Physics 513, Autumn Quarter 2017
Electrodynamics: Homework Assignment 1
Due October 5 either 11:00am in class or 10:45am in the instructor's mailbox.

1. Helmholtz theorem. A 3-dimensional vector field **V** satisfies
$\nabla \cdot \mathbf{V} = s$ and $\nabla \times \mathbf{V} = \mathbf{c}$ (where $\nabla \cdot \mathbf{c} = 0$).

a. Show that **V** has solutions $\mathbf{V} = -\nabla \phi + \nabla \times \mathbf{A}$ .and volume integrals
$\phi(\mathbf{r}) = \frac{1}{4\pi} \int \frac{s(\mathbf{r}\prime)}{r(\mathbf{r},\mathbf{r}\prime)} d\tau\,'$ and $\mathbf{A}(\mathbf{r}) = \frac{1}{4\pi} \int \frac{\mathbf{c}(\mathbf{r}\prime)}{r(\mathbf{r},\mathbf{r}\prime)} d\tau\,'$.

Two subtleties:
b. Why do we demand $\nabla \cdot \mathbf{c} = 0$?
c. To show (a), you may want to invoke the Divergence Theorem. But $\nabla(1/r)$ is singular at r=0. How can "get around" this singularity?

2. The scalar potential Φ in terms of charge density ρ over all space is given by Jackson eqn 1.17. However, you can find the potential even if the charge density is only known within some closed surface *S* and the boundary values of the potential (or their derivatives) are known on the surface *S*.

Find an expression for the potential in terms of the potential Φ within *S* and Φ and $\nabla\Phi$ on *S*. Hint: you may use Green's Theorem (Jackson eqn 1.35)

3. Show that the two electrostatic Maxwell equations (Jackson eqns 1.13 and 1.14) follow from Coulomb's Law.

4. Surface singularities.

a. Two closely separated charge layers form a sheet, not necessarily a planar sheet, with dipole moment per unit area **d**. Outside the sheet what is the potential arising from this dipole layer?

b. In the case where **d** is uniform and normal to the surface, find the potential. The expression in terms of Ω, the solid angle subtended by the sheet, is simple.

c. What is the discontinuity of the potential on crossing the sheet?

d. What is the discontinuity of the normal derivative of the potential on crossing the sheet?

e. Modify the result of problem (2) to express the potential in terms of the surface charge and the surface dipole layer.

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