30 Years of Hot Quark Soup

What have we learned about high temperature QCD?

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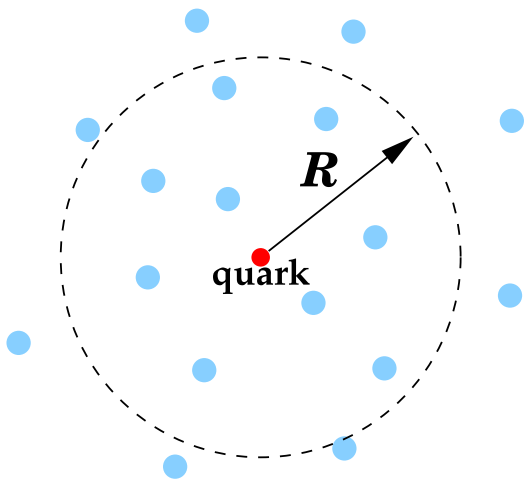
Asymptotic Freedom

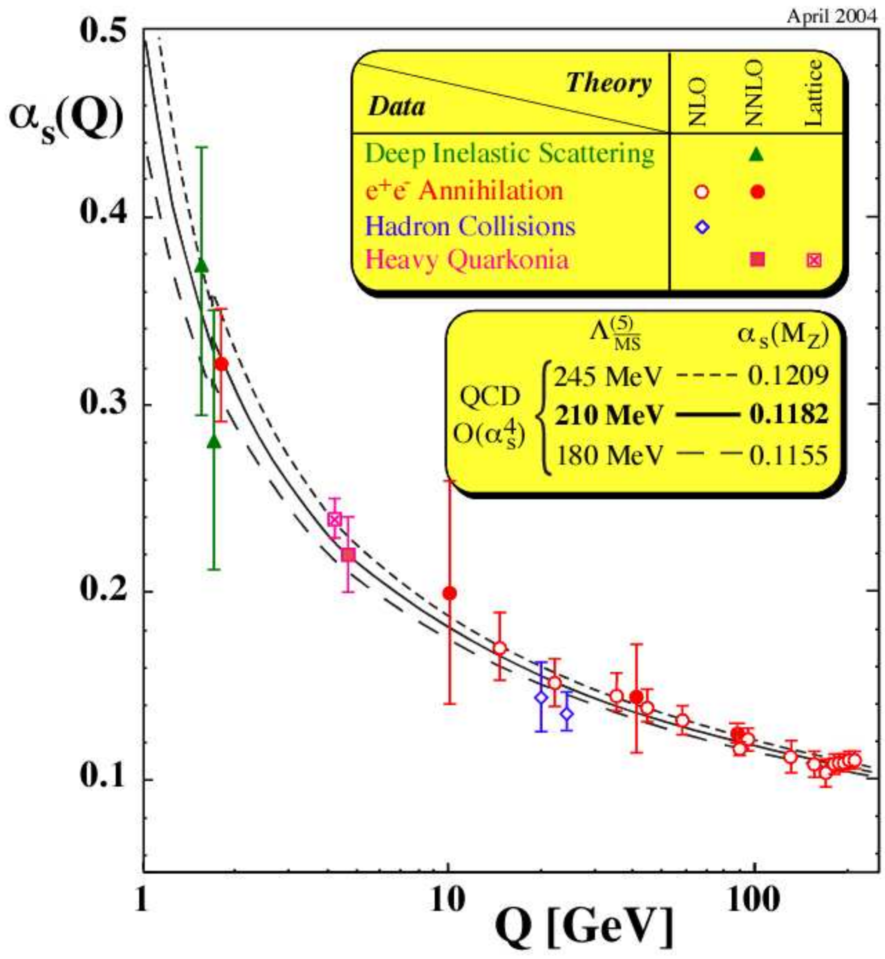
Gross, Politzer, Wilczek, 1973

Self-interacting spin-1 gluons $\Rightarrow$ anti-screening of color charge by vacuum fluctuations (unlike QED)



$$-\frac{d\alpha_s(R)}{d\ln R} \equiv \beta(\alpha_s) < 0$$

$$\alpha_s(Q) \sim \frac{2\pi}{9} \Big/ \ln\left(\frac{Q}{\Lambda_{QCD}}\right)$$



from: S. Bethke, hep-ex/0407021

Extreme QCD

High temperature and/or high baryon density is relevant for:

early universe cosmology
neutron star cores
heavy ion collisions

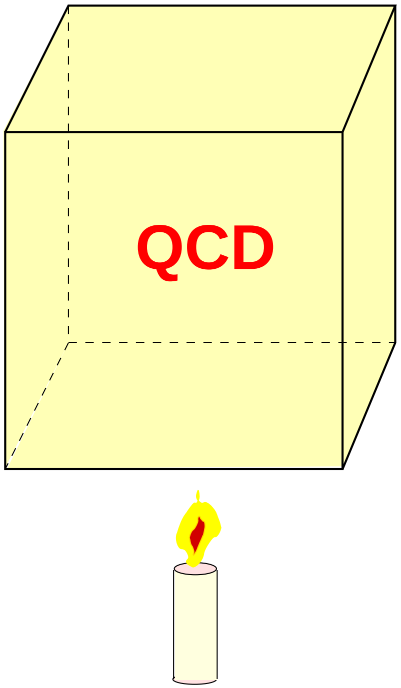
and is theoretically interesting!

