Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}};\quad d\tau = dx\,dy\,dz$

Gradient: $\quad \nabla t = \dfrac{\partial t}{\partial x}\hat{\mathbf{x}} + \dfrac{\partial t}{\partial y}\hat{\mathbf{y}} + \dfrac{\partial t}{\partial z}\hat{\mathbf{z}}$

Divergence: $\nabla \cdot \mathbf{v} = \dfrac{\partial v_x}{\partial x} + \dfrac{\partial v_y}{\partial y} + \dfrac{\partial v_z}{\partial z}$

Curl: $\quad \nabla \times \mathbf{v} = \left(\dfrac{\partial v_z}{\partial y} - \dfrac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\dfrac{\partial v_x}{\partial z} - \dfrac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\dfrac{\partial v_y}{\partial x} - \dfrac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$

Laplacian: $\quad \nabla^2 t = \dfrac{\partial^2 t}{\partial x^2} + \dfrac{\partial^2 t}{\partial y^2} + \dfrac{\partial^2 t}{\partial z^2}$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}};\quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient: $\quad \nabla t = \dfrac{\partial t}{\partial r}\hat{\mathbf{r}} + \dfrac{1}{r}\dfrac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \dfrac{1}{r\sin\theta}\dfrac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$

Divergence: $\nabla \cdot \mathbf{v} = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}(r^2 v_r) + \dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial \theta}(\sin\theta\,v_\theta) + \dfrac{1}{r\sin\theta}\dfrac{\partial v_\phi}{\partial \phi}$

Curl: $\quad \nabla \times \mathbf{v} = \dfrac{1}{r\sin\theta}\left[\dfrac{\partial}{\partial \theta}(\sin\theta\,v_\phi) - \dfrac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}}$

$$+ \dfrac{1}{r}\left[\dfrac{1}{\sin\theta}\dfrac{\partial v_r}{\partial \phi} - \dfrac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \dfrac{1}{r}\left[\dfrac{\partial}{\partial r}(r v_\theta) - \dfrac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian: $\quad \nabla^2 t = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial t}{\partial r}\right) + \dfrac{1}{r^2\sin\theta}\dfrac{\partial}{\partial \theta}\left(\sin\theta\dfrac{\partial t}{\partial \theta}\right) + \dfrac{1}{r^2\sin^2\theta}\dfrac{\partial^2 t}{\partial \phi^2}$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}};\quad d\tau = s\,ds\,d\phi\,dz$

Gradient: $\quad \nabla t = \dfrac{\partial t}{\partial s}\hat{\mathbf{s}} + \dfrac{1}{s}\dfrac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}} + \dfrac{\partial t}{\partial z}\hat{\mathbf{z}}$

Divergence: $\nabla \cdot \mathbf{v} = \dfrac{1}{s}\dfrac{\partial}{\partial s}(s v_s) + \dfrac{1}{s}\dfrac{\partial v_\phi}{\partial \phi} + \dfrac{\partial v_z}{\partial z}$

Curl: $\quad \nabla \times \mathbf{v} = \left[\dfrac{1}{s}\dfrac{\partial v_z}{\partial \phi} - \dfrac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\dfrac{\partial v_s}{\partial z} - \dfrac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \dfrac{1}{s}\left[\dfrac{\partial}{\partial s}(s v_\phi) - \dfrac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$

Laplacian: $\quad \nabla^2 t = \dfrac{1}{s}\dfrac{\partial}{\partial s}\left(s\dfrac{\partial t}{\partial s}\right) + \dfrac{1}{s^2}\dfrac{\partial^2 t}{\partial \phi^2} + \dfrac{\partial^2 t}{\partial z^2}$

FUNDAMENTAL CONSTANTS

$\epsilon_0 = 8.85 \times 10^{-12}\, \text{C}^2/\text{Nm}^2$ (permittivity of free space)

$\mu_0 = 4\pi \times 10^{-7}\, \text{N/A}^2$ (permeability of free space)

$c = 3.00 \times 10^8\, \text{m/s}$ (speed of light)

$e = 1.60 \times 10^{-19}\, \text{C}$ (charge of the electron)

$m = 9.11 \times 10^{-31}\, \text{kg}$ (mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$$

$$\begin{cases} \hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{r}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{r}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \tan^{-1}\left(\sqrt{x^2 + y^2}/z\right) \\ \phi = \tan^{-1}(y/x) \end{cases}$$

$$\begin{cases} \hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \end{cases}$$

Cylindrical

$$\begin{cases} x = s\cos\phi \\ y = s\sin\phi \\ z = z \end{cases}$$

$$\begin{cases} \hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{s}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{s}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\begin{cases} s = \sqrt{x^2 + y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases}$$

$$\begin{cases} \hat{\mathbf{s}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$\mathbf{F} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{r^2}\hat{r}$ Coulomb's Law $\mathbf{F} = Q\mathbf{E} + Q\mathbf{v} \times \mathbf{B}$ Lorentz force

$\mathbf{E} = \dfrac{1}{4\pi\varepsilon_0}\iiint \dfrac{\rho(\mathbf{r}')}{r^2}\hat{r}\, d\tau'$ electric field from continuous charge distribution

$$\oiint \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\varepsilon_0} Q_{\text{encl}} \text{ Gauss' Law (integral), and } \nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0} \rho \text{ Gauss' Law (differential)}$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0 \text{ and } \nabla \times \mathbf{E} = 0 \text{ (statics)}$$

$$V(\mathbf{r}) = -\int_{\wp}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l} \text{ Electrostatic potential, and } \mathbf{E} = -\nabla V \text{ (statics)}$$

