Electrodynamics I: Assignment 7
Due no later than November 23 at 4:00 pm Pacific time.
On-line submission procedure:

1. **Scan your solutions as a single PDF file**
2. **Name your file HW7-*lastname*.pdf**
3. **Attach your file to an email…**
4. **… with subject line HW7-*lastname* …**
5. **… and send the email to ljrosenberg@phys.washington.edu**

1. 4. It's odd that Jackson does not explicitly present this: Show that the electrostatic energy U of an electric dipole **p** in an electric field **E** is $U = -\mathbf{p} \cdot \mathbf{E}$.

2. Statistical mechanics and a dilute gas of polar molecules. (Part a below is simple, part b is a challenge problem from Landau & Lifshitz.) Suppose a dilute gas in thermal equilibrium consists of molecules having dipole moment **p**, the gas having molecule number density *n*. The gas is threaded with a uniform electric field **E**. (a) Show that the gas acquires polarization density of $\mathbf{P} = -n\overline{U}\mathbf{E}/E^2$ where $\overline{U}$ is the average of the energy from (1) above. (b) Recall the probability a molecule having potential energy in the range U to $U+dU$ is proportional to e^{-U/k_BT}, so the average energy is then formally $\overline{U} = \frac{\int_{-pE}^{+pE} Ue^{-U/k_BT}dU}{\int_{-pE}^{+pE} e^{-U/k_BT}dU}$ where the specific limits arise from U required to be in the range $-pE$ to $+pE$. Show that this leads to an electric susceptibility $\chi_E = n\left\{\alpha + \frac{p^2}{3\epsilon_0 k_B T}\right\}$. (This leads to a polarization where the difference between it and that in 2(a) is an additional induced polarization $n\alpha\epsilon_0\mathbf{E}$; this defines α.) Hint: The gas is dilute which means the exponential can be expanded, keeping the lowest-order term.

3. Electromagnetism and thermodynamics. The problem above demonstrates that the dielectric could well have temperature dependence. (There's an annoying tendency in electrodynamics to be sloppy and sometimes use *U* for the free energy instead of *F*. Let's use thermodynamic notation.)

Recall from your thermodynamics the free energy is $F = U - TS = \frac{1}{2}\iiint \mathbf{E} \cdot \mathbf{D}\, dv$ where *F* is the free energy and *U* is the total energy. Also recall the first law of thermodynamics $\delta W = \delta U - T\delta S$ where the increment of work δW for a change $\delta\mathbf{D}$ is given by $\delta W = \iiint E \cdot \delta D\, dv$. Finally recall the heat transferred is $\delta Q = T\delta S$. Find the heat transferred during a change $\delta\mathbf{D}$ in the field at constant temperature in terms of *T*, the dielectric constant $\varepsilon(T)$ and field quantities. Suppose the electric susceptibility has the form of problem 2; does the temperature of a thermally insulated dielectric increase when the field is applied, does it decrease, or does it remain unchanged?

4. Consider a cylindrical bar electret of length *L* made up of material with a uniform permanent "frozen in" dipole density **P** oriented along the bar axis. Consider a closed integration path entering one face of the bar and exiting the other. It's easy to evaluate the closed line integral of **P** along this path by direct integration: *PL*. But the non-zero loop integral implies a non-zero curl of **P**. Identify where the curl of P is non-zero and directly evaluate the curl (you might need to "soften" the abrupt transition of polarization at the boundaries). Then evaluate the loop integral by integrating the curl and check you get *PL*.