# Electrodynamics I: Assignment 8.
# Due November 21 at 11:00am in class or 10:45am in the instructor's mailbox.

1. For the nested cones of Assignment 7, problem 4, find the potential between the cones from integrations of Laplace's equation.

2. The quadrupole-moment tensor as first introduced in class from a Taylor expansion as $Q_{ij} = \iiint \rho(\vec{r}')x_i'\, x_j' dv'$. Verify the claims we made about it in class:

a. Show it's proportional to Jackson Eqn. 4.9

b. Show the delta-function term in Jackson Eqn 4.9 doesn't contribute to the quadrupole term in the potential.

c. Recall in the discussion of degrees-of-freedom, we introduced a new quadrupole tensor $Q_{ij}' = Q_{ij} - \frac{1}{3}\mathrm{Tr}(Q)$; show it's traceless.

3. Suppose a charge distribution has azimuthal symmetry. Further suppose the quadrupole tensor is referenced in its principal axis. Find the number of degrees of freedom in the quadrupole tensor.

4. A charge distribution $\rho(r)$ has a finite spatial extent.

a. Show that the dipole moment of this charge distribution is

$$\mathbf{P} = -3\varepsilon_0 \iiint \mathbf{E}(\mathbf{r})dv$$

b. Now suppose the charge in problem (a) is entirely outside some volume $V$ of finite spatial extent. Show that the electric field **E**($p$) at a point $p$ within the charge-free volume is

$$\mathbf{E}(p) = \frac{1}{V}\iiint \mathbf{E}(\mathbf{r})dv$$

Variants of this are classic done in numerous texts. Hint: Since the charge distribution is finite in extent, there is some sphere which contains all the charge. One way is to start by replacing **E**(**r**) with the integral expression over charge density.

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