Collins & Perry, 1975: *Superdense matter: Neutrons or Asymptotically Free Quarks?*

The quark model implies that superdense matter (found in neutron-star cores, exploding black holes, and the early big-bang universe) consists of quarks rather than of hadrons. Bjorken scaling implies that the quarks interact weakly. An asymptotically free gauge theory allows realistic calculations taking full account of strong interactions.

Equally true at high temperature.

Equilibrium thermodynamics: qualitative



low temperature: "confined phase"
gas of hadrons (baryons & mesons)

high temperature: "deconfined phase"
soup of quarks and gluons

Quarkless QCD:

Sharp distinction between confined (glueballs) and deconfined (unbound gluons).

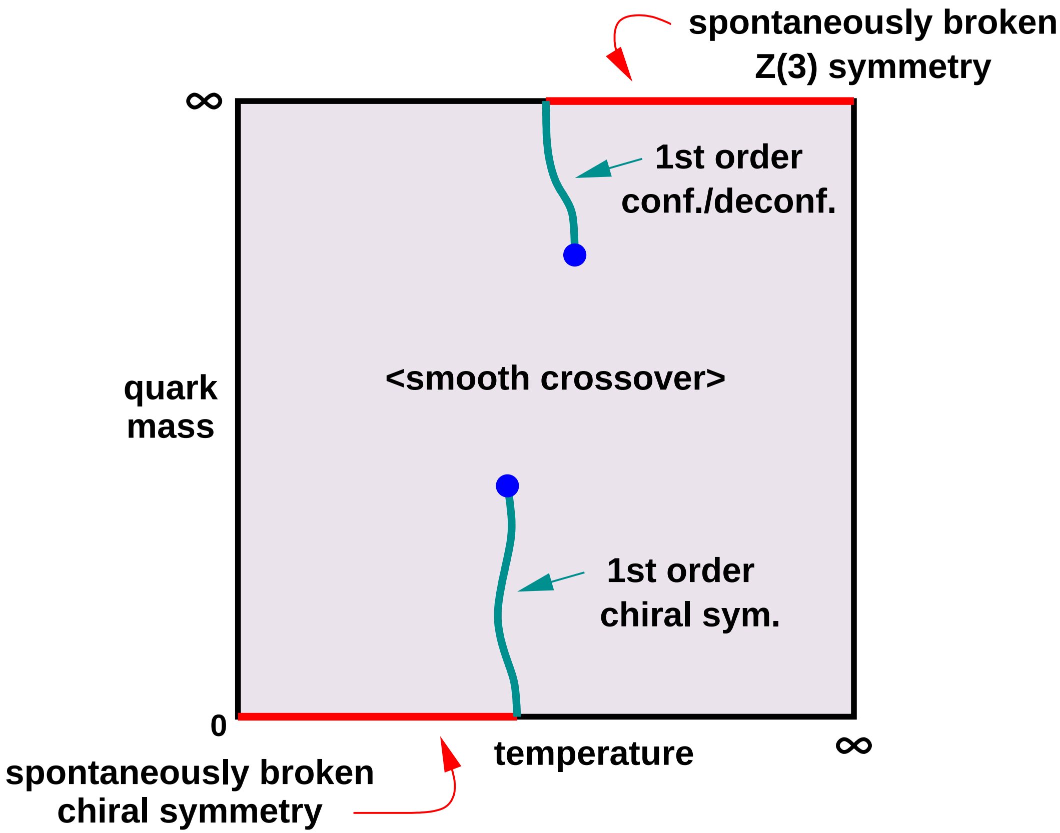
Confinement/deconfinement phase transition:

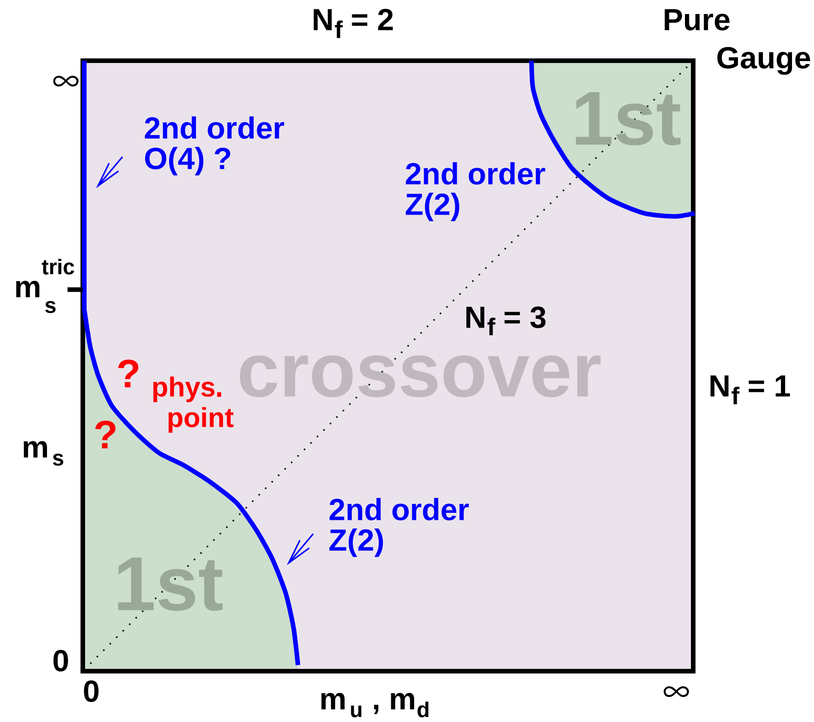
= condensation of electric flux tubes (L. Susskind, 1979)

