Physics 513, Autumn Quarter 2018
Electrodynamics: Homework Assignment 1. Due October 4 either 11:00am in class or 10:45am in the instructor's mailbox.

This is a relatively straightforward problem set.

1. Helmholtz theorem. A 3-dimensional vector field $\mathbf{V}$ satisfies $\nabla \cdot \mathbf{V} = s$ and $\nabla \times \mathbf{V} = \mathbf{c}$ (where $\nabla \cdot \mathbf{c} = 0$).

a. Show that $\mathbf{V}$ has solutions $\mathbf{V} = -\nabla\phi + \nabla\times\mathbf{A}$ and volume integrals $\phi(\mathbf{r}) = \frac{1}{4\pi}\int \frac{s(\mathbf{r}')}{r(\mathbf{r},\mathbf{r}')}d\tau'$ and $\mathbf{A}(\mathbf{r}) = \frac{1}{4\pi}\int \frac{\mathbf{c}(\mathbf{r}')}{r(\mathbf{r},\mathbf{r}')}d\tau'$.

Two subtleties:
b. Why do we demand $\nabla \cdot \mathbf{c} = 0$?
c. To show (a), you may want to invoke the Divergence Theorem. But $\nabla(1/r)$ is singular at r=0. How can you "get around" this singularity?

2. The scalar potential Φ in terms of charge density ρ over all space is given by Jackson eqn 1.17. However, you can find the potential even if the charge density is only known within some closed surface *S* and the boundary values of the potential (or their derivatives) are known on the surface *S*.

Find an expression for the potential in terms of the charge density ρ within the volume bounded by *S* and Φ and $\nabla\Phi$ on *S*. Hint: you might use Green's Theorem (Jackson eqn 1.35)

3. Jackson problem 1.7. Two long, cylindrical conductors of radii a_1 and a_2 are parallel and separated distance *d*, which is large compared with either radius. Show that the capacitance of this [transmission line, as we shall see] is given approximately by $C \approx \pi\epsilon_0 \left(ln\frac{d}{a}\right)^{-1}$, where *a* is the geometrical mean of the two radii.

4. Variant of Jackson problem 1.9. Calculate the attractive force per unit length between the conductors of problem 3 under the condition: (a) fixed charge per length on the conductor; (b) fixed potential difference across the conductors.

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