1. A variant of Jackson problem 1.12. Consider a system of point charges \( \{ q_i \} \), and potentials \( \{ \Phi_i \} \) where \( \Phi_i \) is the potential at the position of \( q_i \) taking into account all the other charges except the charge \( q_i \). Derive a form of Green’s Reciprocation Theorem:

\[
\sum_i \Phi_i q'_i = \sum_i \Phi'_i q_i
\]

where the primed notation indicates a different set of charges and its associated potentials at the same points.

2. Recall in class the surface term in Jackson equation 1.44 can be written as

\[
\Phi(r) = - \int \Phi_s \sigma_{is} dA'
\]

where \( \sigma_{is} \) is the surface charge induced by a unit Green’s function point charge. Show that this expression may be obtained from the result of problem 1. Perhaps you could show the discrete-charge version of the above expression then simply extend the result to a continuous surface.

3. Consider a plane geometry. (NB., the Green’s functions for non-planar geometries are discussed in Jackson chapter 3.)

   a. What is the Green’s function for this geometry? Use cylindrical coordinates with the plane containing the origin, \( z \) the distance above the plane, and \( \rho \) the cylindrical radial coordinate. Hint: the similar image-charge problem should be reviewed.

   b. With this Green’s function, what is the corresponding induced charge on the plane?
c. Suppose the plane has potential \( \Phi_s = \frac{q_0}{2\pi \varepsilon_0 \left( \frac{\rho^2 + z_0^2}{z_0^3} \right)^{3/2}} \frac{1}{\rho} \frac{e^{-\rho/z_0}}{\rho} \),
where \( q_0 \) and \( z_0 \) are constants. Using the Green’s Function from (a), find the potential at position \((\rho=0, z=z_0)\).

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