= spontaneous $Z(3)$ symmetry breaking (N. Weiss, 1981)

= first order transition (B. Svetitsky, L. Yaffe, 1982)

Phase diagram with quarks





from: P. Petreczky, hep-lat/0409139

three flavors, equal mass

three flavors, unequal mass

Equilibrium thermodynamics: analytic

Equation of state at asymptotically high temperature
(p = pressure, $\alpha_s(T) \equiv g(T)^2/4\pi$, no quarks):

$$\frac{p}{T^4} = \frac{\pi^2}{45} \times 8$$ (Stefan-Boltzmann $\times$ 8 gluons)

$$- g(T)^2 \times \frac{1}{6}$$ (Shuryak, Chin, 1977)

$$+ g(T)^3 \times \frac{2}{3\pi}$$ (Debye screening, Kapusta, 1979)

$$+ g(T)^4 \ln g(T) \times \frac{3}{2\pi^2}$$ (Toimela, 1983)

$$+ g(T)^4 \times 0.0304$$ (Arnold, Zhai, 1994)

$$- g(T)^5 \times 0.2247$$ (Zhai, Kastening; Braaten, Nieto, 1995)

$$- g(T)^6 \ln g(T) \times 0.1426$$ (Kajantie, Laine, Rummukainen, Schröder, 2002)

$$+ g(T)^6 \times (C_{\mathrm{pert}} + C_{\mathrm{non-pert}})$$ (Schröder, Vuorinen, *in progress*)

$$+ O[g(T)^7]$$

Lesson 1:

Relevant physics is organized by scale. (K. Wilson)

High temperature $\Rightarrow$ multiple relevant scales:

$\boldsymbol{\lambda} \sim \mathbf{1}/\boldsymbol{T}$	de Broglie wavelength of thermal fluctuations making dominant contribution to pressure, weakly-coupled, perturbative series in $g(T)^2$.
$\boldsymbol{\lambda_D} \sim \mathbf{1}/(\boldsymbol{gT})$	Debye screening scale, $O(g^3)$ contribution to pressure, perturbative series in $g(\lambda_D^{-1})$.
$\boldsymbol{\xi} \sim \mathbf{1}/(\boldsymbol{g^2T})$	correlation length of non-perturbative static magnetic fluctuations, $O(\xi^{-3}) = O(g^6)$ contribution to pressure.

Can separate contributions from each scale using “effective field theory.”

(Braaten, Nieto, 1995)

Lesson 2:

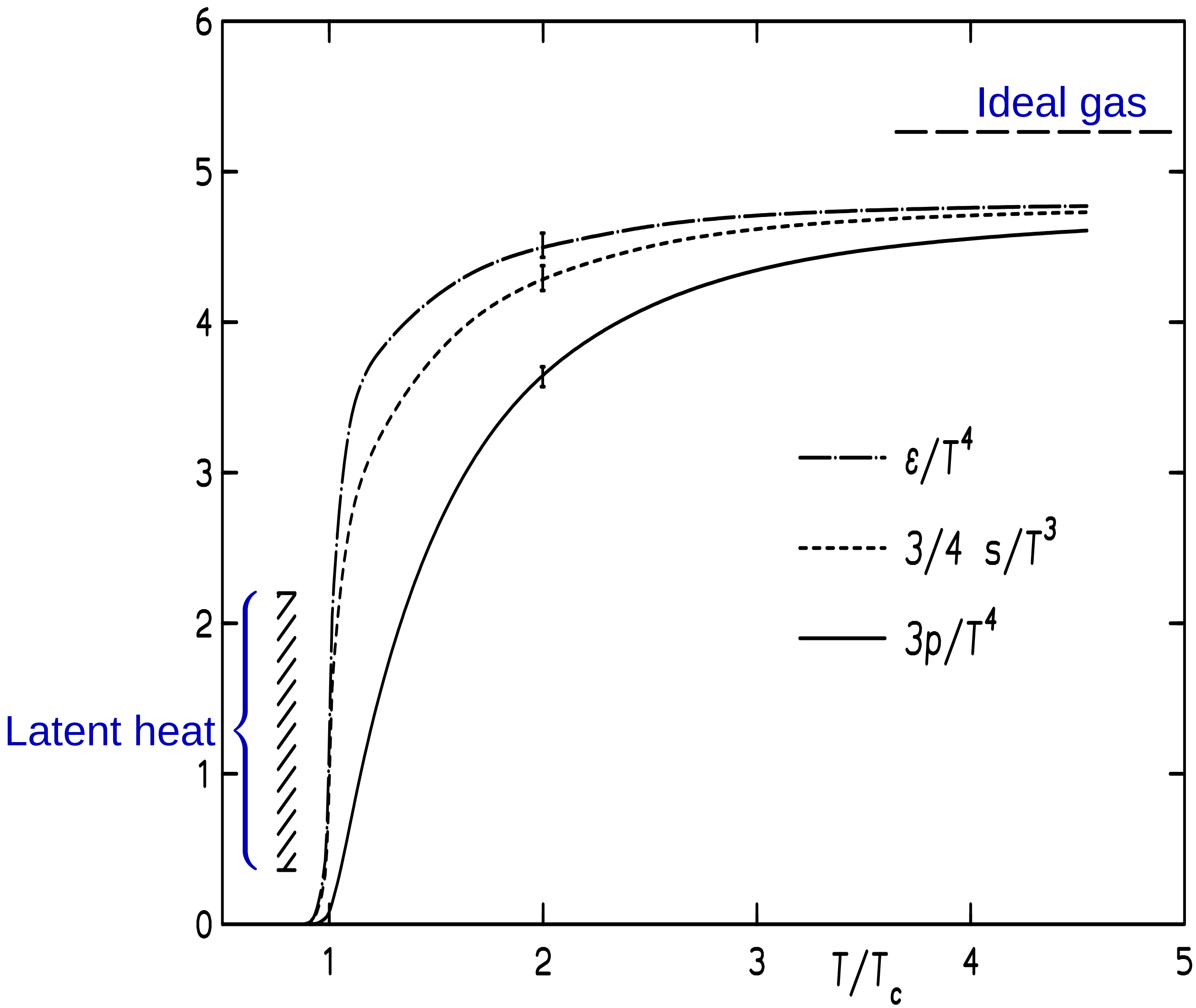
Natural expansion parameter is $g(T)$, not $\alpha_s(T)$.

