

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}}; \quad d\tau = dx dy dz$

Gradient: $\nabla t = \frac{\partial t}{\partial x} \hat{\mathbf{x}} + \frac{\partial t}{\partial y} \hat{\mathbf{y}} + \frac{\partial t}{\partial z} \hat{\mathbf{z}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Curl: $\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{\mathbf{z}}$

Laplacian: $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

Spherical. $d\mathbf{l} = dr \hat{\mathbf{r}} + r d\theta \hat{\boldsymbol{\theta}} + r \sin\theta d\phi \hat{\boldsymbol{\phi}}; \quad d\tau = r^2 \sin\theta dr d\theta d\phi$

Gradient: $\nabla t = \frac{\partial t}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial t}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{r \sin\theta} \frac{\partial t}{\partial \phi} \hat{\boldsymbol{\phi}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta v_\theta) + \frac{1}{r \sin\theta} \frac{\partial v_\phi}{\partial \phi}$

Curl: $\nabla \times \mathbf{v} = \frac{1}{r \sin\theta} \left[\frac{\partial}{\partial \theta} (\sin\theta v_\phi) - \frac{\partial v_\theta}{\partial \phi} \right] \hat{\mathbf{r}}$
 $+ \frac{1}{r} \left[\frac{1}{\sin\theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\phi) \right] \hat{\boldsymbol{\theta}} + \frac{1}{r} \left[\frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\boldsymbol{\phi}}$

Laplacian: $\nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 t}{\partial \phi^2}$

Cylindrical. $d\mathbf{l} = ds \hat{\mathbf{s}} + s d\phi \hat{\boldsymbol{\phi}} + dz \hat{\mathbf{z}}; \quad d\tau = s ds d\phi dz$

Gradient: $\nabla t = \frac{\partial t}{\partial s} \hat{\mathbf{s}} + \frac{1}{s} \frac{\partial t}{\partial \phi} \hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z} \hat{\mathbf{z}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{1}{s} \frac{\partial}{\partial s} (s v_s) + \frac{1}{s} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$

Curl: $\nabla \times \mathbf{v} = \left[\frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right] \hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s} \right] \hat{\boldsymbol{\phi}} + \frac{1}{s} \left[\frac{\partial}{\partial s} (s v_\phi) - \frac{\partial v_s}{\partial \phi} \right] \hat{\mathbf{z}}$

Laplacian: $\nabla^2 t = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial t}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$

FUNDAMENTAL CONSTANTS

$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$	(permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$	(permeability of free space)
$c = 3.00 \times 10^8 \text{ m/s}$	(speed of light)
$e = 1.60 \times 10^{-19} \text{ C}$	(charge of the electron)
$m = 9.11 \times 10^{-31} \text{ kg}$	(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{cases} \quad \begin{cases} \hat{\mathbf{x}} = \sin \theta \cos \phi \hat{\mathbf{r}} + \cos \theta \cos \phi \hat{\boldsymbol{\theta}} - \sin \phi \hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin \theta \sin \phi \hat{\mathbf{r}} + \cos \theta \sin \phi \hat{\boldsymbol{\theta}} + \cos \phi \hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \cos \theta \hat{\mathbf{r}} - \sin \theta \hat{\boldsymbol{\theta}} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \tan^{-1}(\sqrt{x^2 + y^2}/z) \\ \phi = \tan^{-1}(y/x) \end{cases} \quad \begin{cases} \hat{\mathbf{r}} = \sin \theta \cos \phi \hat{\mathbf{x}} + \sin \theta \sin \phi \hat{\mathbf{y}} + \cos \theta \hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} = \cos \theta \cos \phi \hat{\mathbf{x}} + \cos \theta \sin \phi \hat{\mathbf{y}} - \sin \theta \hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} = -\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{y}} \end{cases}$$

Cylindrical

$$\begin{cases} x = s \cos \phi \\ y = s \sin \phi \\ z = z \end{cases} \quad \begin{cases} \hat{\mathbf{x}} = \cos \phi \hat{\mathbf{s}} - \sin \phi \hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin \phi \hat{\mathbf{s}} + \cos \phi \hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\begin{cases} s = \sqrt{x^2 + y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases} \quad \begin{cases} \hat{\mathbf{s}} = \cos \phi \hat{\mathbf{x}} + \sin \phi \hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} = -\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{y}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \hat{\mathbf{r}} \quad \text{Coulomb's Law} \quad \mathbf{F} = Q\mathbf{E} + Q\mathbf{V} \times \mathbf{B} \quad \text{Lorentz force}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \iiint \frac{\rho(\mathbf{r}')}{r^2} \hat{\mathbf{r}} \, d\tau' \quad \text{electric field from continuous charge distribution}$$

$$\oiint \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{encl}} \text{ Gauss' Law (integral)} \quad \oint \mathbf{E} \cdot d\mathbf{l} = 0 \text{ (statics)}$$

$$V(\mathbf{r}) = -\int_{\varphi}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l} \text{ Electrostatic potential, and } \mathbf{E} = -\nabla V \text{ (statics)}$$

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \text{ Poisson's equation, and } \nabla^2 V = 0 \text{ Laplace's equation (regions of no charge)}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \text{ and } V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \iiint \frac{\rho(\mathbf{r}')}{r} d\tau' \text{ (setting reference point at infinity)}$$

$$\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0} \hat{\mathbf{n}} \text{ and } \frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} = \frac{1}{\epsilon_0} \sigma \text{ boundary conditions}$$

$$D_{\text{above}}^{\perp} - D_{\text{below}}^{\perp} = \sigma_f$$

$$V(\mathbf{b}) - V(\mathbf{a}) = \frac{W}{Q} \quad W = \frac{1}{2} \sum_{i=1}^n q_i V(\mathbf{r}_i) \text{ and } W = \frac{1}{2} \iiint \rho V d\tau$$

$$Q = CV \quad W = \frac{1}{2} CV^2 \quad W = \frac{1}{2} Q^2 / C \text{ capacitors}$$

$$\nabla^2 V = -\frac{1}{\epsilon_0} \rho \text{ Poisson's equation} \quad \nabla^2 V = 0 \text{ Laplace's equation}$$

Maxwell's Equations

In general:

$$\begin{cases} \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{cases}$$

In matter:

$$\begin{cases} \nabla \cdot \mathbf{D} = \rho_f \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \end{cases}$$

Auxiliary Fields

Definitions:

$$\begin{cases} \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M} \end{cases}$$

Linear media:

$$\begin{cases} \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}, & \mathbf{D} = \epsilon \mathbf{E} \\ \mathbf{M} = \chi_m \mathbf{H}, & \mathbf{H} = \frac{1}{\mu} \mathbf{B} \end{cases}$$

Potentials

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A}$$

Lorentz force law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Energy, Momentum, and Power

$$\text{Energy:} \quad U = \frac{1}{2} \int (\epsilon_0 E^2 + \frac{1}{\mu_0} B^2) d\tau$$

$$\text{Momentum:} \quad \mathbf{P} = \epsilon_0 \int (\mathbf{E} \times \mathbf{B}) d\tau$$

$$\text{Poynting vector:} \quad \mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B})$$