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \text{ Poisson's equation, and } \nabla^2 V = 0 \text{ Laplace's equation (regions of no charge)}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \text{ and } V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho(\mathbf{r}')}{r} d\tau' \text{ (setting reference point at infinity)}$$

$$\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\varepsilon_0} \hat{\mathbf{n}} \text{ and } \frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} = \frac{1}{\varepsilon_0} \sigma \text{ boundary conditions}$$

$$V(\mathbf{b}) - V(\mathbf{a}) = \frac{W}{Q} \quad W = \frac{1}{2} \sum_{i=1}^{n} q_i V(\mathbf{r}_i) \text{ and } W = \frac{1}{2} \iiint \rho V \, d\tau$$

$$P = \frac{\varepsilon_0}{2} E^2 \text{ electrostatic pressure}$$

$$Q = CV \quad W = \frac{1}{2} CV^2 \quad W = \frac{1}{2} Q^2 / C \text{ capacitors}$$

$$\nabla^2 V = -\frac{1}{\varepsilon_0} \rho \text{ Poisson's equation} \quad \nabla^2 V = 0 \text{ Laplace's equation}$$

$$V(x,y) = \sum_{n=0}^{\infty} (A_n e^{+kx} + B_n e^{-kx})(C_n \sin ky + D_n \cos ky) \text{ solution to Laplace's equation in}$$
Cartesian coordinates in two dimensions

$$\int_0^a \sin(n\pi \frac{y}{a}) \sin(n'\pi \frac{y}{a}) dy = \begin{cases} 0 & n \neq n' \\ \frac{a}{2} & n = n' \end{cases} \text{ orthogonality of sines}$$

$$\int_0^a \sin(n\pi \frac{y}{a})\,dy = \begin{cases} 0 & \text{n even} \\ \dfrac{2a}{n\pi} & \text{n odd} \end{cases} \qquad e^{+kx} + e^{-kx} = 2\cosh kx$$

$$\int_0^\pi \cos^2\theta \sin\theta\, d\theta = 2/3$$

$P_0(x)$	$= 1$
$P_1(x)$	$= x$
$P_2(x)$	$= (3x^2 - 1)/2$
$P_3(x)$	$= (5x^3 - 3x)/2$
$P_4(x)$	$= (35x^4 - 30x^2 + 3)/8$
$P_5(x)$	$= (63x^5 - 70x^3 + 15x)/8$

TABLE 3.1   Legendre Polynomials.

$$V(r,\theta) = \sum_{\ell=0}^{\infty}\left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right)P_\ell(\cos\theta)$$

solution to Laplace's equation in spherical coordinates and azimuthal symmetry

$$\int_0^\pi P_\ell(\cos\theta)P_{\ell'}(\cos\theta)\,d\cos\theta = \begin{cases} 0 & \ell \neq \ell' \\ \dfrac{2}{2\ell+1} & \ell = \ell' \end{cases} \quad \text{orthogonality of Legendre polynomials}$$

$$V(r,\theta) = -E_0\left(r - \frac{R^3}{r^2}\right)\cos\theta \quad \text{potential of neutral conducting sphere in uniform E-field}$$

$$\frac{1}{\mathfrak{R}} = \frac{1}{r}\sum_0^\infty \left(\frac{r'}{r}\right)^n P_n(\cos\alpha) \quad \text{1/r expansion in Legendre polynomials}$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\sum_0^\infty \frac{1}{r^{n+1}}\iiint (r')^n P_n(\cos\alpha)\rho(\mathbf{r}')d\tau' \quad \text{multipole expansion}$$

$$\vec{p} = \iiint \vec{r}'\rho(\vec{r}')d\tau' \quad \vec{p} = \sum_1^n q_i\vec{r}_i' \quad V_{\text{dip}}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\bullet\hat{r}}{r^2} \quad \text{dipole moment}$$

$$\vec{E}_{\text{dip}} = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$

$$\mathbf{N} = \vec{\mathbf{p}}\times\vec{\mathbf{E}} \qquad \vec{\mathbf{F}} = \left(\vec{\mathbf{p}}\bullet\vec{\nabla}\right)\vec{\mathbf{E}} \qquad U = -\vec{\mathbf{p}}\bullet\vec{\mathbf{E}} \quad \text{torque, force, energy of dipole in E field}$$

$$\sigma_b = \vec{\mathbf{P}}\bullet\hat{\mathbf{n}} \qquad \rho_b = -\vec{\nabla}\bullet\vec{\mathbf{P}} \quad \text{bound surface and volume charge}$$

$$V(r,\theta) = \frac{P}{3\varepsilon_0}\frac{R^3}{r^2}\cos\theta \text{ for r>R}, \quad V(r,\theta) = \frac{P}{3\varepsilon_0}r\cos\theta \text{ for r<R} \quad \text{potential of uniformly-}$$

polarized sphere of radius R

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} \quad \vec{\nabla}\bullet\vec{D} = \rho_f \quad \oiint\vec{D}\bullet\hat{n}\,dA = Q_{f\,enc}$$

$$\vec{D}_{above}^{\parallel} - \vec{D}_{below}^{\parallel} = \vec{P}_{above}^{\parallel} - \vec{P}_{below}^{\parallel} \quad D_{above}^{\perp} - D_{below}^{\perp} = \sigma_f \quad \text{boundary conditions}$$

$$\vec{P} = \varepsilon_0\chi_E\vec{E} \quad \vec{D} = \varepsilon\vec{E} \quad \varepsilon_r = 1 + \chi_E = \varepsilon/\varepsilon_0$$

$$W = \frac{1}{2}\iiint\vec{D}\bullet\vec{E}\,d\tau \quad \vec{F} = -\vec{\nabla}W \quad F = \frac{1}{2}V^2\frac{dC}{dx}$$

$$G = 6.67\times10^{-11} \text{ N m}^2/\text{kg}^2 \quad g = 9.8 \text{ m/s}^2$$