Non-perturbative physics persists on large length scales, even when $g(T) \ll 1$. Due to large phase-space density,

$$n_{\mathrm{Bose}}(E) = \frac{1}{e^{E/T}-1} \sim \frac{T}{E} \sim \frac{1}{g^2},$$

for $E \sim g^2 T$.

Equilibrium thermodynamics: numerical

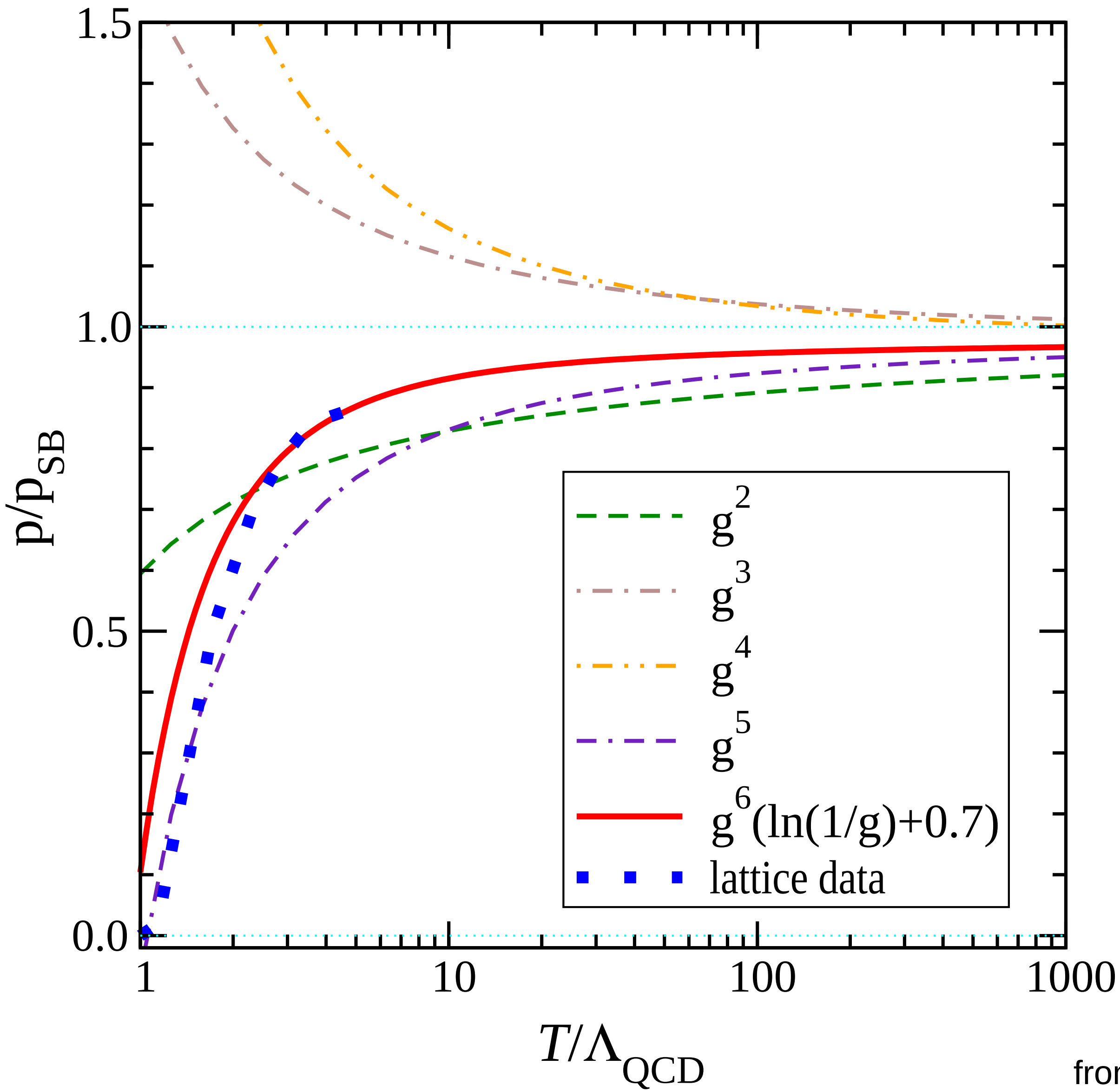


from: G. Boyd, *et.al.*, hep-lat/9602007

Monte Carlo simulation
SU(3), no quarks
Lattice size: up to 8×32^3
continuum extrapolation

$\varepsilon =$ energy density
$s =$ entropy density
$p =$ pressure

Approach to asymptopia



Numerical lattice data compared with successive orders of perturbation theory.

Unknown $O(g^6)$ constant fit to data.

from: K. Kajantie, *et.al.*, hep-ph/0211321

Lesson 3:

Utility of asymptotic high temperature expansion is strongly dependent on desired precision & order of truncation.

