Large N Volume Independence in Confining and Conformal Theories

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Outline:

- Large N equivalences
- Large N volume independence
- Example 1: Yang-Mills
- Example 2: QCD(adj)
- Example 3: N=4 SYM

Large N equivalences

Differing finite N gauge theories can have identical* large N limits:

Gauge group independence $U(N) \sim O(N) \sim Sp(N)$	Lovelace 1982
Volume independence $\mathbb{R}^d \sim \mathbb{R}^{d-1} \times S^1 \sim \mathbb{R}^{d-2} \times (S^1)^2 \sim \dots$	Eguchi & Kawai 1982, Bhanot, Heller & Neuberger 1982, Gonzalez-Arroyo & Okawa 1983, ...
Orbifold projections $U(2N) \sim U(N) \times U(N)$, *etc.*	Bershadsky & Johansen 1998, Schmaltz 1998, Strassler 2001, Kovtun, Ünsal & L.Y. 2003, ...
Orientifold projections *antisymmetric ~ adjoint matter*	Armoni, Shifman & Veneziano 2003, ...

Proof: Comparisons of loop equations or large N coherent state dynamics

*With important caveats...

Large N volume independence

$SU(N)$ gauge theory on toroidal compactifications of $\mathbb{R}^d$:

no volume dependence in leading large N behavior of topologically trivial single-trace observables (or their connected correlators)

provided

no spontaneous breaking of center symmetry or translation invariance

Proof: comparison of large N loop equaions (EK) or $N=\infty$ classical dynamics (LY)

Example 1: $SU(N)$ YM on $\mathbb{R}^3 \times S^1$

circumference L

- Z_N center symmetry, order parameter = Wilson line Ω
- $L > L_c$: unbroken center symmetry

  $\langle \mathrm{tr}\, \Omega^n \rangle = 0$

  confined phase
- $L < L_c$: broken center symmetry

  $\langle \mathrm{tr}\, \Omega^n \rangle \neq 0$

  deconfined plasma phase

  failure of EK reduction

Center-symmetry stabilization

- Unwanted symmetry breaking? Fix it!
  - quenched EK: doesn't work — Bringoltz, Sharpe 2008
  - twisted EK: doesn't work — Teper, Vairinhos 2007; Azeyanagi, Hanada, Hirata, Ishikawa 2008
  - adding massless adjoint fermions: works — Kovtun, Unsal, Yaffe 2007
  - adding explicit stabilizing terms: works — Unsal, Yaffe 2008

$$S_{\rm YM} \longrightarrow S_{\rm YM} + \Delta S$$

$$\Delta S = \int_{\mathbb{R}^3\times S^1} \frac{1}{L^4} \sum_{n=1}^{\lfloor N/2 \rfloor} c_n \, |\mathrm{tr}\, \Omega^n|^2$$

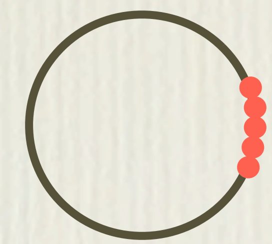
$\{c_n\}$ sufficiently positive, $O(1)$ as $N \to \infty$

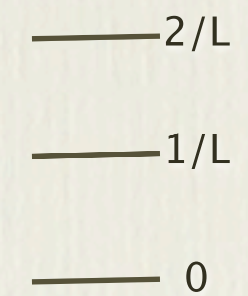
deformation prevents symmetry breaking but has
no effect on $N = \infty$ center symmetric dynamics

Dimensional Reduction ?

- small L, asymptotic freedom $\Rightarrow$ heavy, weakly coupled KK modes

- usual case: broken center symmetry

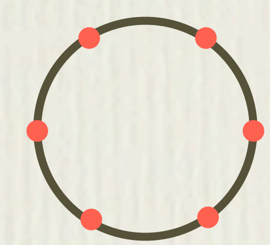




$\langle \mathrm{tr}\,\Omega^n \rangle \neq 0 \Leftrightarrow$ eigenvalues clump

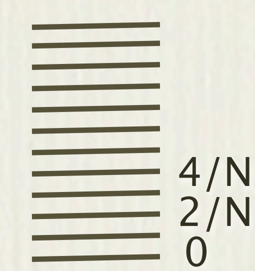
$m_{KK} = 1/L,\ 2/L, \ldots$, perturbative control when $L\Lambda \ll 1$

integrate out $\Rightarrow$ *3d* effective theory, L-dependent



- center-symmetric case:

