

Electrodynamics I: Assignment 6

**Due no later than November 16 at 4:00 pm
Pacific time.**

On-line submission procedure:

- 1. Scan your solutions as a single PDF file**
- 2. Name your file HW6-*lastname*.pdf**
- 3. Attach your file to an email...**
- 4. ... with subject line HW4-*lastname* ...**
- 5. ... and send the email to
ljrosenberg@phys.washington.edu**

1. The quadrupole-moment tensor, first introduced in class from a Taylor expansion, was $Q_{ij} = \iiint \rho(\vec{r}') x'_i x'_j dv'$. Verify the claims we made about it in class:

- Show it's proportional to Jackson Eqn. 4.9
- Show the delta-function term in Jackson Eqn 4.9 doesn't contribute to the quadrupole term in the potential.
- Recall in the discussion of degrees-of-freedom, we introduced a new quadrupole tensor $Q'_{ij} = Q_{ij} - \frac{1}{3}\text{Tr}(Q)$; show it's traceless.

2. Suppose the properties of a certain medium are not completely known, it may not even be a linear dielectric. But suppose the polarization density $\mathbf{P}(\mathbf{r})$ near a boundary is known. What are the boundary conditions, corresponding to Jackson eqn's 4.40, in terms of the polarization-density discontinuity $\delta\mathbf{P}$ across the boundary?

4. A parallel-plate capacitor consists of two plates of area A separated by a distance d . The capacitor is filled with an anisotropic dielectric having a tensor constitutive relation $D_\alpha = \varepsilon_{\alpha\beta} E_\beta$. Unfortunately, the dielectric is oriented so its principal axes are not aligned with the capacitor axes: the first principal axis makes an angle θ to the normal of a plate, the second makes an angle $\pi/2-\theta$ to the normal of a plate, and third is tangential to the plates. Assuming

no fringe fields, find the capacitance. (You might recall that in coordinates aligned with the principal axes, the tensor is diagonal.)

4. Show from basic energy relations that the dielectric-constant tensor of the previous problem is symmetric. (This is a restatement that the contraction of a fully anti-symmetric tensor with a fully symmetric tensor vanishes.)