Gluons at 2–3 $\times T_c$ are not yet truly weakly interacting. Non-perturbative $O(g^6)$ contribution is required to match lattice data.

Equilibrium properties: current frontier

Non-zero baryon chemical potential

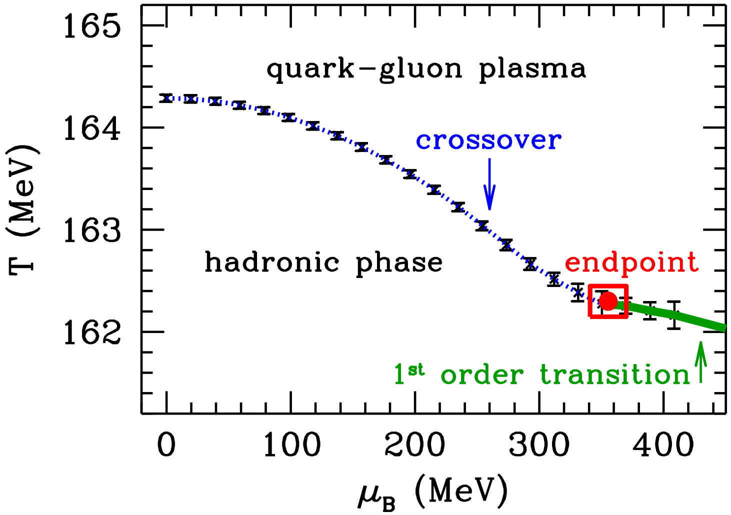
Spectral functions of mesonic excitations

Effects of topological defects (monopoles, vortices, instantons)

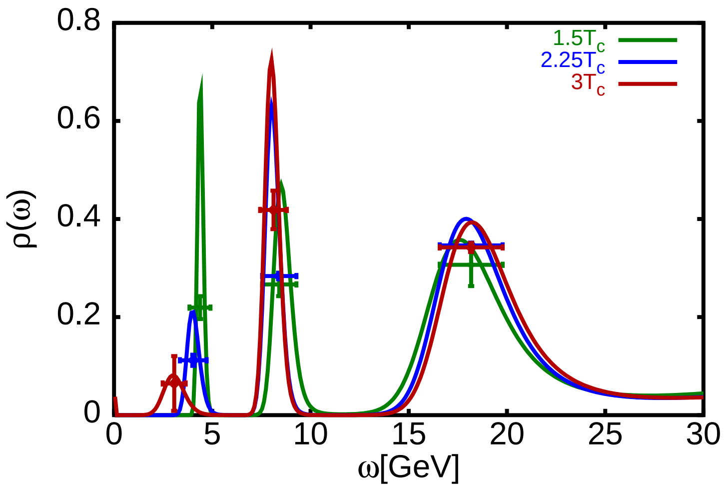
Improved simulations with dynamical fermions

Gauge/string duality

⋮



from: Z. Fodor, S.D. Katz, hep-lat/0402006



Charmonia vector spectral functions above T_c, from: S. Datta, *et.al.*, hep-lat/0312037

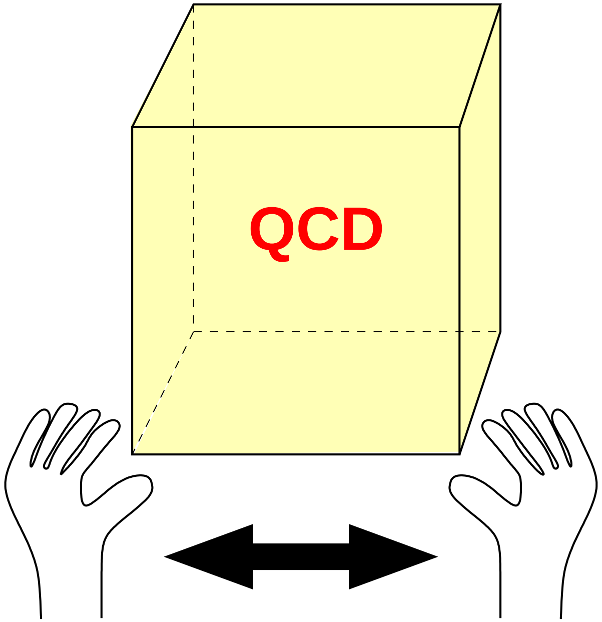
Near-equilibrium dynamics

elementary excitations?

collective excitations?

transport properties?

equilibration dynamics?



