

Final Exam

Printed Name \_\_\_\_\_  
*last* *first*

- If you need more space than is available to answer any part of a problem, use the **back side of the same page** to complete your answer. Scratch paper will not be graded.
- Show your work in enough detail so that the grader can follow your reasoning and your method of solution.
- Feel free to ask for an equation. I have a copy of Jackson and can look up an equation for you.

POINTS

1. \_\_\_\_\_/25

2. \_\_\_\_\_/25

3. \_\_\_\_\_/25

4. \_\_\_\_\_/25

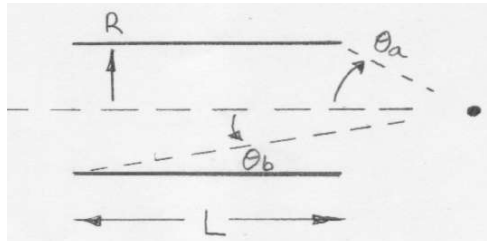
Total \_\_\_\_\_/100

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**I. (25 points) Magnetic field calculation.**

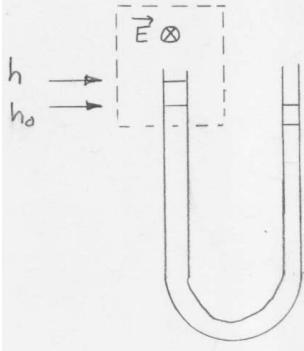
**a. Circular loop.** Consider a circular loop of radius  $R$  carrying current  $I$ . Find the magnetic field  $\mathbf{B}$  everywhere on the loop axis.

**b. Solenoid.** Consider a cylinder of length  $L$  and radius  $R$  carrying solenoidal surface currents  $\mathbf{K}$ . Find the magnetic field  $\mathbf{B}$  everywhere on the solenoid axis. Express your answer in terms of  $\theta_a$  and  $\theta_b$  (the angles to the edges of the solenoid face as “seen” by the field point) shown below.



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**II. (25 points) Forces on dielectrics.** Consider a “U-tube” containing a liquid linear dielectric of permittivity  $\epsilon$  and mass density  $\rho$ . One arm of the tube is between plates of a capacitor. (You can assume the capacitor is ideal, so the electric field  $\mathbf{E}_0$  between the plates is uniform.) See the sketch below.



- With the cross-section of the U-tube circular, find the height the liquid rises  $h-h_0$  when the electric field is applied. Hint: recall the electric field within a dielectric cylinder with axis at right angles to a uniform applied field  $E_0$  is  $2E_0/(1+\epsilon/\epsilon_0)$ .
- How would your result change if the U-tube cross section is a rectangle with a very long edge parallel to the applied field, and a very short edge normal to the electric field?
- How would your result change if the U-tube cross section is a rectangle with a very short edge parallel to the applied field, and a very long edge normal to the electric field?

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**III. (25 points) Mutual Inductance.** Consider two coaxial thin circular wires of radii  $a$  and  $b$  with their centers a distance  $d$  apart and  $d \gg a$  and  $b$ . Find the mutual inductance.

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**IV. (25 points) Image currents, magnetic materials, inductance.**

**a.** A long thin wire carrying current  $I$  lies a distance  $d$  in vacuum from the surface of a semi-infinite slab of linear permeable material (with permeability  $\mu$ , and as usual  $\mu > \mu_0$ ). Find the force per unit length on the wire. Hint: Choose a sensible location of the image currents.

**b.** The wire is replaced with a circular loop of thin wire placed on the surface of the semi-infinite slab. What is the ratio of the loop's self-inductance when on the slab surface to the loop's self-inductance when in free space? Hint: (i) You may want to consider part a to infer the location of the image current. (ii) You may want to consider the location of sources of B field to infer the "shape" of the B field.

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