$\langle \mathrm{tr}\,\Omega^n \rangle = 0 \Leftrightarrow$ eigenvalues repel



$m_{KK} = 1/NL,\ 2/NL, \ldots$, perturbative control when $NL\Lambda \ll 1$

$SU(N) \Rightarrow U(1)^{N-1}$ Higgsing

$$m_W = \frac{2\pi}{NL}, \quad m_\gamma \sim m_W\, e^{-8\pi^2/Ng^2(m_W)} \implies \frac{m_\gamma}{m_W} \sim (NL\Lambda)^{11/6}$$

topological defects (monopoles) $\Rightarrow$ mass gap, confinement

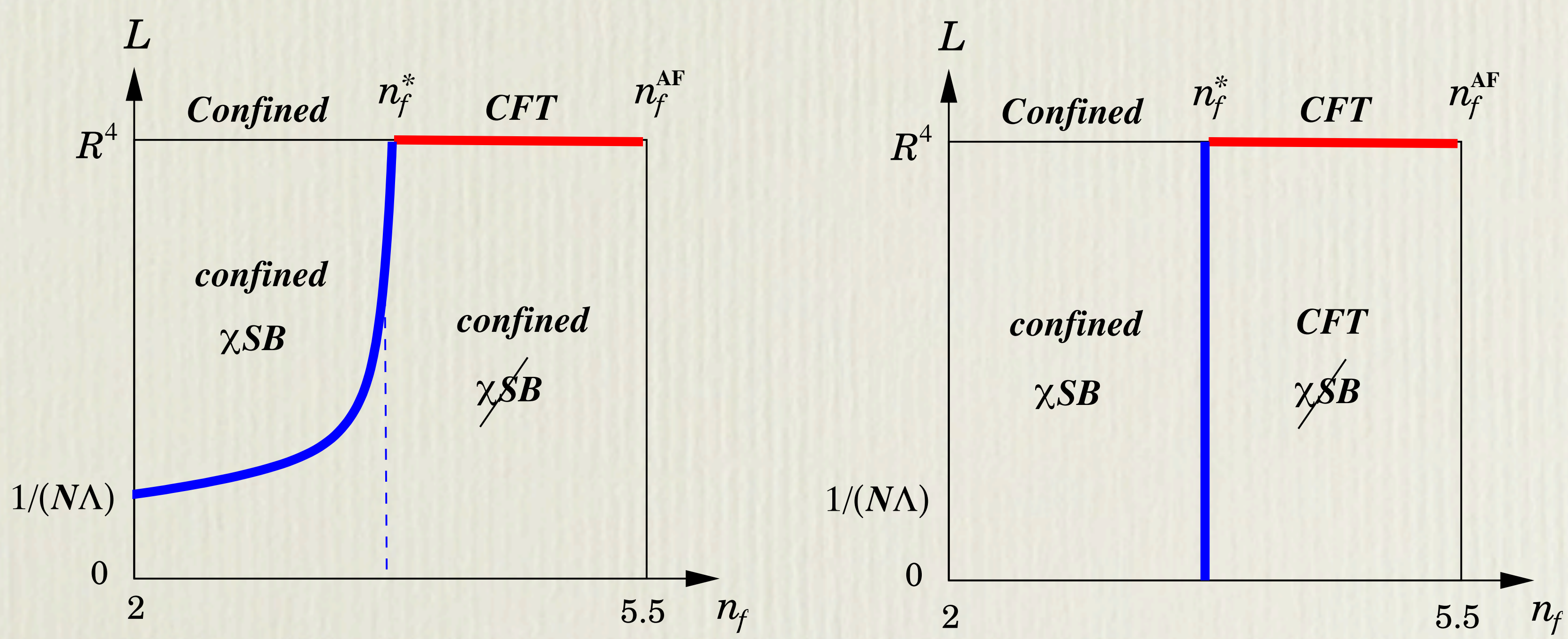
Large N vs. small L

- unbroken center symmetry $\Rightarrow$ relevant scale is NL, not L.
- $L \to 0$, N fixed $\Rightarrow NL\Lambda \ll 1$
  - KK modes: decouple
  - IR physics = Abelian 3*d* dynamics, semiclassical confinement
  - volume dependence
- $N \to \infty$, L fixed $\Rightarrow NL\Lambda \gg 1$
  - KK spectrum $\Rightarrow$ continuum
  - IR physics = non-Abelian 4*d* dynamics