Very hot QCD $=$ “simple” condensed matter system — dynamical properties should be calculable from first principles.

Are they? Which properties?

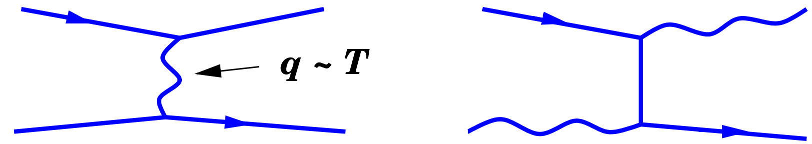
Quarks & gluons: good quasiparticles

typical energy, momentum	$\sim T$
number density	$\sim T^3$
thermal dispersion relation	$E(\mathrm{p}) \sim \sqrt{\mathrm{p}^2 + m_{\mathrm{th}}^2(T)}$
thermal mass, gluons	$m_{\mathrm{th}}(T) = g(T)\,T\,\sqrt{\frac{1}{2}+\frac{n_f}{12}}$
thermal mass, quarks	$m_{\mathrm{th}}(T) = g(T)\,T\sqrt{\frac{1}{3}}$
thermal width	$\Gamma \sim g(T)^2\,T\,\ln g(T)^{-1}$

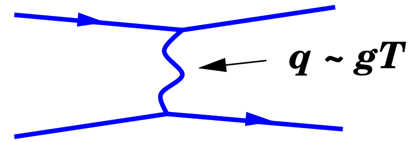
High temperature $\Rightarrow$ weak coupling, $g(T) \ll 1$
$\Rightarrow$ thermal width $\ll$ energy
$\Rightarrow$ long-lived, particle-like excitations

Relevant scattering processes

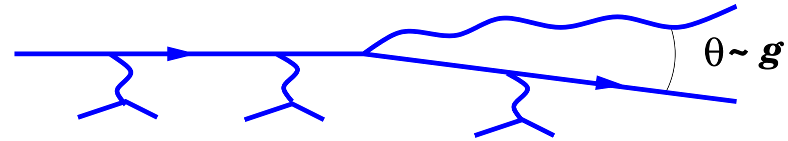
large angle



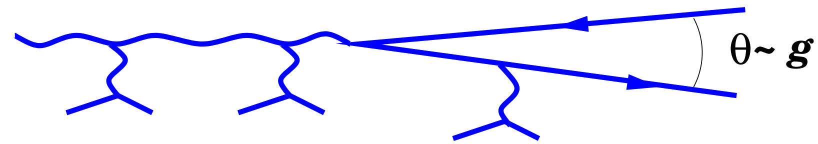
small angle



near-collinear bremsstrahlung



near-collinear pair creation



process	mean free time	change in: direction	momentum	color	species
large angle	$1/(g^4 T \ln 1/g)$	$O(1)$	$O(1)$	$O(1)$	$O(1)$
small angle	$1/(g^2 T \ln 1/g)$	$O(g)$	$O(g)$	$O(1)$	—
near-collinear	$1/(g^4 T)$	$O(g)$	$O(1)$	$O(1)$	$O(1)$

Hydrodynamic excitations

Low frequency, long wavelength dynamics of hot QCD: same as any neutral fluid.

Collective excitations $=$ hydrodynamic modes

propagating sound waves: $\boldsymbol{\omega(\mathbf{k}) \sim v_s|\mathbf{k}| - i\gamma_s\,\mathbf{k}^2}$

speed of sound ↗ ↑ sound attenuation

diffusive relaxation of fluctuations in:

(transverse) momentum density $\boldsymbol{\pi(\mathbf{k}) \sim \exp -\gamma_\eta \mathbf{k}^2 t}$

shear viscosity/enthalpy ↗

baryon (or flavor) density $\boldsymbol{n(\mathbf{k}) \sim \exp -D\mathbf{k}^2 t}$

diffusivity ↗

Transport coefficients:

Depend on equilibration dynamics

$\Rightarrow$ sensitive to quasiparticle scattering processes

$\Rightarrow$ may be calculated using appropriate effective theory for quasiparticle dynamics $=$ effective kinetic theory

$$(\partial_t + \mathrm{v} \cdot \nabla)\, f(x, \mathrm{p}) = -C[f] \qquad \text{(Boltzmann equation)}$$

Collision term $C[f]$ must correctly reproduce all relevant quasiparticle scattering processes.

Ex: Shear viscosity ($n_f = 3$):

$$\eta = \frac{\pi^4 T^3}{g(T)^4} \left[\frac{1.095}{\ln \frac{2.41}{g(T)}} + O\left(\frac{1}{\ln^3 \frac{1}{g(T)}} \right) \right] \quad \text{(Arnold, Moore, Yaffe, 2001,2003)}$$

More lessons:

Transport coefficients (at leading order in α_s) are insensitive to non-perturbative long-distance dynamics.

