILDING UP ANYWHERE; NO DISPLACEMENT CURRENT. $\vec{\nabla}\cdot\vec{J} = 0$.

SLIP IN AN IDEAL CAPACITOR (NO RADIATION! "QUASI-STATIC")



CHARGE IS BUILDING UP ON THE PLATES: $Q = It$; $\sigma = \frac{tI}{a}$

$$E = \frac{1}{\varepsilon_0}\frac{It}{a}; \quad \frac{d\vec{D}}{dt} = \frac{I}{a}; \quad I_D = I.$$

⇒ BY ADDING THE DISPLACEMENT CURRENT TO I, CURRENT IS CONSERVED THROUGHOUT THE WHOLE SYSTEM.

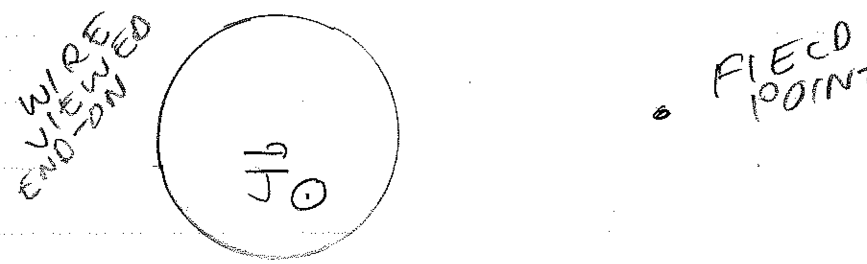
A THOUGHT PROBLEM: RE-WRITE THE BIOT-SAVART FIELD:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\iiint \vec{J}(\vec{r}\,')\times\vec{\nabla}'\frac{1}{|\vec{r}-\vec{r}\,'|}\,dv'$$

$$= \frac{\mu_0}{4\pi}\iiint \frac{1}{|\vec{r}-\vec{r}\,'|}\,\vec{\nabla}'\times\vec{J}(\vec{r}\,')\,dv'$$

$$+ \frac{\mu_0}{4\pi}\oiint \frac{1}{|\vec{r}-\vec{r}\,'|}\,\vec{J}(\vec{r}\,')\times\hat{n}'\,da',$$

APPLY THIS TO A WIRE OF FINITE CROSS-SECTION CARRYING UNIFORM CURRENT DENSITY:



INSIDE THE WIRE $\vec{\nabla}\times\vec{J}=0$. AT THE SURFACE $\vec{\nabla}\times\vec{J}\neq 0$. DOES THIS MEAN YOU CAN IGNORE CONTRIBUTIONS OF BULK CURRENTS FOR THE $\vec{B}$ FIELD AT THE FIELD POINT?

COMMENT: GAUGES II. THE LORENTZ CONDITION.

RECALL $\vec{A} = \frac{\mu_0}{4\pi}\iiint \frac{\vec{J}}{|\vec{r}-\vec{r}\,'|}\,dv'$, so

$$\vec{\nabla}\cdot\vec{A} = \frac{\mu_0}{4\pi}\iiint \underbrace{\vec{\nabla}\cdot\frac{\vec{J}}{|\vec{r}-\vec{r}\,'|}}\,dv'$$

$$\underbrace{\frac{1}{|\vec{r}-\vec{r}\,'|}\vec{\nabla}\cdot\vec{J}}_{\to 0 \text{ Q: WHY?}} + \vec{J}\cdot\underbrace{\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}\,'|}}_{-\vec{\nabla}'\frac{1}{|\vec{r}-\vec{r}\,'|}}$$

SIMILARLY

$$\vec{\nabla}'\cdot\frac{\vec{J}}{|\vec{r}-\vec{r}\,'|} = \frac{1}{|\vec{r}-\vec{r}\,'|}\vec{\nabla}'\cdot\vec{J} + \vec{J}\cdot\vec{\nabla}'\frac{1}{|\vec{r}-\vec{r}\,'|}$$

COMBINING:

$$\vec{\nabla}\cdot\vec{A} = \frac{\mu_0}{4\pi}\iiint \frac{1}{|\vec{r}-\vec{r}\,'|}\vec{\nabla}'\cdot\vec{J}\,dv'$$

+ SURFACE TERM $\to 0$ FOR BOUNDED CURRENTS

FROM CONTINUITY

$$\vec{\nabla}'\cdot\vec{J} + \frac{\partial}{\partial t}\rho = 0,$$

$$\vec{\nabla}\cdot\vec{A} = -\varepsilon_0\mu_0\frac{\partial}{\partial t}\frac{1}{4\pi\varepsilon_0}\iiint\frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,dv'$$

FINALLY, WE IDENTIFY THE ELECTROSTATIC POTENTIAL

$$\Phi(\vec{r}') = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} \, dv', \quad \text{SO}$$

$$\vec{\nabla} \cdot \vec{A} + \varepsilon_0 \mu_0 \frac{\partial \Phi}{\partial t} = 0,$$

THE LORENTZ CONDITION.

N.B., $\varepsilon_0 \mu_0$ WILL BE SHOWN TO BE $1/c^2$; SO THE LORENTZ CONDITION IS OFTEN SHOWN AS

$$\vec{\nabla} \cdot \vec{A} + \frac{1}{c^2} \frac{\partial}{\partial t} \Phi = 0.$$

THIS WILL BE VERY USEFUL WHEN YOU GET TO RADIATION. IT ALSO PROVIDES THE MAGNETIC FIELD IN QUASI-STATIC PROBLEMS.

(SEE HOMEWORK).