Example 2: massless QCD(adj), $\mathbb{R}^3\times S^1$

- $N_\mathrm{f} \geq 1$ massless adjoint rep. fermions:
  periodic boundary conditions ➡ stabilized center symmetry

$$V_{1-\mathrm{loop}}(\Omega) = \frac{2}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \,(-1+N_\mathrm{f})\, |\mathrm{tr}\,\Omega^n|^2$$



(a) $N < \infty$ (b) $N = \infty$

- $n_\mathrm{f} > n_\mathrm{f}^*$ = conformal window lower limit: IR CFT on $\mathbb{R}^4$

  compactify on $\mathbb{R}^3\times\mathbf{S}^1$ ➡ correlation length ~ NL, not L.

Massive QCD(adj), $\mathbb{R}^3 \times S^1$

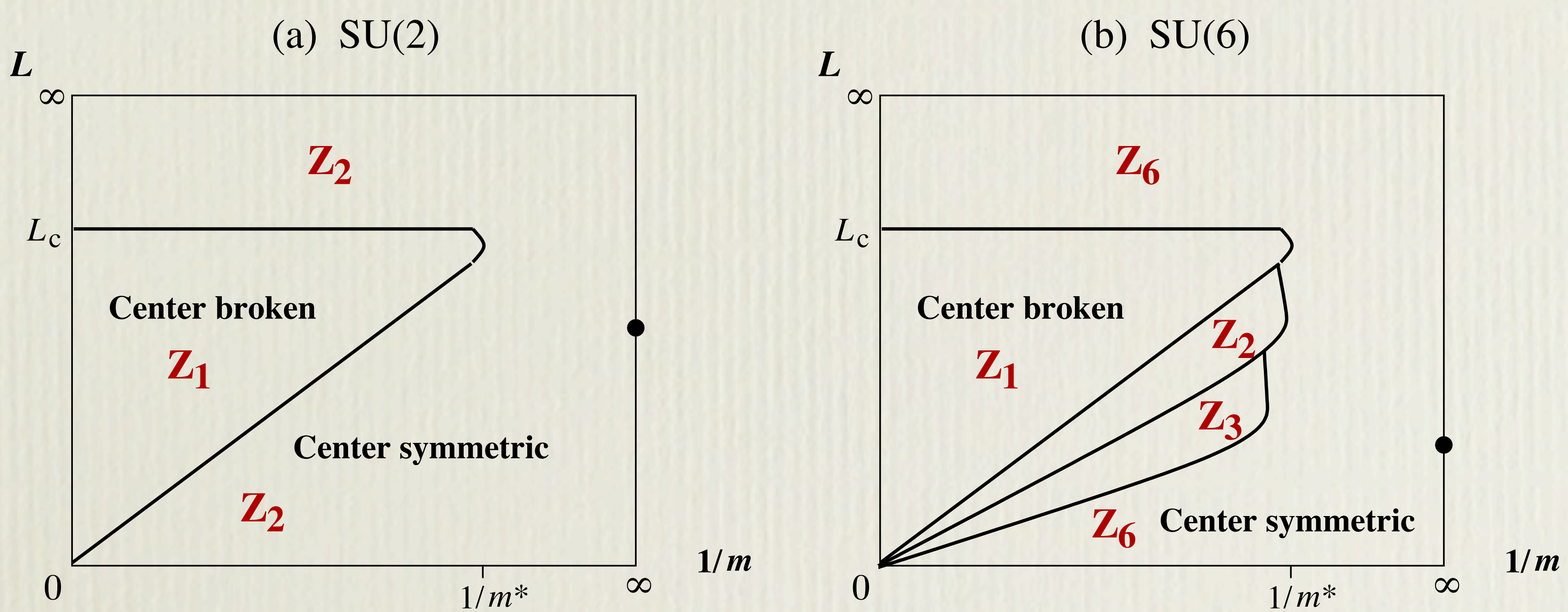
- Suppose $1/L \gg m \gg \Lambda$: negligible change to $V_{\text{1-loop}}$? No:

$$\frac{1}{n^4}(-1+N_f) \Longrightarrow \frac{1}{n^4}\left(-1+N_f\, f(nLm)\right),$$

$$f(z) = \tfrac{1}{2}\, z^2 K_2(z) \sim \begin{cases} 1\,, & z \ll 1\,; \\ e^{-z}\,, & z \gg 1. \end{cases}$$

