

Electrodynamics: Homework Assignment 8.

This is a short problem set because of Thanksgiving. Note the unusual due date and due location. Due November 21 at 11:00am in the instructor's mailbox.

1. Consider a cylindrical bar of length L made up of material with a uniform permanent “frozen in” dipole density \mathbf{P} oriented along the axis of the bar. Consider a closed-loop integration path entering one face of the bar and exiting the other. It's easy to evaluate the line integral of this path by direct integration: PL . But the non-zero loop integral implies a non-zero curl of \mathbf{P} . Identify where the curl of \mathbf{P} is non-zero and evaluate the curl (you might need to “soften” the abrupt transition of polarization at the boundaries). Then evaluate the loop integral by integrating the curl and check you get PL . Note this exercise implies the \mathbf{D} vector for this problem also has a curl.

2. Electromagnetism and thermodynamics. If the dielectric constant does not depend on temperature, thermodynamics does not much matter. But suppose the dielectric does have temperature dependence; this is very common. (There's an annoying convention in electrodynamics to use U for the free energy instead of F . Let's use thermodynamic notation.)

Recall from lecture and your thermodynamics the free energy is $F = U - TS = \frac{1}{2} \iiint \mathbf{E} \cdot \mathbf{D} dv$ where F is the free energy and U is the total energy. Also recall the first law of thermodynamics $\delta W = \delta U - T\delta S$ where the increment of work δW for a change $\delta \mathbf{D}$ is given by $\delta W = \iiint \mathbf{E} \cdot \delta \mathbf{D} dv$. Finally recall the heat transferred is $\delta Q = T\delta S$. Find the heat transferred during a change $\delta \mathbf{D}$ in the field at constant temperature in terms of T , the dielectric constant $\epsilon(T)$ and field quantities.

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