Physics 513, Autumn Quarter 2018
Electrodynamics: Homework Assignment 1.
Due October 4 either 11:00am in class or 10:45am in the instructor’s mailbox.

This is a relatively straightforward problem set.

1. Helmholtz theorem. A 3-dimensional vector field \( \mathbf{V} \) satisfies
\[ \nabla \cdot \mathbf{V} = s \quad \text{and} \quad \nabla \times \mathbf{V} = \mathbf{c} \quad (\text{where} \quad \nabla \cdot \mathbf{c} = 0). \]

   a. Show that \( \mathbf{V} \) has solutions \( \mathbf{V} = -\nabla \phi + \nabla \times \mathbf{A} \) and volume integrals
   \[ \phi(r) = \frac{1}{4\pi} \int \frac{s(r')}{r(r,r')} \, d\tau' \quad \text{and} \quad \mathbf{A}(r) = \frac{1}{4\pi} \int \frac{c(r')}{r(r,r')} \, d\tau'. \]

   Two subtleties:
   b. Why do we demand \( \nabla \cdot \mathbf{c} = 0? \)
   c. To show (a), you may want to invoke the Divergence Theorem. But \( \nabla(1/r) \) is singular at \( r=0 \). How can you “get around” this singularity?

2. The scalar potential \( \Phi \) in terms of charge density \( \rho \) over all space is given by Jackson eqn 1.17. However, you can find the potential even if the charge density is only known within some closed surface \( S \) and the boundary values of the potential (or their derivatives) are known on the surface \( S \).

   Find an expression for the potential in terms of the charge density \( \rho \) within the volume bounded by \( S \) and \( \Phi \) and \( \nabla \Phi \) on \( S \). Hint: you might use Green’s Theorem (Jackson eqn 1.35)

3. Jackson problem 1.7. Two long, cylindrical conductors of radii \( a_1 \) and \( a_2 \) are parallel and separated distance \( d \), which is large compared with either radius. Show that the capacitance of this [transmission line, as we shall see] is given approximately by
\[ C \approx \pi \varepsilon_0 \left( \ln \frac{d}{a} \right)^{-1}, \] where \( a \) is the geometrical mean of the two radii.
4. Variant of Jackson problem 1.9. Calculate the attractive force per unit length between the conductors of problem 3 under the condition: (a) fixed charge per length on the conductor; (b) fixed potential difference across the conductors.

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