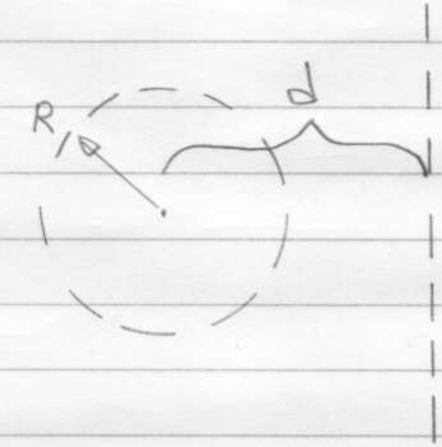
1\. CHALLENGE PROBLEM (SMYTHE)
WHAT'S THE CAPACITANCE OF A SYSTEM CONSISTING OF A CONDUCTING SPHERE AND AN INFINITE CONDUCTING PLANE?

FRANKLY, I CAN'T FIND AN EXACT SOLUTION. I CAN FIND AN APPROXIMATE SOLUTION WITH IMAGE CHARGES.



START ADDING IMAGE CHARGES:

1. CHARGE $+q_1$ AT THE CENTER OF THE SPHERE. THIS MAKES THE SPHERE, BUT NOT THE PLANE, AN EQUIPOTENTIAL.

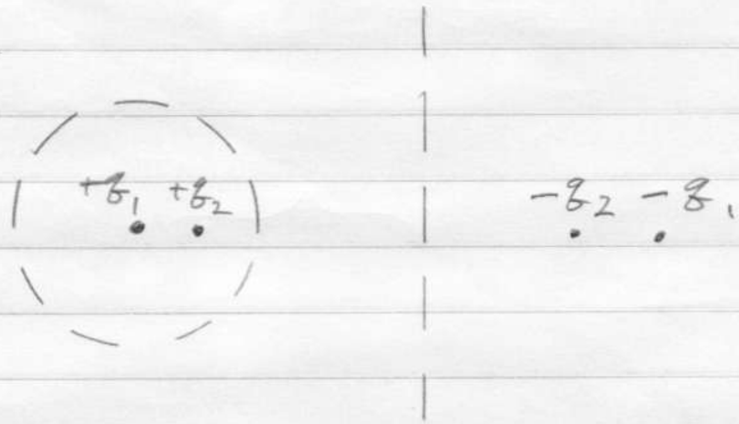
2. CHARGE $-q_1$ TO RIGHT OF PLANE. THE PLANE IS AN EQUIPOTENTIAL, BUT NOT THE SPHERE.

3. CHARGE $+q_2$ INSIDE SPHERE. THE SPHERE IS AN EQUIPOTENTIAL, BUT NOT THE PLANE.

4. CHARGE $-q_2$ TO RIGHT OF THE PLANE. THE PLANE IS AN EQUIPOTENTIAL, THE SPHERE IS NOT.

5. ETC., etc.

SO FAR, WE HAVE IMAGE CHARGES:



THE MAGNITUDE AND POSITIONS OF THE CHARGES ARE KNOWN FROM THE SIMPLER CHARGE-ABOVE-PLANE AND CHARGE-IN-SPHERE IMAGE CHARGE PROBLEMS:

$$q_1 = q_1, \quad q_2 = \frac{R}{2d} q_1, \quad q_3 = \frac{(R/2d)^2}{1-(R/2d)^2} q_1$$

$+q_1$ LOCATED AT CENTER OF SPHERE

$-q_1$ AT $2d$ (FROM CENTER OF SPHERE)

$+q_2$ AT $R^2/2d$

$-q_2$ AT $2d-(R^2/2d)$

$+q_3$ AT $R^2/\{2d-(R^2/2d)\}$

etc

etc.

THE TOTAL CHARGE ON THE SPHERE IS

$$q = q_1 \left(1 + R/2d + (R/2d)^2/\{1-(R/2d)^2\} + \cdots\right)$$

A SUBTLETY IS THAT ONLY q_1 SETS THE SPHERE POTENTIAL. THE OTHER CHARGES MAKE THE SPHERE POTENTIAL ZERO: e.g., THE PAIR $-q_1, +q_2$ MAKE THE SPHERE SURFACE ZERO POTENTIAL, SAME FOR ALL OTHER CHARGE-PAIRS. THE POTENTIAL OF THE SPHERE IS THEREFORE

$$\Phi = q_1/4\pi\varepsilon_0 \; 1/R$$

WITH RESULTING CAPACITANCE

$$C = q/\Phi = 4\pi\varepsilon_0 R\left(1 + \frac{R}{2d} + \cdots\right)$$

THE PLANE INCREASES THE CAPACITANCE OF THE SPHERE.

LIMITS:

$R/2d \rightarrow 0$, $C \rightarrow 4\pi\varepsilon_0 R$

$R/2d \rightarrow 1/2$, $C \rightarrow \infty$