University of Washington
Physics 513
Graduate Electrodynamics I

Autumn Quarter 2018
December 12, 2018

Final Exam

Printed Name ____
last *first*

- If you need more space than is available to answer any part of a problem, use the **back side of the same page** to complete your answer. Scratch paper will not be graded.
- Show your work in enough detail so that the grader can follow your reasoning and your method of solution.
- This is an open-book exam; you may refer to Jackson in paper or electronic form.

POINTS ARE TOTALED ON THE BACK OF THE EXAM

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I. (40 points) The vector potential and magnetic field.

A sphere of radius R is carries a fixed uniform surface charge density σ. The sphere rotates about the polar angle $\theta=0$ diameter at constant angular velocity ω. (For experts, all speeds are well below the speed of light.)

a. Currents. Find the current densities everywhere.

b. Vector potential expansion: general. Write the vector potential inside and outside the sphere in terms of a general expansion into spherical harmonics.

c. Vector potential expansion: orthogonality. In part b, apply orthogonality to collapse the sum.

d. Vector potential and magnetic field B: inside. Find the vector potential **A** and the magnetic field **B** inside the sphere.

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II. (20 points) Magnetization fields. A very long cylinder of radius *R* is uniformly magnetized with **M** in the axial direction.

a. Find the value and location of all the bound currents and thereby find the **B** and **H** fields inside and outside the cylinder (outside for distances small compared to the length of the cylinder).

b. Find the value and location of all the effective magnetic charge densities and thereby find the **B** and **H** fields inside and outside the cylinder for field points not very far from the center of the cylinder compared to the length of the cylinder.

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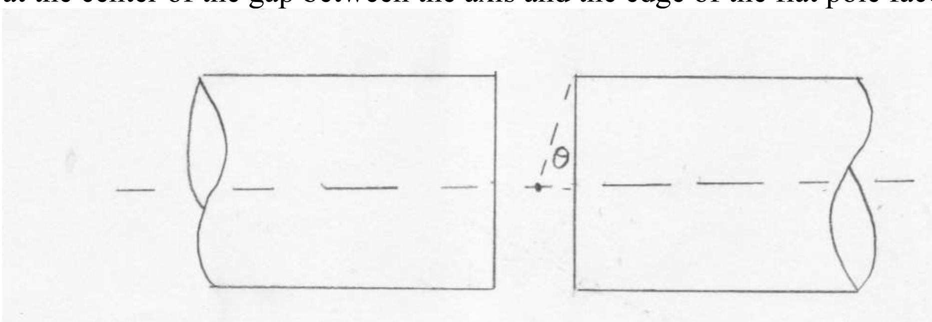
III. (20 points) The field in a permanent-magnet gap. Consider a magnet consisting of two long magnetized cylinders arranged to have a gap, as shown. The magnetization **M** is uniform, in the axial direction, and in the same direction in both cylinders. For a field point in the center of the gap, find the magnetic field **B** in terms of the angle θ, the angle at the center of the gap between the axis and the edge of the flat pole face.



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IV. (20 points) Force and energy relations in magnetostatics.

Recall a starting point of magnetostatics: the mechanical torque **N** exerted on a magnetic moment $\boldsymbol{\mu}$ in a magnetic field **B** (Jackson eqn. 5.1). In addition to this, we also need force and energy relations.

a. Show that the force on an infinitesimal magnetic dipole is given by $\mathbf{F} = \nabla(\boldsymbol{\mu}\cdot\mathbf{B})$.

b. Show that the energy *U* of an infinitesimal dipole is given by $U = -\boldsymbol{\mu}\cdot\mathbf{B}$. What constraints are placed on **B** for this relation to hold?

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POINTS TOTALS

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4. ____________________/20

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