Electrodynamics I: Assignment 1.
Due October 3 at 11:00am in class or 10:45am in the instructor's mailbox.

1. In expressions like Jackson eqn 1.5 the field point is $\mathbf{x}$ and a source point is $\mathbf{x}$'. The vector operator ∇, acting on field coordinates, has components $\partial/\partial x_i$. In terms of ∇, what's the corresponding vector operator ∇', acting on source coordinates, with components $\partial/\partial x'_i$? We'll frequently use this identity.

2. Helmholtz theorem. A 3-dimensional vector field **V** satisfies
$\nabla \cdot \mathbf{V} = s$ and $\nabla \times \mathbf{V}=\mathbf{c}$ (where $\nabla \cdot \mathbf{c} = 0$).

a. Show that **V** has solutions $\mathbf{V} = -\nabla\phi+\nabla\times\mathbf{A}$ and volume integrals
$\phi(\mathbf{r}) = \frac{1}{4\pi}\int \frac{s(\mathbf{r}\prime)}{r(\mathbf{r},\mathbf{r}\prime)} d\tau\,'$ and $\mathbf{A}(\mathbf{r}) = \frac{1}{4\pi}\int \frac{\mathbf{c}(\mathbf{r}\prime)}{r(\mathbf{r},\mathbf{r}\prime)} d\tau\,'$.

This theorem allows us to introduce vector and scalar potentials in electromagnetism.

Two subtleties:
b. Why do we demand $\nabla \cdot \mathbf{c} = 0$?
c. To show (a), you may want to invoke the Divergence Theorem. But $\nabla(1/r)$ is singular at r=0. How can you evade this singularity?

3. Show that electrostatic Maxwell equations (Jackson eqns 1.13 and 1.14) follow from Coulomb's Law.

4. Dipole surface singularity. Two closely separated charge layers form a sheet, not necessarily a planar sheet, with dipole moment per unit area **d**.

a. Outside the sheet, what's the potential arising from this dipole layer?

b. In the case where **d** is uniform and normal to the surface, find the potential. potential in terms of Ω, the solid angle subtended by the sheet at the field point, is particularly simple and interesting. We'll use this geometric construction when establishing the integral form of Gauss's Law.

c. What is the discontinuity of the potential on crossing the sheet?

d. What is the discontinuity of the normal derivative of the potential on crossing the sheet?