

### Mid-Term Exam II

- The exam is due via email by Friday, December 11, at 4 pm PST. Points will be deducted for a late submission.
- Use a separate sheet of paper for each problem solution; you will therefore have a minimum of 3 pages in your submitted PDF file. Assemble the PDF pages in problem-order (1, 2, 3).
- This is an open-book exam; you may refer to the Jackson textbook, your lecture notes and anything on the course web site.
- Show your work in enough detail so the grader can follow your reasoning and your method of solution.
- Take up to an hour and 20 minutes to complete the exam, please don't continue beyond that.
- Feel free to contact me with questions.
- Email-submission instructions:
  1. Scan your solutions as a single PDF file
  2. Name your file `midterm2-lastname.pdf`
  3. Attach your file to an email
  4. ... with subject line `midterm2-lastname`
  5. Send that email to `ljrosenberg@phys.washington.edu`

**I. (35 points) Dipole magnetic field.**

Consider a spherical surface of radius  $R$  with a point dipole of magnetic moment  $\mathbf{m}$  at its center. Find the average magnetic field  $\langle \mathbf{B} \rangle$  within the sphere.

**II. (35 points) Solution to Poisson and Laplace equations.**

Consider a long straight wire (in the  $z$ -direction) of radius  $R$  carrying a uniform current density  $J$ . Use the magnetic Poisson equation in Coulomb gauge (Jackson eqn. 5.101) to find the vector potential and magnetic field everywhere.

**III. (30 points) Gauge transformations.**

Consider a static point charge  $e$  at  $r=0$ . We started this quarter by using for its potentials  $\Phi = \frac{1}{4\pi\epsilon_0} \frac{e}{r}$  and  $\mathbf{A} = 0$ . (a) Are these in either Coulomb or Lorentz gauge? Explain. (b) Show that these familiar potentials are related to less-familiar (“Hamiltonian” or “Weyl” gauge) potentials  $\Phi = 0$  and  $\mathbf{A} = -\frac{1}{4\pi\epsilon_0} \frac{et}{r^2} \hat{\mathbf{r}}$  by a gauge transformation. (c) What is a suitable scalar gauge function for this gauge transformation? (d) Are these less-familiar potentials in either Coulomb or Lorentz gauge? Explain.