Electrodynamics I: Assignment 7.
Due November 14 at 11:00am in class or 10:45am in the instructor's mailbox.

1. a. Find the "exterior-problem" Green's function in spherical coordinates with a spherical boundary of radius *R*. b. Verify your Green's function satisfies the boundary conditions in *r* and *r*' at *R* and the surface at infinity. c. Verify your Green's function's reciprocal properties of *r* and *r*' are satisfied. d. Verify or argue your Green's functions satisfies Laplace's equation.

2. Find the electrostatic potential inside a very long grounded cylinder of radius *R* whose axis coincides with the z-axis and containing a point charge *q* at the origin. For those I confused by the integration bounds I used in class, you may find clarity from the normalization (Jackson equation 3.95) $\int_0^{\xi_0} [J_0(c\xi)]^2 \xi \, d\xi = \frac{\xi_0^2}{2}[J_1(c\xi_0)]^2$. Notice how a higher-order Bessel function appears in the normalization.

3. Variant of Jackson problem 3.9. A hollow right circular cylinder of radius *R* has its axis coincident with the z-axis and its ends at *z*=0 and *z*=*L*. The potential of the end faces is zero, the potential of the cylindrical surface is a constant Φ_0. Find the potential inside the cylinder. You will probably want to choose the periodic coordinate in the z-direction, meaning that separation constant has the opposite-from-usual sign, and the Bessel functions become Modified Bessel functions (Jackson equations 3.100-101).

4. Work out the problem we discussed in class: Consider nested concentric cones whose apexes coincide. The inner cone, defined by half-angle β, is at ground. The outer cone, defined by half-angle α, is at potential Φ_0. a. Show that the Legendre functions of the first kind (the kind regular at the origin) cannot provide a solution. b. Find the potential everywhere between the cones. c. Verify your solution satisfies the boundary conditions. Jackson does not tabulate the irregular solutions of Legendre's equation. Here's a table from Abramowitz & Stegun:

8.4. Explicit Expressions

$(x=\cos\theta)$

8.4.1 $P_0(z)=1 \qquad P_0(x)=1$

8.4.2

$$Q_0(z)=\frac{1}{2}\ln\left(\frac{z+1}{z-1}\right) \qquad Q_0(x)=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$$=xF(\tfrac{1}{2},1;\tfrac{3}{2};x^2)$$

8.4.3 $P_1(z)=z \qquad P_1(x)=x=\cos\theta$

8.4.4

$$Q_1(z)=\frac{z}{2}\ln\left(\frac{z+1}{z-1}\right)-1 \qquad Q_1(x)=\frac{x}{2}\ln\left(\frac{1+x}{1-x}\right)-1$$

8.4.5

$$P_2(z)=\tfrac{1}{2}(3z^2-1) \qquad P_2(x)=\tfrac{1}{2}(3x^2-1)$$

$$=\tfrac{1}{4}(3\cos 2\theta+1)$$

8.4.6

$$Q_2(z)=\frac{1}{2}P_2(z)\ln\left(\frac{z+1}{z-1}\right) \qquad Q_2(x)=$$

$$-\frac{3z}{2} \qquad \left(\frac{3x^2-1}{4}\right)\ln\left(\frac{1+x}{1-x}\right)-\frac{3x}{2}$$

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