

E&M: Homework Assignment 10.

Due December 6 either 11:00am in class or 10:45am in the instructor's mailbox.

1. (Currents 1: From 2017.) A capacitor is constructed from two circular plates of radius a with a small gap d . The centers of the plates are connected between the plates by a straight thin wire of resistance R . Charges $+Q$ and $-Q$ are then placed uniformly on the plates and a current begins to flow through the plates and wire. The gap is small so ignore fringe fields, and the inductance of the system is small but R is large so it is quasi-static (ignore the inductance of the system). The charge density on the plates is uniform but obviously decreases in time. Caveat: This is a dynamical problem in the quasi-static approximation, so there will arise unavoidable conceptual difficulties.

- Find the charge on the plates as the current flows.) And find the current flowing through the wire. (This comes from RC circuit theory.)
- Find the magnetic field between the plates as the current flows.
- Something seems wrong: There are axial currents flowing through the wire and non-axial currents flowing through the plates. Why, therefore, is the magnetic field in (b) solely in the azimuthal direction?

2. (Currents 2: From 2017.) Suppose a system has non-stationary currents \mathbf{J} arising at some time, where the variation in \mathbf{J} is so slow that magnetic effects are small. What is the characteristic time for stationary currents to reestablish themselves in terms of the conductivity and dielectric constant of the medium? This is called the "relaxation time" of the medium.

3. (Currents 3: from 2017.) Thought problem: There's a saying "Irrotational currents cannot source magnetic fields." However, the infinite, straight, current-carrying wire is an irrotational current sourcing a magnetic field. Find under what circumstances this saying is correct.

4. (Scalar Potential 1: From 2017.) Recall the magnetic scalar potential $B = -\mu_0 \nabla \Phi_m$ where the scalar potential had a geometric interpretation $\Phi_m = \frac{I}{4\pi} d\Omega$ representing the change in the solid angle subtended by a current loop as seen at a field point due to a small translation of the loop. Suppose the loop is a circle of radius R carrying current I . Use this formalism to find the magnetic field along the loop axis. You might ponder under what circumstances you can use a scalar magnetic potential to derive the magnetic field.