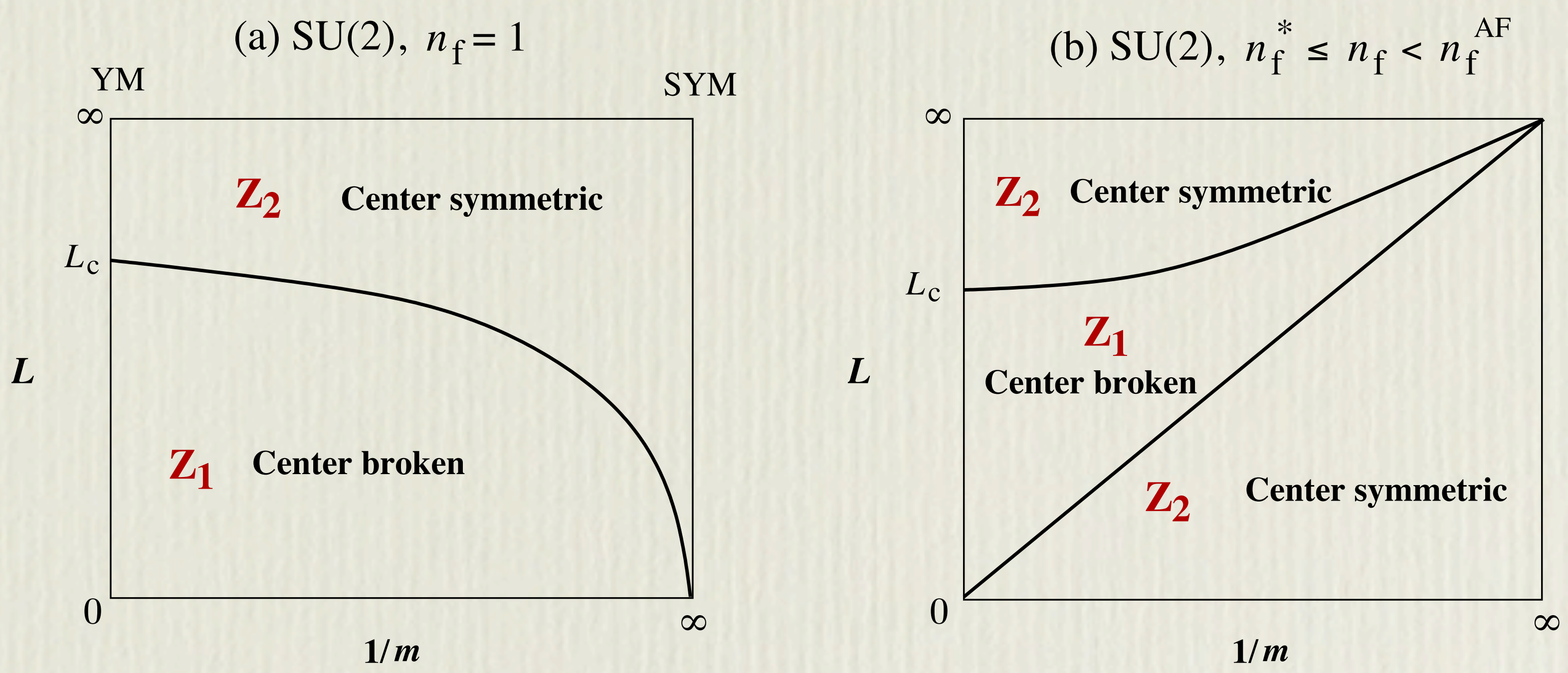
small $m \neq 0$ ➡ high winding loops become unstable