Near-collinear bremsstrahlung/pair-creation is relevant at leading order. Requires quantum-mechanical treatment of multiple soft scattering during emission (“LPM effect”). Produces $1 \leftrightarrow 2$ collision terms in effective kinetic theory.

Effective kinetic theory captures essential dynamics in a very efficient fashion. Direct diagrammatic evaluation is virtually impossible.

Far-from-equilibrium dynamics

Use non-Abelian Boltzmann-Vlasov kinetic theory

“hard” (high momentum) excitations
$\Rightarrow$ particles with phase space distribution $f(x, \mathbf{p})$,

“soft” (long wavelength) excitations
$\Rightarrow$ classical non-Abelian gauge field

Any anisotropic hard particle distribution
$\Rightarrow$ exponentially growing unstable gauge field modes

(S. Mrówczyński, 1993)

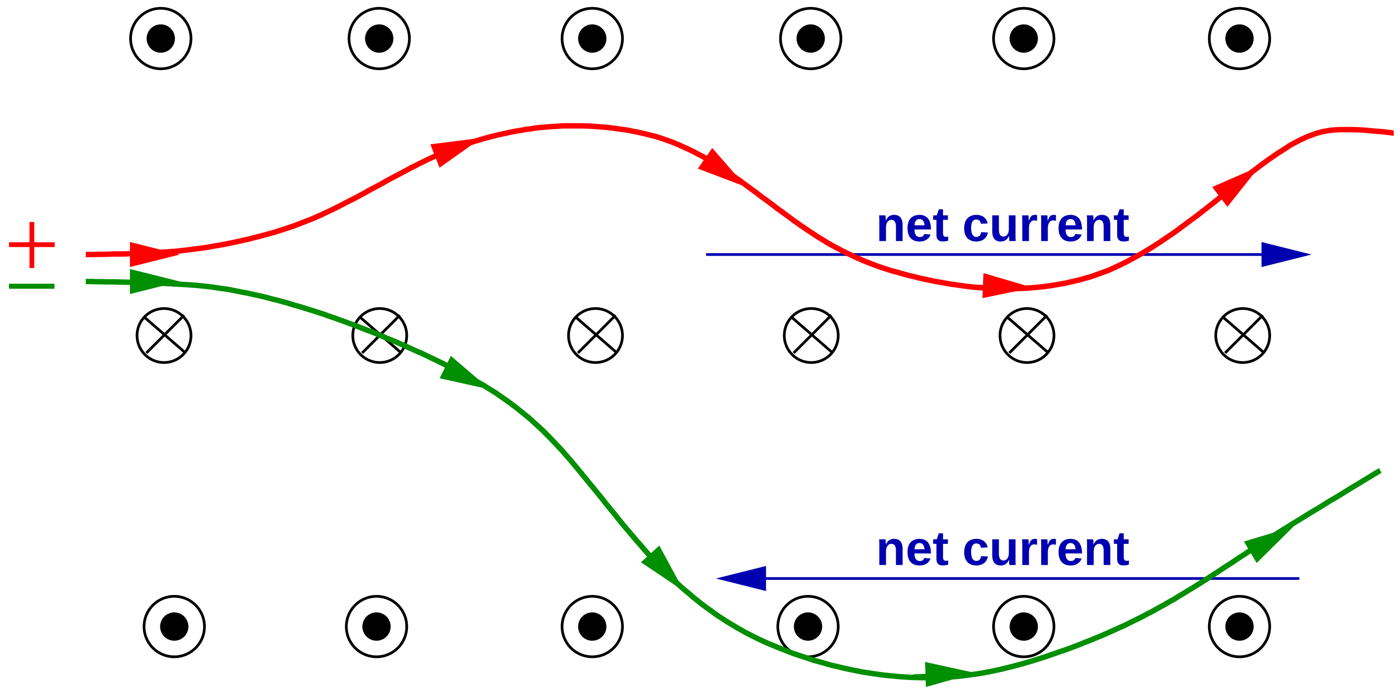
Analogous to magnetic instabilities in conventional plasmas

(E. Weibel, 1959)

But: ultrarelativistic particles, 8 gluon fields,
non-Abelian gluon self-interactions, $\alpha_{\mathrm{s}} \gg \alpha_{\mathrm{EM}}$.

