Electrodynamics I: Assignment 1
Due no later than October 12 at 4:00 pm Pacific time.
On-line submission procedure:

1. **Scan your solutions as a single PDF file**
2. **Name your file HW1-*lastname.*pdf**
3. **Attach your file to an email…**
4. **… with subject line HW1-*lastname* …**
5. **… and send the email to**

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1. Discrete transformations. In lecture we briefly mentioned the invariance of electrodynamics under the interchange of the sign of the charge: “C” parity. Electrodynamics is also invariant under parity “P” and time-reversal “T” operations. What we mean by “invariant” under a transformation in this context is that the dynamical equations maintain the same form under the transformation.

a. Show this invariance of electrodynamics under “P” and “T” transformations by explicitly demonstrating the invariance of one of the dynamical Maxwell equations of I.1a or I.1b.

b. Recall Faraday rotation: a material is threaded by a magnetic field **H**. Polarized plane waves propagating in the material in a direction perpendicular to **H** have their polarization rotated as they traverse the material. You might expect from T-invariance that on reversing time (“running the film backwards” whilst preserving the direction of **H**) the reversed plane wave would reverse its rotation, but the rotation direction is preserved. This seems to naively violate T-invariance. What is the source of this apparent T-violation? (This is very useful in optical and microwave applications: There’s a 3-port device called a “circulator” where all electromagnetic power incident on port-1 exits port-2 and all power incident on port-3 exits port 1.You might expect from T-invariance that all power incident on port-2 exits port-1 since that is the time-reversed situation, but instead all power exits port-3.)

2. Helmholtz theorem. Suppose a 3-dimensional vector field **V** had divergence and curl
$\mathbf{\nabla}\cdot\mathbf{V}=s$ and $\mathbf{\nabla}\times\mathbf{V}=\mathbf{c}$ (where $\mathbf{\nabla}\cdot\mathbf{c}=0$) where s is a scalar function and **c** is a vector function.

a. Show that **V** has solutions $\mathbf{V}=-\mathbf{\nabla}\phi+\mathbf{\nabla}\times\mathbf{A}$ where
$\phi(\mathbf{r})=\frac{1}{4\pi}\int\frac{s(\mathbf{r}')}{r(\mathbf{r},\mathbf{r}')}d\tau'$ and $\mathbf{A}(\mathbf{r})=\frac{1}{4\pi}\int\frac{\mathbf{c}(\mathbf{r}')}{r(\mathbf{r},\mathbf{r}')}d\tau'$
where the volume integrals extend over all space.

Two subtleties:
b. Why do we demand $\nabla\cdot\mathbf{c}=0$?
c. To show (a), you may want to invoke the Divergence Theorem. But $\nabla(1/r)$ is singular at r=0. How can "get around" this singularity?

This is an important theorem. It allows us to construct fields from potentials. It also explains why the static electric field only needs the gradient of the scalar potential and why the dynamical magnetic field only needs the curl of the vector potential.

3. In expressions like Jackson eqn 1.5 the field point is **x** and a source point is **x**'. The vector operator ∇, acting on field coordinates has components $\partial/\partial x_i$. In terms of ∇, what's the corresponding vector operator ∇' acting on source coordinates with components $\partial/\partial x'_i$? This is a frequently-used identity.

4. Surface singularities. There are two main types of surface singularities: a "charge layer" and a "dipole layer".

a. Dipole layer. Two closely-separated charge layers of equal and opposite sign of charge densities form a sheet, not necessarily a planar sheet, with dipole moment per unit area **d**. Outside the sheet what's the potential arising from this dipole layer?

b. In the case where **d** is uniform and normal to the surface, find the potential. The potential in terms of Ω, the solid angle subtended by the sheet as “viewed” by the field point, is simple. We’ll use this same reasoning to find Ampère’s law from the current-loop’s geometry.

c. What is the discontinuity of the potential on crossing the sheet?

d. What is the discontinuity of the normal derivative of the potential on crossing the sheet?