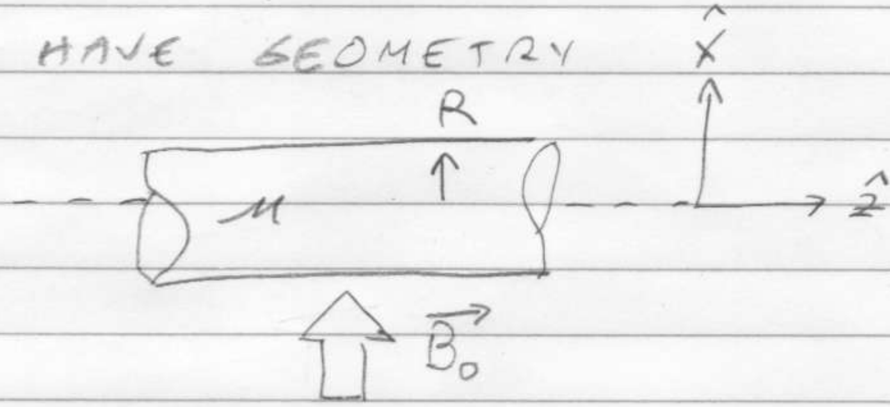
EXAMPLE

HOMEWORK #10, PROBLEM 2 WITHOUT THE CURRENTS.

THIS IS A 2D PROBLEM. WE HAVE A LONG CYLINDER OF PERMEABILITY μ AND A UNIFORM MAGNETIC FIELD $\vec{B}_0$ AT RIGHT ANGLES TO THE AXIS. FIND $\vec{A}$ EVERYWHERE.

WE HAVE GEOMETRY



SINCE BY SYMMETRY THERE CAN ONLY BE $\vec{B}$ FIELDS ALONG $\hat{x}$ & $\hat{y}$, $\vec{A} \sim \hat{z}$.
WE THEN HAVE LAPLACE'S EQUATION

$$\nabla^2 A_z = 0$$

(NOTE, FOR PROBLEM 2, WE HAVE POISSON'S EQUATION INSIDE THE CYLINDER.)

WE CONSIDER SEPARATE SOLUTIONS INSIDE & OUTSIDE THE CYLINDER:

$$A_z\Big|_{r<R} = \sum_\ell \left(a_\ell \cos \ell\theta + b_\ell \sin \ell\theta\right) r^\ell$$

FOR THE FIELD OUTSIDE, NOTE THAT $\vec{B}_0 = B_0 \hat{x}$ IS THE FIELD FROM THE POTENTIAL $A = B_0 Y = B_0 r \sin\theta$. THEREFORE

$$A_z\Big|_{r>R} = B_0 r \sin\theta + \sum_\ell \left(c_\ell \cos \ell\theta + d_\ell \sin \ell\theta\right) \times 1/r^\ell$$

NOTE INSIDE THERE'S (NON-SINGULAR) r^ℓ AND OUTSIDE THERE'S (NON-SINGULAR) $1/r^\ell$; WE REMOVE THE SINGULAR SOLUTIONS.

NOTICE $A_z\big|_{r>R}$ HAS THE CORRECT ASYMPTOTIC BEHAVIOR FOR $r \to \infty$.

LET'S APPLY BOUNDARY CONDITIONS:

1. CONTINUITY OF $\vec{B}$ ACROSS THE CYLINDER SURFACE AT $r = R$ IMPLIES CONTINUITY OF A_z ACROSS THE SURFACE.

2. CONTINUITY OF THE TANGENTIAL COMPONENT OF $\vec{H}$ IMPLIES

$$\frac{1}{\mu}\frac{d}{dr} A_z\Big|_{r<R} = \frac{1}{\mu_0}\frac{d}{dr} A_z\Big|_{r>R}$$

ACROSS THE SURFACE AT $r = R$.

FOR $\ell \neq 1$, THESE BOUNDARY CONDITIONS GIVE $a_\ell = b_\ell = c_\ell = d_\ell = 0$ AND $a_1 = c_1 = 0$.

FOR $\ell = 1$, THE TERM $B_0 r \sin\theta$ CONTRIBUTS TO $\sin \ell\theta$ TERMS FOR $\ell = 1$. AND THESE BOUNDARY CONDITIONS LEAD TO

$$b_1 R = B_0 R + d_1 / R$$

$$\text{AND } \frac{1}{\mu} b_1 = \frac{1}{\mu_0} B_0 + \frac{1}{\mu_0} d_1 (-1) \frac{1}{R^2}$$

THUS

$$A_z \big|_{r<R} = \frac{2\mu}{\mu_0 + \mu} B_0 r \sin\theta$$

$$A_z \big|_{r>R} = \left(1 + \frac{\mu - \mu_0}{\mu + \mu_0} \frac{R^2}{r^2}\right) B_0 r \sin\theta$$