Plasma instabilities: physical origin

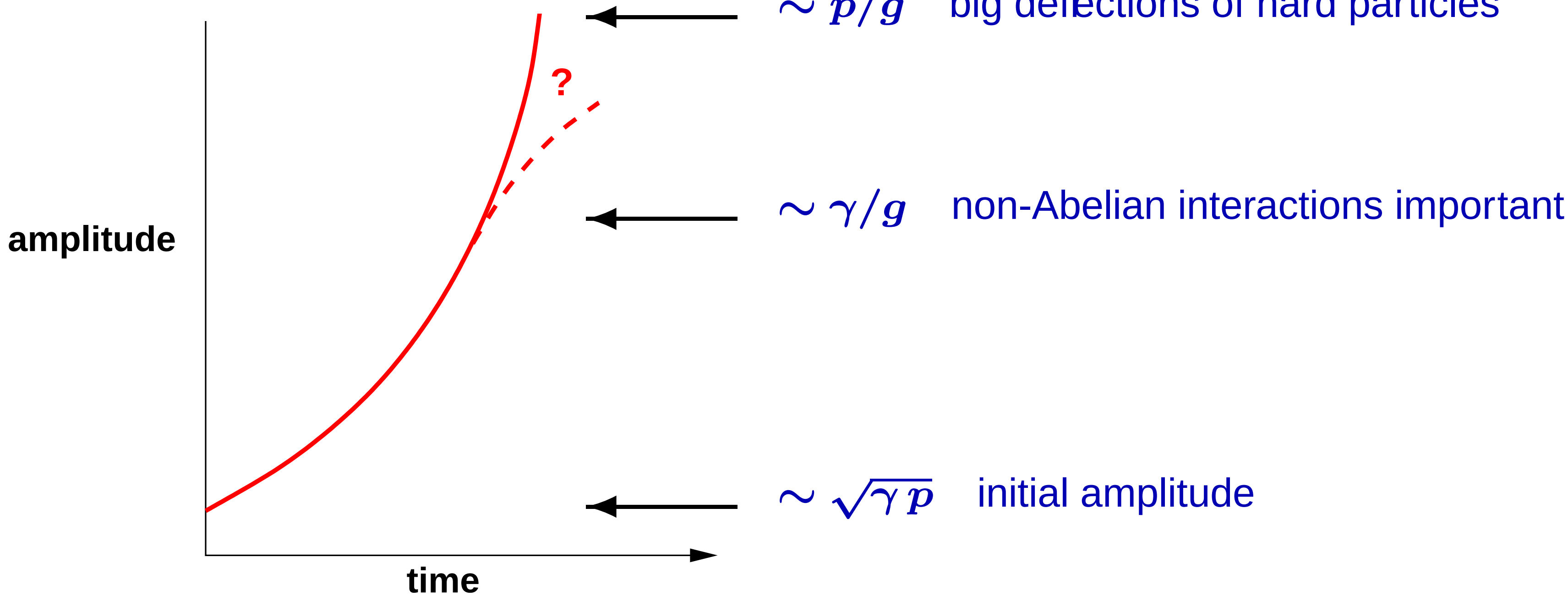
Trapping of particle trajectories by inhomogeneous magnetic field



Field generated by trapped particle currents augments trapping field $\Rightarrow$ instability.

QCD plasma instabilities

Growth rate $\gamma \sim g\sqrt{n/p}$ is faster than all perturbative scattering rates ($p \equiv$ typical momentum of hard particles).

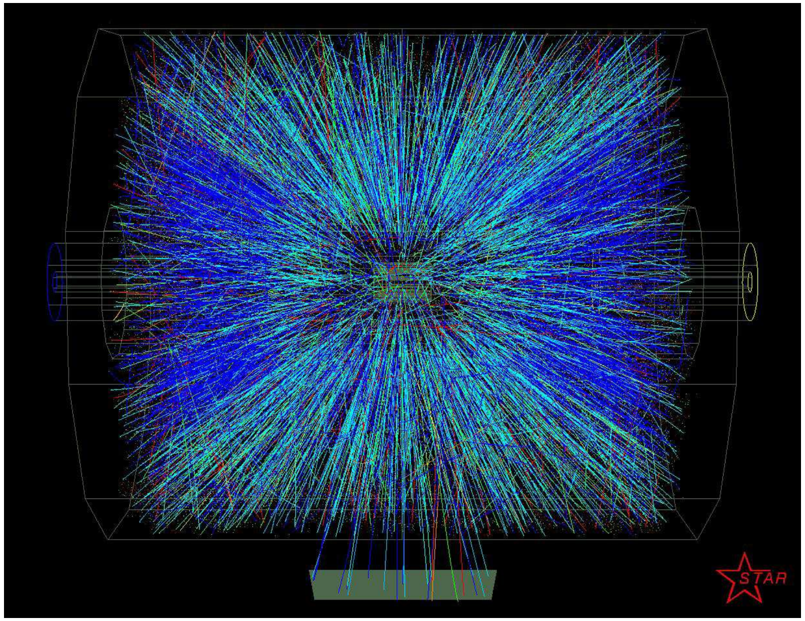


\# e-foldings for instabilities to grow big: $O(\ln 1/g)$ — provided growth continues past non-Abelian threshold.

3d numerical simulations: in progress.

RHIC thermalization

Hydrodynamic modeling of heavy ion collisions works surprisingly well, provided one assumes that quarks and gluons thermalize in ~ 0.6 fm/c.



Fast thermalization $=$ big puzzle. Hard to understand using perturbative scattering.

May be result of non-perturbative, non-Abelian plasma instabilities.

Conclusions

Dynamics of hot QCD reveals a fascinating interplay of relativistic quantum field theory, statistical mechanics, condensed matter physics, kinetic theory, hydrodynamics, and plasma physics.

We've come a long way...

Many interesting open questions remain,
especially in non-equilibrium dynamics,
even at asymptotically high temperature.

Acknowledgments: Thanks to my collaborators Peter Arnold and Guy Moore, and to many others from whom I've learned.