- $2 \le n_f < n_f^*$: non-uniform $m \to \infty$ and $L \to 0$ limits:



see also: Hollowood & Myers

Massive QCD(adj), $\mathbb{R}^3 \times S^1$



- $n_f = 1$: non-uniform $m \to 0$ and $L \to 0$ limits.
- $n_f^* \le n_f < n_f^{AF}$: non-uniform $m \to 0$ and $L \to \infty$ limits.
- Require: $NLm < O(1)$ for center stabilization,

  $NL\Lambda \gg 1$ for volume independence

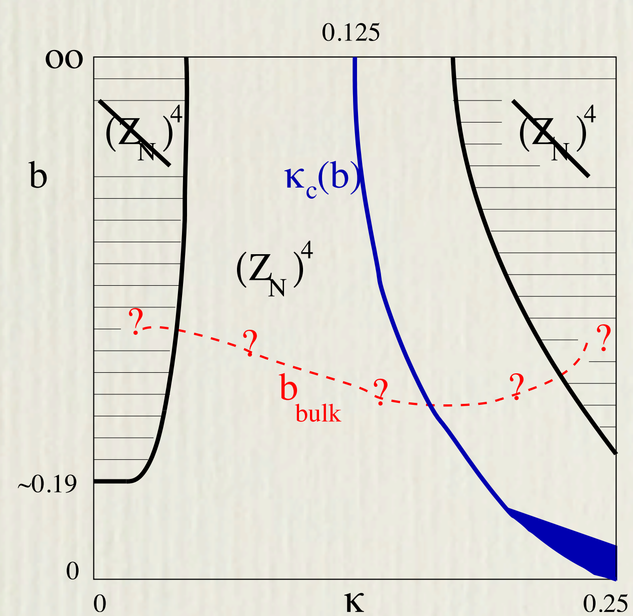
➡ m/Λ must vanish as $N \to \infty$ for volume independence with fermion induced center stabilization

Wednesday, February 1, 12

Massive QCD(adj), T^4

- Eigenvalues of commuting $\Omega_1,\ldots,\Omega_4$ ➡ N particles on dual 4-torus
  - small mass ➡ particles repel
  - large mass ➡ particles attract
- $O(1)$ eigenvalue fluctuations for all L
  - N finite: unbroken center symmetry
  - $N = \infty$: unbroken center symmetry for even when $m \gg \Lambda$!
- One site lattice theory:

Bringoltz & Sharpe: arXiv:0906.3538



Example 3: N=4 SYM on $\mathbb{R}^3 \times S^1$

- Moduli space = $\mathbb{R}^{6N}/S_N$ on $\mathbb{R}^4$,

  $= [\mathbb{R}^{6N} \times (\hat{S}^1)^N]/S_N$ on $\mathbb{R}^3 \times S^1$

  Wilson line eigenvalues

- "Usual" $\Omega = 1$ compactification: Seiberg, 1997

  spontaneously broken center symmetry

  $1/L$ = relevant scale, no volume independence

  IR physics = N=8 superconformal $3d$ SYM theory, enhanced $SO(8)_R$ symmetry

- $\mathrm{tr}\, \Omega^n = 0$ compactification:

  unbroken center symmetry

  $1/NL$ = relevant scale

  $N < \infty$: IR physics = $U(1)^{N-1}$ massless $3d$ Abelian, no superpotential

  $N = \infty$: IR physics = $4d$ non-Abelian, L independent

Concluding remarks

- Volume independence is a remarkable consequence of the large N limit in an interesting class of non-Abelian gauge theories:
  - ⋆ Necessary & sufficient: unbroken center & translation symmetry.
  - ⋆ Does not require confining phase or continuity of phases.
  - ⋆ Allows one to trade $V \to \infty$ limit for $N \to \infty$ limit.
- Compactification produces rich phase structure in QCD(adj).
- Promising practical utility for studying large N limit of Yang-Mills, QCD(adj), and real QCD $\to$ QCD($\begin{smallmatrix}\square\\\square\end{smallmatrix}$) ~ QCD(adj).
  - ⋆ Numerical work underway: Bringoltz, Sharpe; Catterall, Galvez, Ünsal; Azeyanagi, Hanada, Yacoby, ...