Electrodynamics: Homework Assignment 7. Due November 15 either 11:00am in class or 10:45am in the instructor's mailbox.

1. The quadrupole tensor as first introduced in class from a Tayor expansion as
$Q_{ij} = \iiint \rho(\vec{r}')x_i' x_j' dv'$.
a. Show that this is proportional to Jackson eqn 4.9
b. Show that the delta-function term in Jackson eqn 4.9 does not contribute to the quadrupole term in the potential.
c. Recall that in class we introduced a new quadrupole tensor. $Q_{ij}' = Q_{ij} - \frac{1}{3}\mathrm{Tr}(Q)$. Show that this is traceless. This led to our surprising result in class that the quadrupole tensor, when referenced to its principal axes, has 2 degrees of freedom.

If you need a refresher on tensor notation, see Appendix V of Slater & Franck, "Mechanics". Someone asked in class about counting degrees of freedom when the quadrupole term in the potential is expanded into spherical versus rectangular coordinates; this is tricky but see Slater & Franck, end of appendix VI, "Electromagnetism".

2. Suppose a charge distribution is symmetric about some axis, say the z-axis. Show that the quadrupole term in the potential referenced to this principal axis depends on a single degree of freedom.

3. Find the potential for a spherically-symmetric charge distribution where the charge density can vary with radius. This could be approached with Gauss's law. But the intent of this problem is for you to use the multipole expansion. If you wish, but it isn't required, you can demonstrate that the Gauss's law method is equivalent.

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